1

$$f(x)=x^3+x^2$$

$$f'(x)=3x^2+2x$$

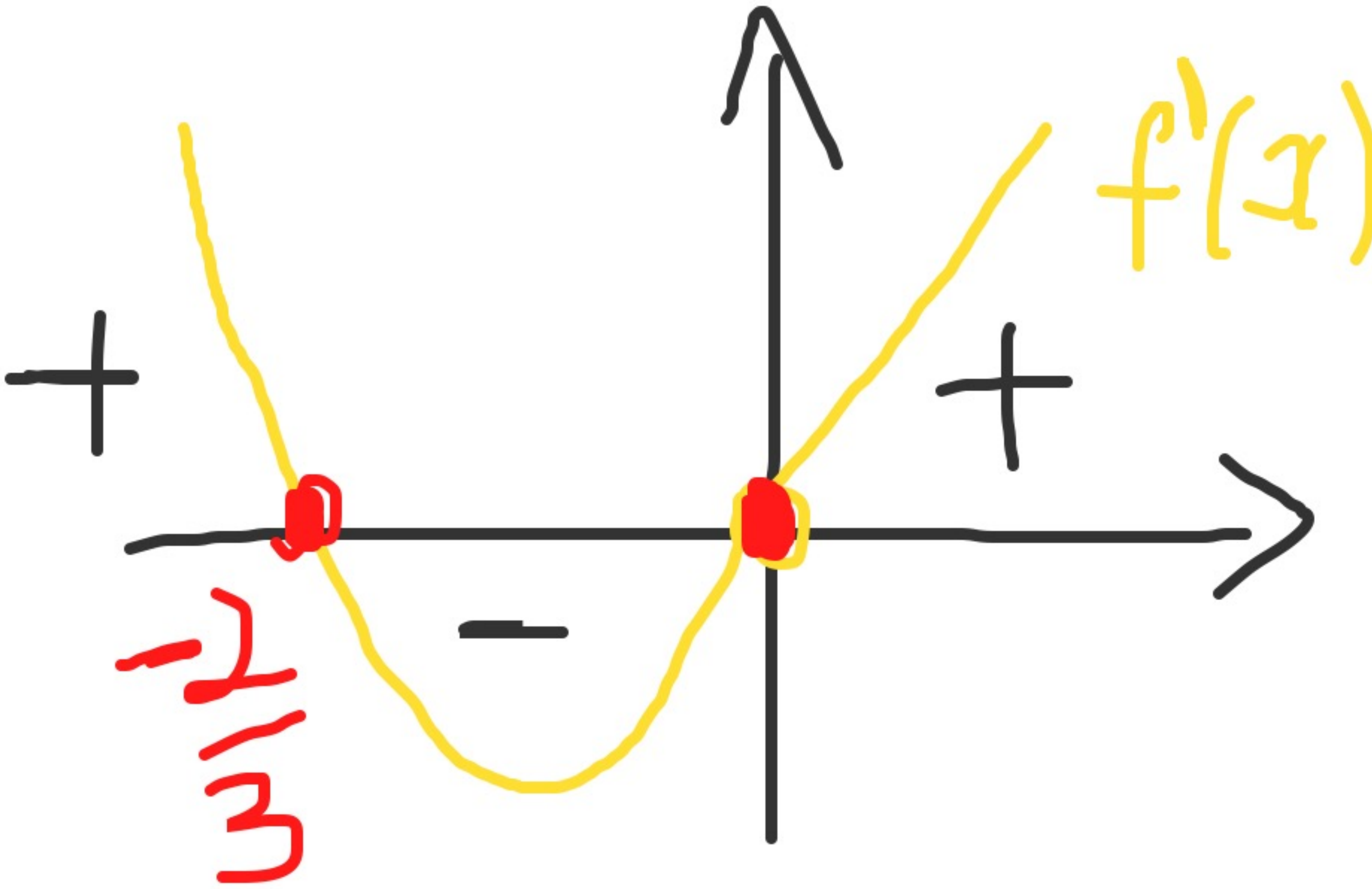
$$f''(x)=6x+2$$

- $f'(x)=0 \Leftrightarrow 3x^2+2x=0$

$\Leftrightarrow x(3x+2)=0$

$\therefore \boxed{f'(x)=0} \Leftrightarrow \underline{x=0}$

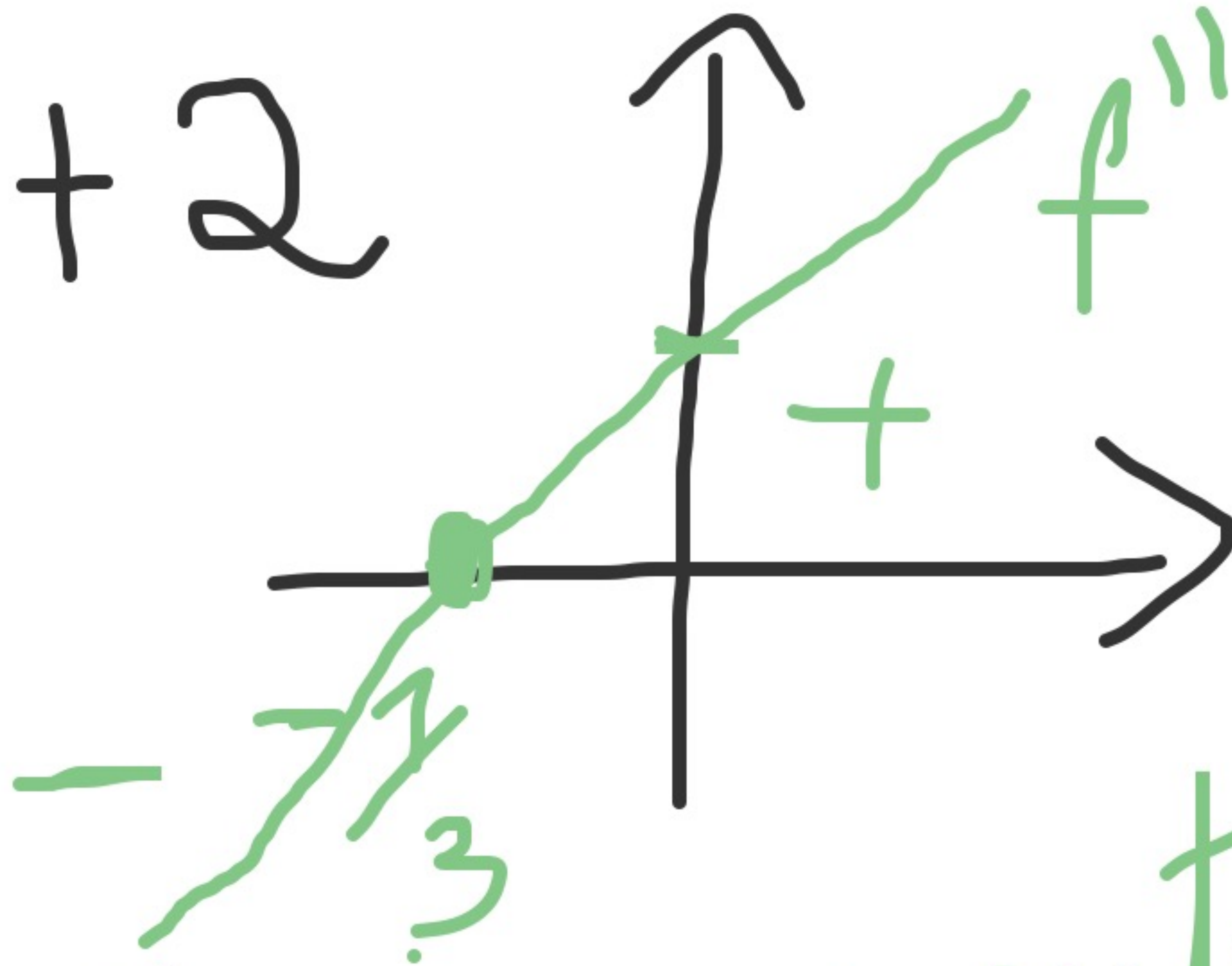
ou $x=-\frac{2}{3}$ ptos críticos



Intervalos de crescimento e decrescimento de f:

- $f'(x) > 0$ se $x \in (-\infty, -\frac{2}{3}) \cup (0, +\infty)$
- $f'(x) < 0$ se $x \in (-\frac{2}{3}, 0)$

$$f''(x) = 6x + 2$$



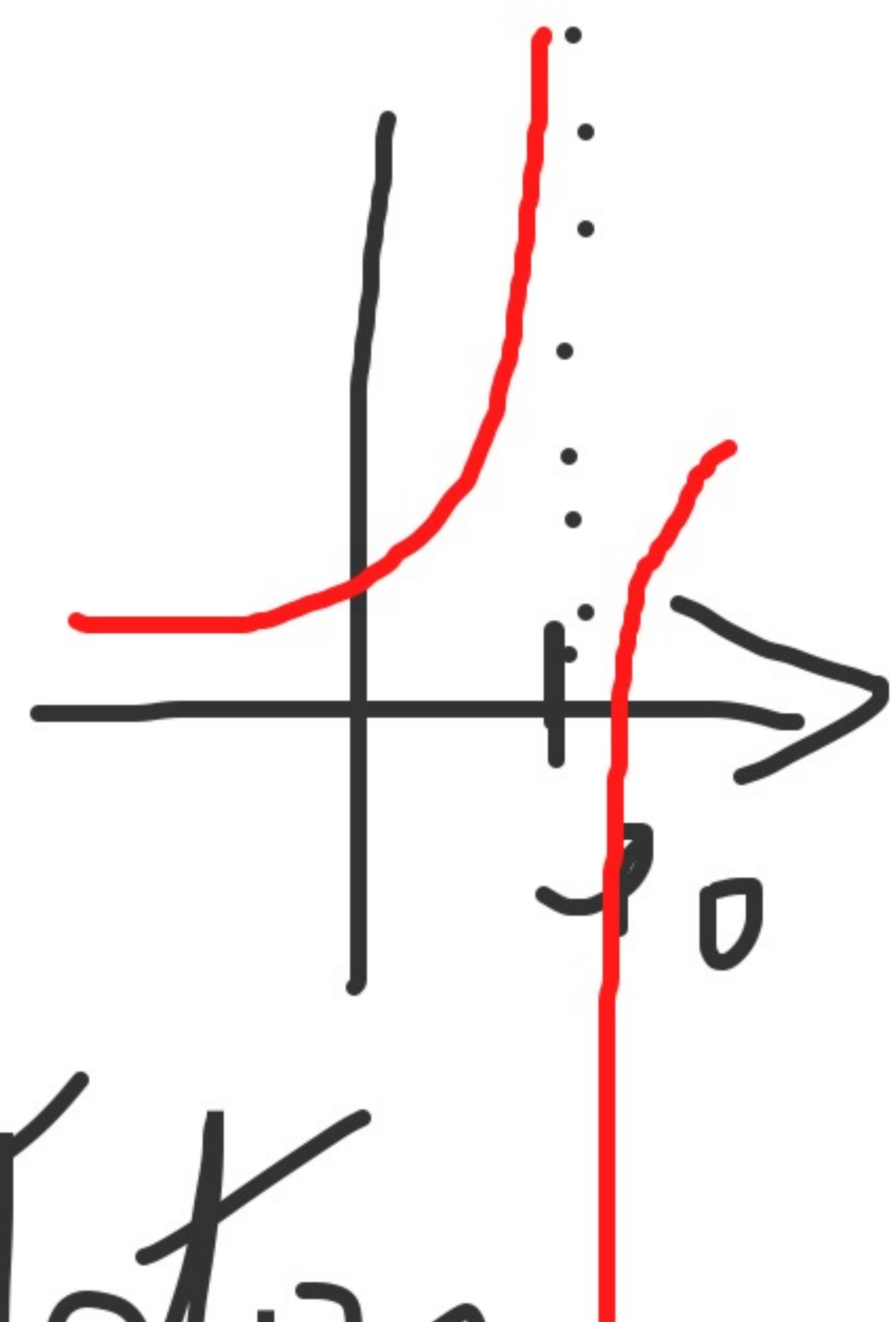
$$f'\left(-\frac{1}{3}\right) = 0$$

$\therefore$ $x = -\frac{1}{3}$ é o único ponto de inflexão de f.

$$\underline{\underline{f(x) = x^3 + x^2}}$$

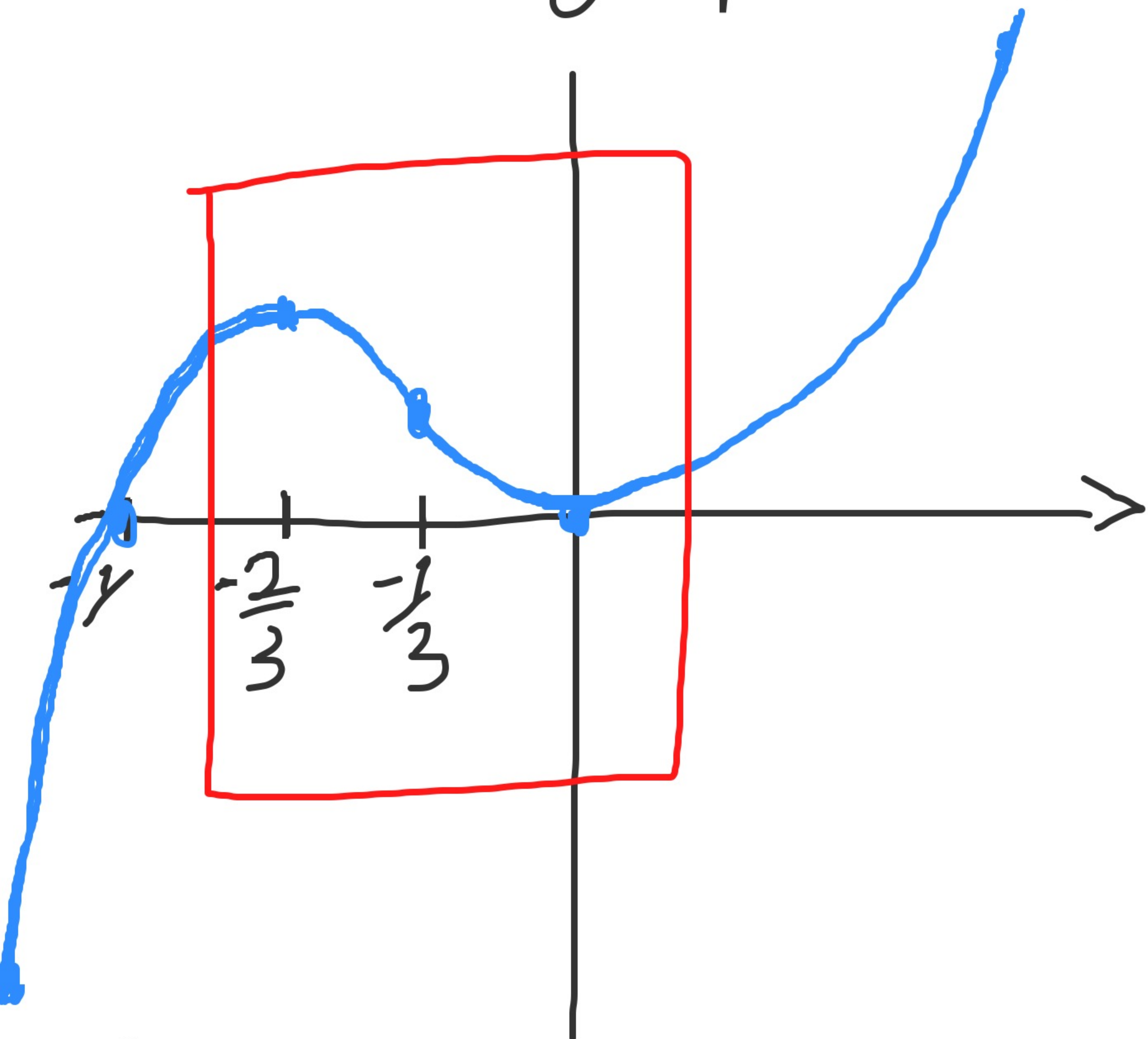
- $\lim_{x \to -\infty} f(x) = -\infty$

- $\lim_{x \to +\infty} f(x) = +\infty$



- f não tem assíntotas

· Esboço do gráfico de f



$$x^3+x^2=0 \Leftrightarrow x^2(x+1)=0$$

$$f(x) = x^4 + 2x^3 + 3x^2$$

$$\boxed{f'(x) = 4x^3 + 6x^2 + 6x}$$

$$f''(x) = 12x^2 + 12x + 6$$

- Pontos críticos e intervalos de crescimento e decrescimento

$$f'(x) = 0 \iff$$

$$\boxed{2x}\,(\underbrace{2x^2+3x+3}_{\neq 0}) = 0$$

$$\iff \boxed{x = 0} \quad \Delta = 9 - 4 \cdot 2 \cdot 3 < 0$$

Portanto $x = 0$ é o único ponto crítico.

Como $f'(x) = 2x \cdot p(x)$ com $p(x) > 0, \forall x \in \mathbb{R}$, temos que

- $\boxed{x < 0 \Rightarrow f'(x) < 0}$

- $\boxed{x > 0 \Rightarrow f'(x) > 0}$

- Pontos de inflexão e comportamento do.

derivada segunda:

$$f''(x) = 12x^2 + 12x + 6$$

$\cdot\ f''(x) = 0 \iff$

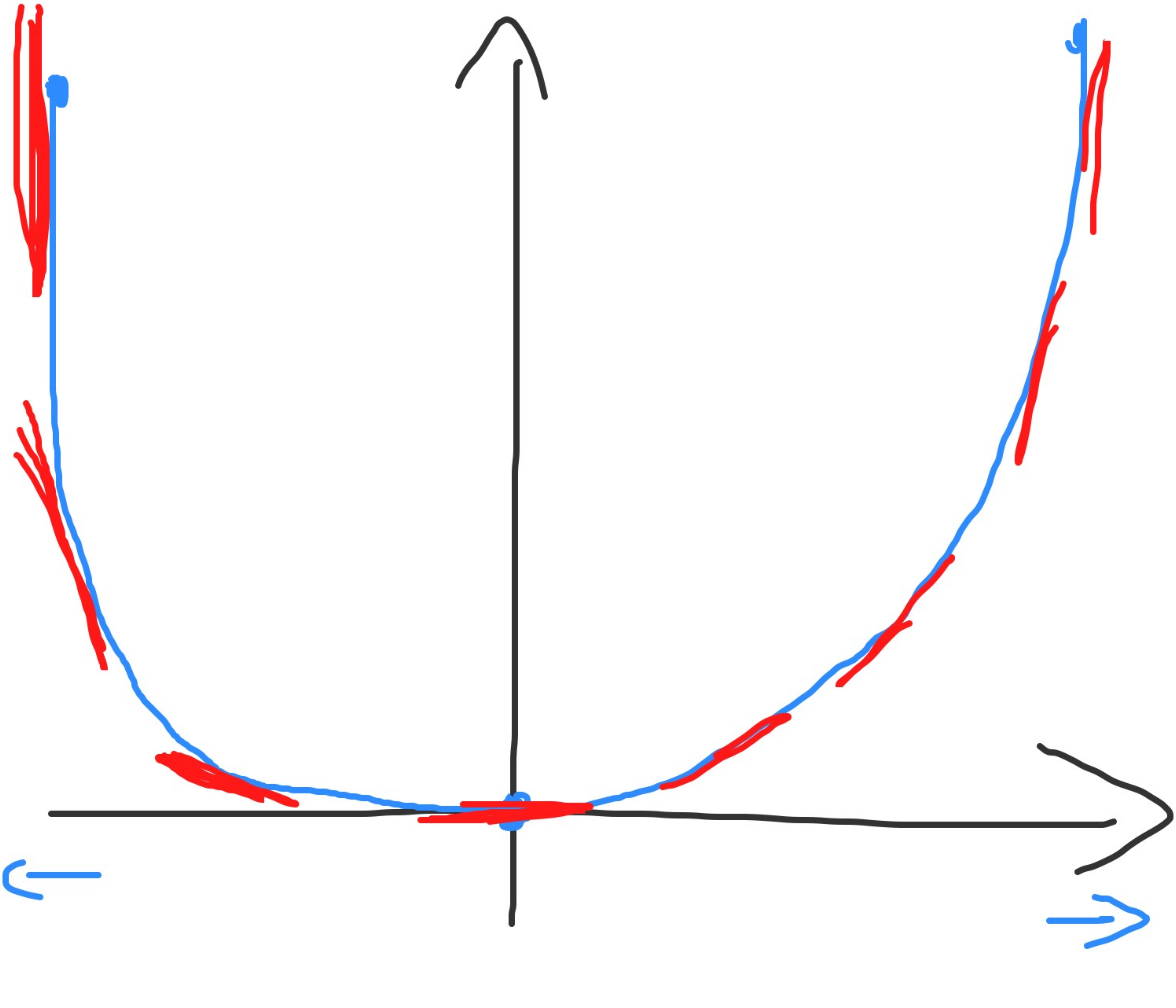
$$\underline{6}\,(\underbrace{2x^2 + 2x + 1}_{\Delta = 4 - 4.2.1 < 0}) = 0$$

>0

$$\Rightarrow f''(x) > 0, \forall x \in \mathbb{R}$$

$\therefore f'$ é crescente

$$\cdot \lim_{x \to \pm\infty} f(x) = +\infty$$



$$\boxed{f(x) = \frac{1}{(x-2)^2}}$$

$x - 2 \to 0$ ssi $x \to 2$

$$f'(x) = \frac{0 \cdot (x-2)^2 - 2(x-2)}{(x-2)^4}$$

$$= \frac{-2(x-2)}{(x-2)^4}$$

$$= \frac{-2}{(x-2)^3} \neq 0$$

- $(x-2)^3 > 0$ si $x > 2$
- $(x-2)^3 < 0$ si $x < 2$

- $f'(x) > 0$ se $x < +2$

- $f'(x) < 0$ se $x > 2$

- $\lim\limits_{x \to \pm\infty} f'(x) = 0$

- Assíntotas de f:

$$\lim_{x \to \pm\infty} f(x) = 0$$

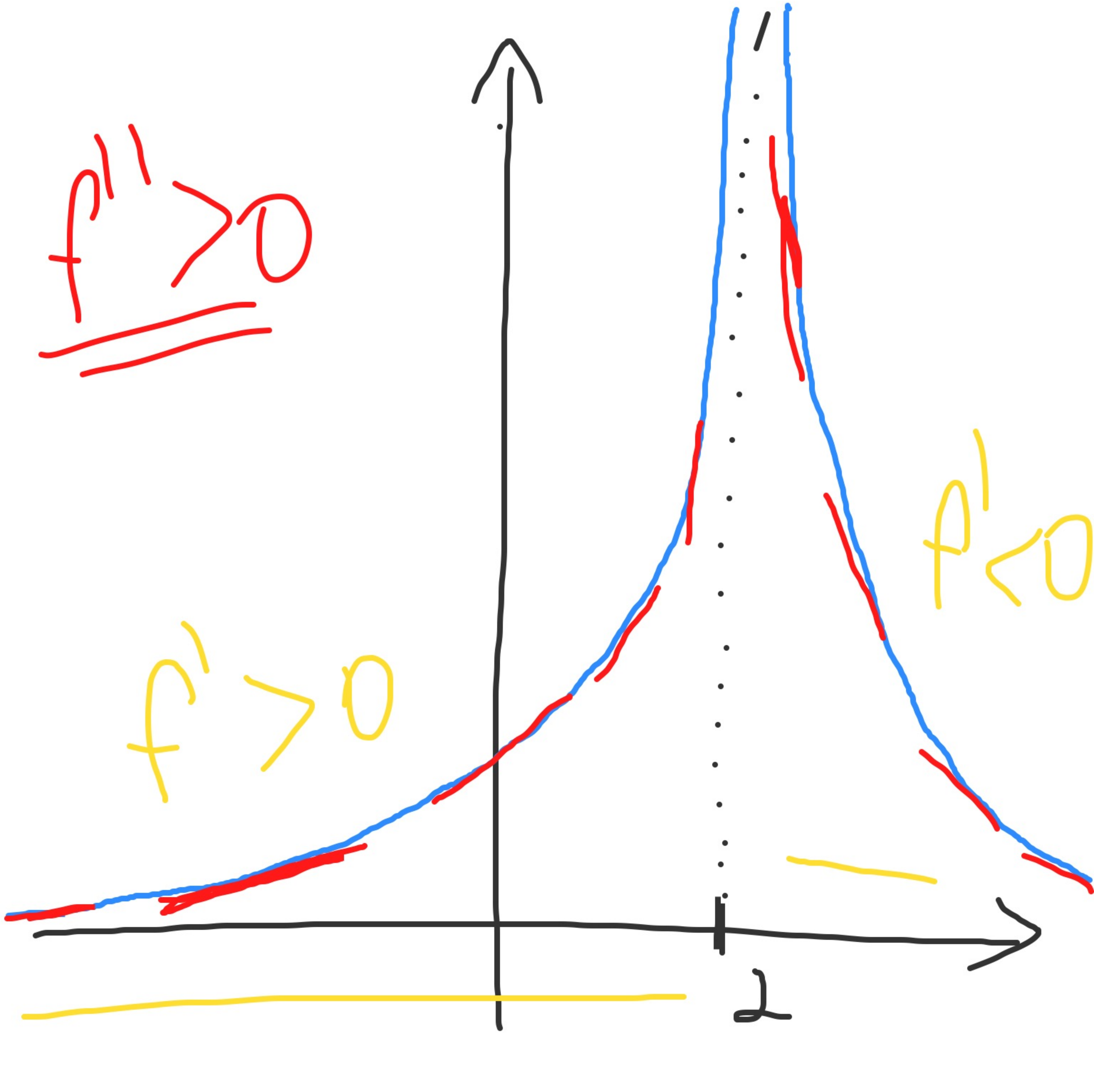
- $\lim\limits_{x \to 2} f(x) = +\infty$

$$f''(x) = \frac{3(x-2)^2}{(x-2)^6} = \frac{3}{(x-2)^4}$$

$$> 0$$

$\Rightarrow f'$ só aumenta!

* f tem assíntota horizontal em $y=0$ e assíntota vertical em $x=2$



f''>0
f'>0
f'<0
1