Regra da cadeia

- função composta:

$f: I \subseteq \mathbb{R} \to \mathbb{R}$,

$g: J \subseteq \mathbb{R} \to \mathbb{R}$

suponha que $\boxed{g(J) \subseteq I}$, a função composta de f e g é $f \circ g$ definida

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por

$$(f \circ g)(x) = f(g(\underline{x}))$$

$\forall \underline{x \in J}$

Exemplo: $f(x) = x^5$

$g(x) = x+1$.

$f : \underline{\mathbb{R}} \to \mathbb{R}$, $g : \underline{\mathbb{R}} \to \mathbb{R}$

$(f \circ g)(x) = g(x)^5 =$

$$= (x+1)^5$$

(2) $f, g : \mathbb{R} \to \mathbb{R}$

$$f(x) = x^2, \quad g(x) = x^3+5$$

$$(f \circ g)(x) = g(x)^2 = (x^3+5)^2$$

2º 1º

$$\neq$$

$$(g \circ f)(x) = f(x)^3 + 5$$

$$= (x^2)^3 + 5 = \boxed{x^6 + 5}$$

$$(x^3+5)^2 = x^6 + 10x^3 + 25$$

3 $f : [0, +\infty) \longrightarrow \mathbb{R}$

$g : \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = \sqrt{x}$, $g(x) = 6x - 7$

Rascunho: p/ compor $f \circ g$:

$(f \circ g)(x) = f(g(x))$

$= \sqrt{g(x)}$

só tem sentido se

$g(x) \geq 0 \iff 6x - 7 \geq 0$

$\iff x \geq 7/6$

Podemos compor
f com g restrita ao
intervalo $\left[\frac{7}{6}, +\infty\right)$

$f: [0, +\infty) \longrightarrow \mathbb{R}$

$g: [7/6, +\infty) \longrightarrow [0, +\infty)$

$(f \circ g)(x) = \sqrt{6x - 7}$

4 $f: (0, +\infty) \longrightarrow \mathbb{R}$

$f(x) = \frac{1}{\sqrt{x}}$

$g: \mathbb{R} \to (0, +\infty)$, $g(x) = e^x$ 6

$$(f \circ g)(x) = \frac{1}{\sqrt{e^x}}$$

Lembrete: $f: I \subseteq \mathbb{R} \to \mathbb{R}$

$\Rightarrow f': I \subseteq \mathbb{R} \to \mathbb{R}$

Teorema (Regra da Cadeia)

$$(f \circ g)'(x) = f'(g(x))\, g'(x)$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

1\] $y(x) = (x+1)^5$

y è composta di f e g,

$f(x) = x^5$ e $g(x) = x+1$

$\boxed{f'(x) = 5x^4}$ e $g'(x) = 1$

$$y'(x) = (f \circ g)'(x)$$

$$= f'(g(x)) \cdot g'(x)$$

$$= 5g(x)^4 \cdot 1 = 5(x+1)^4$$

2) $y(x) = (x^3+5)^2$

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$y(x) = (f \circ g)(x)$, onde

$f(x) = x^2$ e $g(x) = x^3+5$

$\Rightarrow f'(x) = 2x$ e $g'(x) = 3x^2$

$$y'(x) = f'(g(x)) \cdot g'(x)$$
$$= 2g(x) \cdot g'(x)$$
$$= 2(x^3+5) \cdot 3x^2$$
$$= 6(x^3+5) \cdot x^2$$

3| $y(x) = \left(\frac{x-1}{x}\right)^4 = \frac{(x-1)^4}{x^4}$ 3

$f(x) = x^4$

$g(x) = \frac{x-1}{x}$

$\Rightarrow y(x) = (f \circ g)(x)$

$f'(x) = 4x^3$

$g'(x) = \frac{x - 1\cdot(x-1)}{x^2}$

$= \frac{x - x + 1}{x^2}$

$= \frac{1}{x^2}$

$$y'(x) = f'(g(x)) \cdot g'(x)$$

$$= 4g(x)^3 \cdot g'(x)$$

$$= 4\left(\frac{x-1}{x}\right)^3 \cdot \frac{1}{x^2} \; //$$

5. $$y(x) = \left(\frac{4x+1}{x+2}\right)^3 = \frac{(4x+1)^3}{(x+2)^3}$$

$$y(x) = (f \circ g)(x)$$

$f(x) = x^3$ & $g(x) = 4x+1/x+2$

$$f'(x) = 3x^2,$$

$$g(x) = \frac{4x+1}{x+2}$$

$$\Rightarrow g'(x) = \frac{4(x+2) - 1\cdot(4x+1)}{(x+2)^2}$$

$$= \frac{4x+2-4x-1}{(x+2)^2} = \frac{1}{(x+2)^2}$$

$$y'(x) = f'(g(x))\cdot g'(x)$$

$$= 3g(x)^2\cdot g'(x) = 3\left(\frac{4x+1}{x+2}\right)^2 \frac{1}{(x+2)^2}$$

$\boxed{5}$ $y(x) = (4x^5 - 7x^2)^{30}$

$y'(x) = 30(4x^5 - 7x^2)^{29} \times$

$(20x^4 - 14x)$