ORBITAL EPHEMERIDES FOR VISUAL BINARY STARS

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Abstract

A new method to predict ephemerides for binary stars is presented. Instead of solving the conventional Kepler's equation for mean anomaly, is solved a transcendental equation based on x/a relation. This equation present a well determined interval of variation and is solved numerically by Bisection Method.

1 Introduction

Many years ago I started binary star observation and studies with ephemerides calculation in mind. By that early time I was in close correspondence with a friend, Eng. Roberto Frangetto, who patiently introduced me to all this fascinating area of Astronomy. At that time we wrote a couple of articles which can be found in [2].

The method described herein was developed in 1980 and since then some improvements have been made. The basic idea, however, is due to Eng. Roberto Frangetto who regretfully passed away some years ago.

The main characteristic of our method is to avoid Kepler's equation for mean anomaly. Kepler himself realized how difficult it was to solve this equation, and Small [5] tells us as follow:" *but, with respect to the direct solution of the problem from the mean anomaly given to find the true anomaly (Kepler) tells us that he found it impracticable, and that he did not believe there was any geometrical or rigorous method of attaining it*". Everyone knows that Kepler's equation is time consuming when solved by an ordinary numerical iterative process.

In the present method some difficulties are overcome by the use of a transcendental equation based on the relation x/a, that is derived from geometrical considerations and it has a determined interval of variation and is less complicated to be solved. This unusual relation applied to orbit ephemerides calculation is solved very easily by Bisection's Method [3], which locates a zero of a continuous function in a given interval $[a, b]$. Such numerical algorithm has been shown to be very robust and accurate in this specific application.

The intent of this paper is to show all important parts that concern to the derivation of each equation and gives a procedure of solution for a numerical algorithm. In section 2 is presented some aspects of the geometrical relations that concern to a typical visual binary system. In section 3 the orbital elements are introduced and discussed. In section 4 the area relations involving the secondary position in each quadrant is formulated. In section 5 Kepler's second law is introduced as a basis to obtain a full relation involving the secondary position at any ephemerides time. In section 6 a logical analysis is carried out to bring in the clear way of how to define for any ephemerides time, in what quadrant the secondary is. In section 7 a numerical procedure to solve the transcendental equation applying the Bisection Method is presented, and finally in section 8 examples of application are given for some binary star systems in order to show the accuracy of the present method when compared with other results.

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2 Geometrical Relations

Figure(1) shows the appearance of a true and projected binary star orbit in geometrical terms. In this figure A is the primary and is located at the ellipse focus. B the companion or secondary and it orbits A. The line of nodes is the observers meridian through A. The direction given by A–North defines the North direction. Define *position angle* θ as the angle measured Eastwards which gives the position for the line joining A to B. The distance between A and B is defined by the length ρ that is called *separation angle*. If the elements of the binary orbit are known, is then possible to calculate the *position angle* and *separation*. Hence is possible to predict the apparent orbit of any binary star systems.

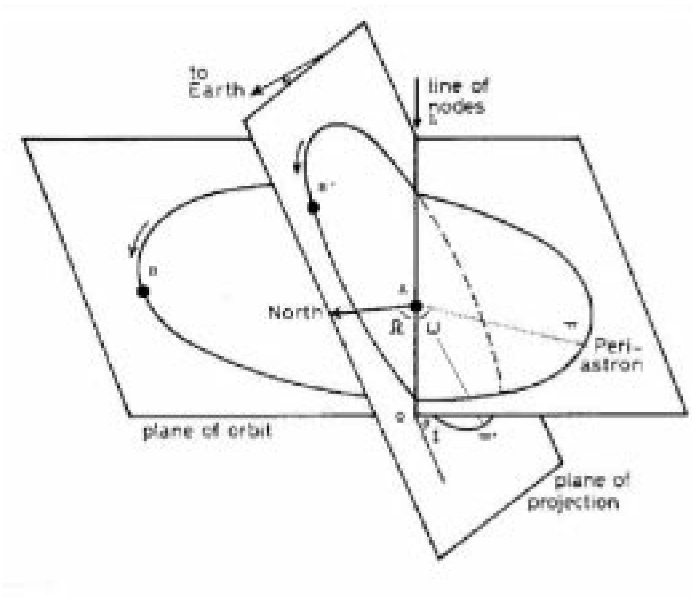

Figure 1: Binary Star True and Projected orbit

Now let consider Figure(2), where the primary star A is located at the focus and its secondary B' is left free to describe an orbit which is shown by the ellipse.

- $A \Rightarrow$ primary.
- $B \Rightarrow$ secondary.
- $B' \Rightarrow$ projection of secondary in the orbital plane.
- $\Pi \Rightarrow$ periastron.
- $\Pi' \Rightarrow$ projection of periastron in the orbital plane.
- North $\Rightarrow$ the place of North direction as in Figure(1).
- $Q \Rightarrow$ projection of ascending node.
- $L \Rightarrow$ projection of descending node
- $LQ \Rightarrow$ line of nodes
- $i \Rightarrow$ inclination of the orbit to the plane of the Sky
- $\theta \Rightarrow$ position angle for the secondary.

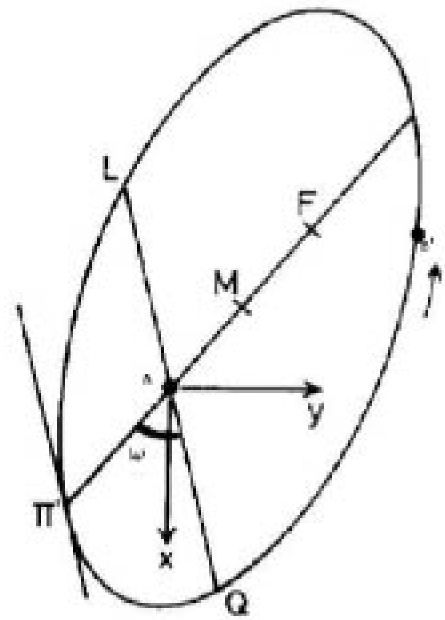

Figure 2: Apparent Orbit Elements

- $\Omega \Rightarrow$ position angle of the ascending node.
- $\omega \Rightarrow$ longitude of the periastron.

Is need to define the longitudes which are reckoned from the ascending node Q, and the true anomaly ϑ which is the angle between the secondary B' and the periastron $\Pi^{'}$, see Figure(2).

3 Orbital Elemets

The orbit of a visual pair can be described if the seven so called *orbital elements* are known. This orbit is related with the relative motion of the secondary B' to the primary star A. The *apparent orbit*, or observed orbit is in general not identical to the *true orbit* as shown in the Figure(2). This happens because, the true orbit is projected into the celestial sphere. As an example, the major axis of the true orbit can be defined by the maximum and minimum true distance between the components (termed apoastron and periastron, respectively). In general they do not project onto the axis of the apparent orbit due to the angle which the orbit is inclined against the plane of sky, angle i [1].

A set of four elements that specify the true orbit and motion is described as follows:

1. $P \Rightarrow$ period of revolution in mean solar years.
2. $T \Rightarrow$ time of periastron passage, in years and fractions thereof.
3. $a \Rightarrow$ major semi axis, expressed in seconds of arc.
4. $e \Rightarrow$ numerical eccentricity of the orbit.

The remaining three elements determine the projection of the true orbit onto the apparent orbit as shown in Figure(2). These orbital elements which are all angles, depend on the orbit orientation relative to the observer. The following definitions specify how these angles elements are defined.

$i \Rightarrow$ inclination of the orbit plane; its value lies between 0° and 180°, direct motion of the companion (position angle increasing) is indicated by $0° < i \leq 90°$, retrograde motion (position angles decreasing) by $90° < i \leq 180°$, and projected entirely onto the line of nodes if $i = 90°$.

$\Omega \Rightarrow$ position of the nodal point which lies between 0° and 180°, it is defined by the line of intersection between the projected orbit plane into the celestial sphere and the plane of sky. There are two nodes which differ by 180°. The node where the orbital motion is directed away from the observer is called the ascending node. The other one in which the orbital motion is directed towards the observer is called the descending node. Ω is measured with respect to the North pole at a specified epoch; therefore, it is exposed to precession and will change slowly with time.

$\omega \Rightarrow$ angle in the plane of the orbit between the line of nodes and the major axis, measured from the nodal point Ω to the point of periastron passage in the direction of companion's motion. It ranges from 0° to 360°, and is sometimes called the longitude of periastron. If $e = 0$ the periastron is undefined and $\omega = 0°$ is then chosen so that T gives the time of nodal passage.

The orbital elements are exposed to changes caused by precession, tangential and radial motion and some aspects of these phenomena can be found in [1].

4 Ellipse Area Relation By Quadrants

Use to define the ellipse and some geometrical values as shown in Figure(3). Lets consider the coordinates transformation by an axis translation from the system (x', y') located at centre of the circle M to (x, y) system located at ellipse focus A.

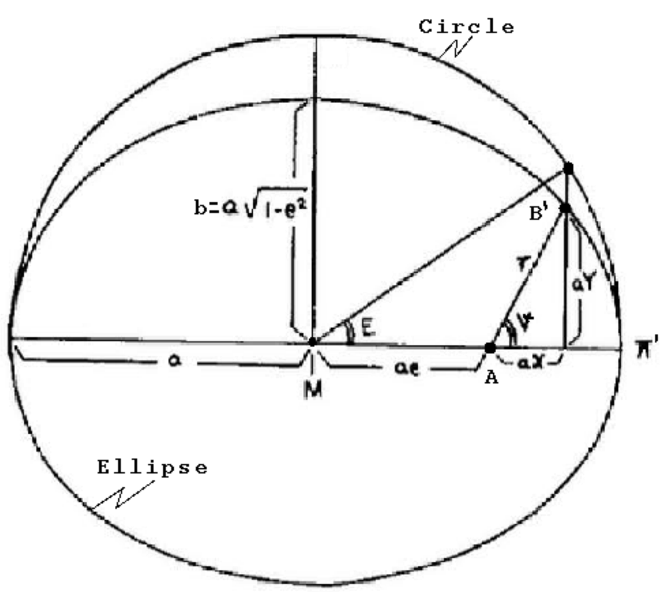

Figure 3: Ellipse geometrical relations

- $E \Rightarrow$ eccentric anomaly.
- $\vartheta \Rightarrow$ true anomaly.

- $A \Rightarrow$ ellipse focus.
- $(x, y) \Rightarrow$ cartesian coordinates system.
- $(x', y') \Rightarrow$ cartesian coordinates system.
- $\rho \Rightarrow$ separation (distance between A and B').
- $b = \sqrt{1 - e^2} \Rightarrow$ minor axis.
- $a \Rightarrow$ major axis.
- $e \Rightarrow$ eccentricity.

The geometrical relation that describes the x and y coordinates for any point on the ellipse curve, in clockwise sense, is given by

$$\begin{aligned} x &= a\cos(E) - a\,e \\ y &= b\sin(E) \end{aligned} \tag{1}$$

Applying *Green-Riemann's theorem for area*, which is given by

$$A = \frac{1}{2}\int\left[x\,\frac{\partial y}{\partial E} - y\,\frac{\partial x}{\partial E}\right]\,dE \tag{2}$$

The area for each quadrant can then be determined as:

1. first quadrant.

$$A = \frac{ab}{2}\left[\frac{\pi}{2} - \arcsin\frac{x}{a} - e\sqrt{1 - \frac{x^2}{a^2}}\right] \tag{3}$$

2. second quadrant.

$$A = \frac{ab}{2}\left[\frac{\pi}{2} + \arcsin\frac{x}{a} - e\sqrt{1 - \frac{x^2}{a^2}}\right] \tag{4}$$

3. third quadrant.

$$A = \frac{ab}{2}\left[3\frac{\pi}{2} - \arcsin\frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}}\right] \tag{5}$$

4. fourth quadrant.

$$A = \frac{ab}{2}\left[3\frac{\pi}{2} + \arcsin\frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}}\right] \tag{6}$$

5 Kepler's Second Law

Kepler's second law is needed in this study because is necessary to know the relation between the secondary position star anywhere on the ellipse for any time interval, and also the position for the last periastron passage. Kepler's second law is stated as follow:

"*The line joining the secondary to the primary sweeps out equal areas in equal times*". Then is possible to write the following relation:

$$\Delta A = K \Delta T \tag{7}$$

and if in P time units the area swept is $\pi * a * b$, then the constant K will be equal to $\pi * \frac{a*b}{P}$.
This equation can be rewritten in a more easy form as:

$$\Delta A = \pi \frac{a*b}{P} \Delta T \tag{8}$$

and

$$\Delta T = Ta - T \tag{9}$$

where $Ta \rightarrow$ ephemerides time, must be $> T$ and ΔT must be in same units as T. Then, the relation of ΔT for each quadrant can be derived as follow:

1. First quadrant.

$$\pi \left[\frac{1}{2} - 2\frac{\Delta T}{P} \right] = \arcsin \frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}} \tag{10}$$

2. Second quadrant.

$$\pi \left[\frac{1}{2} - 2\frac{\Delta T}{P} \right] = -\arcsin \frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}} \tag{11}$$

3. Third quadrant.

$$\pi \left[2\frac{\Delta T}{P} - \frac{3}{2} \right] = -\arcsin \frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}} \tag{12}$$

4. Fourth quadrant.

$$\pi \left[2\frac{\Delta T}{P} - \frac{3}{2} \right] = \arcsin \frac{x}{a} + e\sqrt{1 - \frac{x^2}{a^2}} \tag{13}$$

The above equations give the relation between ΔT and the secondary star position given by x/a. Such equations are transcendental equations and the solution of those equations will be discussed in the following sections.

6 Quadrant Localization

To carry out the solution of the transcendental equations given in Equations(10) through (13) some relations must be presented. It is necessary to know, for any ΔT the quadrant in which the secondary is, in order to solve the correct equation for the respective quadrant. This can be done by substituting $x = a, 0, -a$ and 0 respectively in the Equations (10) through (13) which will give the following relations:

1. Periastron passage, $x = a$ into Equation(10).

$$\Delta T'_1 = 0 \tag{14}$$

2. Passage from the first to the second quadrant, $x = 0$ into Equation(11).

$$\Delta T'_2 = P \left[\frac{1}{4} - \frac{e}{2\pi} \right] \tag{15}$$

3. Passage from the second to the third quadrant, $x = -a$ into Equation(12).

$$\Delta T'_3 = \frac{P}{2} \tag{16}$$

4. Passage from the third to the fourth quadrant, $x = 0$ into Equation(13).

$$\Delta T'_4 = P - P\left[\frac{1}{4} - \frac{e}{2\pi}\right] \tag{17}$$

The following logical relations will help to determine in which quadrant the secondary is

$$\begin{aligned} \Delta T \ &\leq \ \Delta T'_2 \Rightarrow frist\, quadrant \\ \Delta T \ &\leq \ \Delta T'_3 \Rightarrow second\, quadrant \\ \Delta T \ &\leq \ \Delta T'_4 \Rightarrow third\, quadrant \\ \Delta T \ &> \ \Delta T'_4 \Rightarrow fourth\, quadrant \\ \Delta T \ &= \ \Delta T'_1 \Rightarrow periastron \end{aligned} \tag{18}$$

7 Procedure For Ephemerides Calculation

For any given time Ta, the coordinates ρ and θ or x and y can be calculated from the known orbital elements. Some steps will be presented in order to guide the ephemerides calculation.

1. Determination of the quadrant where the secondary is, by one of the relations given in Equations(18).
2. Resolution of the transcendental equations (given by Equation(10) to Equation(13)) by iterative process, which will give the numerical result for x/a for any given ΔT.
3. Projection of the real orbit to apparent orbit in the sky plane of sight.

The first step "1" is quite immediate. No difficulties are present, because it involves only a few easy numerical operations. But the second step "2" demand more elaborate work, and will be treated as follow. The last item will be treated following that.

For example, lets consider the Equation(10) which can be rewritten as:

$$F(\lambda) = \pi\left[\frac{1}{2} - 2\frac{\Delta T}{P}\right] - \arcsin(\lambda) - e\sqrt{1-\lambda^2} \tag{19}$$

where $\lambda = x/a$.

But, it is known and also it is easy to verify that $\lambda = x/a$ range from -1 to $+1$. Then, using the Bisection Method [3] for calculation of the root of this equation, the following iterative algorithm can be written. It is written using FORTRAN language, however can be easily rewritten in any other language.

Making use of F function as defined before is possible to write:

$F(\lambda)$ =...Equation(19)

```
XL=0.0
XR= 1.0
DO 100 I=1,50
```

```
        XM=(XL + XR)/2.0
        IF(F(XM)*F(XL)) 20,30,40
  20    XR=XM
        GOTO 50
  30    XR=XM
  40    XL=XM
  50    IF(XL - XR) 100,200,100
 100    CONTINUE
 200    WRITE(*,*) XM =========> ROOTS, x/a
```

The above algorithm remains the same when applied to the other equations, except the function $F(\lambda)$ must be changed for the respective function for each quadrant considered and the intervals of existence of the roots for $F(\lambda)$.

The last step "3" is the real calculation for the *position angle* and *separation* which is now shown. Is necessary to determine the true anomaly which is given by the following expression in function of $\lambda = x/a$

$$tan(\vartheta) = \pm \frac{\sqrt{(1 - e\lambda^2) - (e - \lambda^2)}}{(\lambda - e)} \tag{20}$$

if ΔT belong to first or second quadrant, then use signal (+)

if ΔT belong to third or fourth quadrant, then use signal (−)

to ϑ in the first or second quadrant, then if ϑ if $> 0°$ add $0°$, if $\vartheta \leq 0°$ add 180o.

to ϑ in the third or fourth quadrant, then if $\vartheta > 0°$ add $180°$, if $\vartheta \leq 0°$ add $360°$.

Such conditions must be respected to eliminate singularities that can arise from the tangent function of the true anomaly given by Equation(20).

The *position angle* can be calculated by [1]

$$\tan(\theta - \Omega) = \tan(\vartheta + \omega)\cos(i) \tag{21}$$

and finally the *separation* is given by

$$\rho = a(1 - e\lambda)\frac{\cos(\vartheta + \omega)}{\cos(\theta - \Omega)} \tag{22}$$

if $\theta > 360°$ subtract $360°$.

if $\theta < 0°$ add $360°$.

if $\rho < 0$ change signal and add $180°$ in θ.

The mathematical relations for the *position angle* and *separation* can be found in [1] where much more information is given. Some comments are necessaries about these expressions. Problems of numerical instability apart of those from iterative process can also arise when some special situation eventually occurs, as for example:

- when $x/a = e \Rightarrow \tan(\vartheta) = \infty$
- when $\theta - \Omega = 90° \Rightarrow \cos(\theta - \Omega) = 0$

These ideas must be kept in mind, and should be implemented in any program to avoid sudden stops or any other problems which can lead the program blow up due to under or overflow errors.

8 Practical Applications and comments

In order to test the present method some ephemerides calculations for θ and ρ are carried out for some binary star systems. The orbital elements are given in the Tables 1, 2 as shown, and also can be found in [4], except for the last case herein presented.

Name	ADS	Σ	α h m	δ deg min	Vis. Mag.
γ Vir	8630	60	12 39	-1 11	3.6 3.6
44 Boo	9494	1903	15 02	$+47$ 51	6.0 6.8
36 Oph	10417	SH243	17 12	-26 32	5.3 5.3
α $Sco*$	10074	xxxx	16 23	-26 13	var 5.5

Table 1: Orbital Elements part 1

Name	a	e	i	T	P	Ω	ω	Calculator
γ Vir	3.6	.877	148.	1836.	171.	29.3	250.	W. Wolf 1949
44 Boo	4.1	.360	84.5	2042.	246.	237.	228.	W.D.Heintz 1963
36 Oph	13.9	.900	99.1	1643.	548.	93.6	90.0	P. Brosche 1960
α $Sco*$	3.27	.000	89.3	1888.	853.	95.1	177.	J. Hopman 1957

Table 2: Orbital Elements part 2

ADS $\Rightarrow$ number in R.G. Aitken's double star catalogue.

Σ $\Rightarrow$ Wilhelm Struve catalogue.

SH $\Rightarrow$ Jones South, John Herschel catalogue.

$*$ $\Rightarrow$ source from Doppelsterm Ephemeriden.

Right ascension α and Declination δ are for 1950.

The results of calculation are shown in the tables 3 to 6 as following

γ Vir				
	J. Meeus		Calculated	
Year	ρ	θ	ρ	θ
1970	4.57	303.6	4.57	303.6
1975	4.26	300.7	4.26	300.7
1980	3.91	297.2	3.91	297.2
1985	3.51	293.1	3.51	293.1
1990	3.05	287.8	3.05	287.8
1995	2.51	280.4	2.51	280.4
2000	1.87	268.4	1.87	268.4

Table 3: Results for γ Vir

The results obtained for γ Vir, 44 Boo and 36 Oph are in complete agreement with the ones presented by Jean Meeus [4]. But, in case of α Sco some discrepancy is noted. Actually, the orbital elements for this calculation are not the same used by DE-Doppelsterm Ephemeriden catalogue. So, the calculation even considering this fact shows quite good agreement. Also, such orbital elements are quite uncertain for this particular binary system.

9 Conclusion

A new approach for ephemerides calculation applied to visual binary stars has been presented. The numerical Bisection Method for root search is shown to be a very good choice for the transcendental equation as derivated

44 Boo				
J. Meeus			Calculated	
Year	ρ	θ	ρ	θ
1970	0.47	324.1	0.47	324.1
1975	xxxxx	xxxxx	0.59	27.6
1980	0.90	28.8	0.90	28.8
1985	xxxxx	xxxxx	1.25	38.8
1990	1.61	44.2	1.61	44.2
1995	xxxxx	xxxxx	1.95	47.7
2000	2.26	50.2	2.26	50.2

Table 4: Results for 44 Boo

36 Oph				
J. Meeus			Calculated	
Year	ρ	θ	ρ	θ
1970	4.52	158.9	4.52	158.9
1975	xxxxx	xxxxx	4.58	156.8
1980	4.64	154.7	4.64	154.7
1985	xxxxx	xxxxx	4.70	152.7
1990	4.77	150.7	4.77	150.7
1995	xxxxx	xxxxx	4.84	148.8
2000	4.91	147.0	4.91	147.0

Table 5: Results for 36 Oph

in the present method. As consequence a very robust and accurate method is designed which is able to predict ephemerides for any given time for any visual binary systems. This approach can also be applied to any other situation where involves orbital ephemerides prediction solving the presented transcendental equations instead of Kepler's equation for mean anomaly.

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α Sco				
J. Meeus			Calculated	
Year	ρ	θ	ρ	θ
1970	xxxxx	xxxxx	2.66	275.6
1975	2.65	275.2	2.59	275.6
1980	2.59	275.2	2.51	275.7
1985	2.52	275.6	2.43	275.7
1990	2.45	275.9	2.35	275.8
1995	2.37	276.1	2.27	275.8
2000	2.30	276.3	2.18	275.9

Table 6: Results for α Sco