Determination of The Orbital Elements of a Visual Binary Star

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Abstract

The observation of a visual binary star consist in measure a set o values which are: ρ, θ and take the time t. Where ρ is the separation between the two components and θ is the position angle, for $\theta = 0^\circ$ indicates the North Celestial Pole. Such observations certainly will contain errors and can be of different origins. However, the effects of such errors can be minimized since it is known that the apparent orbit is an ellipse. By the use of Kepler's second law applied to the apparent ellipse the constant of areal velocity can be determined by means of numerical way using a finite difference approach and also the least square analysis can applied to the set of observations, in order to determine a general equation of a conic which represent the apparent orbit. The orbital elements are then calculated since one know the conic equation of the apparent orbit by means of Kowalsky's method and a numerical procedure is designed in order to calculate the periastron passage using the inverse process of the ephemeris calculation. The orbital period is also calculated using areal's constant of velocity and the apparent ellipse area. By this numerical procedure, as defined in this paper, is possible to calculate the true orbit and then calculate the ephemeris predictions.

1 Introduction

In remembering some aspects of the past, one can get a clear idea of our present time and the possibilities which are now available. How the things have changed so much in too short period of time... I can refer the reader to *Prof. Henry Norris Russell* [9]. He, said referring to the his method *"Experience has shown it to be rapid, two complete orbits having once been computed in one day"*. Just to make known his words was stated in 1933!. What could say *Prof. Russell* now a days ... Certainly he will be happy in know that not only two, but thousands and thousands of orbits could be computed in one day. The main reason in attempt the numerical solution is make use of small computers which are able to make tricky tasks in easy way.

The method devised by this paper does not intend to be a final one, but has as its main purpose show how the numerical procedures can be applied to a set of observations collected by one single observer or various. As is known by people involved with this kind of study, that the measurements of any binary star orbit in function of time, when tabulated (with corrections, if required) will exhibit discrepancies arising from accidental and systematic errors of observations, occasionally, from actual mistakes. If the measurements are plotted, the points which correspond the secondary star will not fall upon an ellipse but will be jointed by a very irregular polygonal line resembling an ellipse.

The present numerical method make use of the least square (LS) analysis in order to fit the set of observations to an ellipse which is a result of the minimization of the residuals among those set of points. As can be noted, the result which comes from such numerical procedure will not be exact, as compared with those ones devised by graphical methods, but will be fast and reliable. The construction of the apparent ellipse is the critical part of the

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entire orbit determination, and keeping the validation of the *law of areas* is the major problem. The "computer"[1] who is skilled in the task of graphical methods has also enough ability to discard any observation which is not good or presents great discrepancy in relation to all the sets of observations. In order to adjust the constant of areal velocity many trials are done until a reasonable apparent orbit is obtained. But, by the numerical procedure all sets of observations are involved in the calculation and inevitably some discrepancy still will arise.

Once the conic equation representing the apparent orbit is obtained by the least squares (LS) method it is possible to start the calculation of the orbital elements by means of the Kowalsky method. The only elements which can not be directly obtained by such method are the orbital period and the periastron passage. So, a inverse ephemeris numerical procedure is designed, in the present paper, in order to calculate the periastron passage and the period is calculated by means of the knowledge of the apparent orbit area. After the orbital elements are available, any classical ephemeris method is used to calculate the orbit over the same period of observation time in order to evaluate the accuracy of the present method.

It must also be stressed that the present work has been conducted only as an exercise in visual binary orbit computing techniques, and must not be interpreted or used as a definitive orbital element procedure of calculation.

About the several other methods it is necessary to know that they will not always work properly and sometimes there is no way to succeed with any method at all. It is also important to say that strange results may arise in such attempts once the "computer" has decided that some measurements are wrong and should be rejected. As I used to say *"the orbit calculation is a cumbersome task and can give weird results* . . ..

By the course of these considerations there is clearly a moral obligation on the part of the "computer" to assure that some conditions must be respected, as for example:

- Depending on the bulk of information available relating to the binary star which intend to compute the orbit, attention must be done in how the orbit represent the present collected data.
- The orbit calculated based on old and new material has to be significant in terms of (C–O)(computed minus observed) residuals and also represent the orbit for all observation data sets.
- If the new elements calculated are really significant and also different from the previously published orbit, it should be published.
- In any circumstances the "computer" should yield just redundant calculations and even more submit them to publication. There is no reason at all to do calculations except if the results are justifiable.

Keeping in mind such conditions all the calculations which can comes from the present technique should be seem as a tool in order to serve an immediate necessity of research. However, the present technique can be improved to be more robust in order to produce even more accurate results and be used extensively.

2 Least Square Method – LSM

The LSM is largely used when one has a set of observations, and in present case the set for the present application must cover at least $270°$ of position angle (θ), another-wise the procedure will fail. This restriction is easily justified since there is not enough points which describe the apparent orbit well, it will be quite difficult to the LSM be able to fit any reasonable ellipse in a set representing an open curve. It is depicted in Figure(1) some situations related to small observations set.

The great objection to this method is that it entirely disregards the time of observation [1]. As one should know the quantity to be measured (ρ) is very small, the observation errors can be large in proportions to this quantity. So, the LSM can yield not perfect ellipse at first time, needing repeated trials in which concern with the preparation of the data set, where sometimes one can have bad data, as can be seen when plotting the original

[1] Who carry on the orbit calculation by graphical or numerical way.

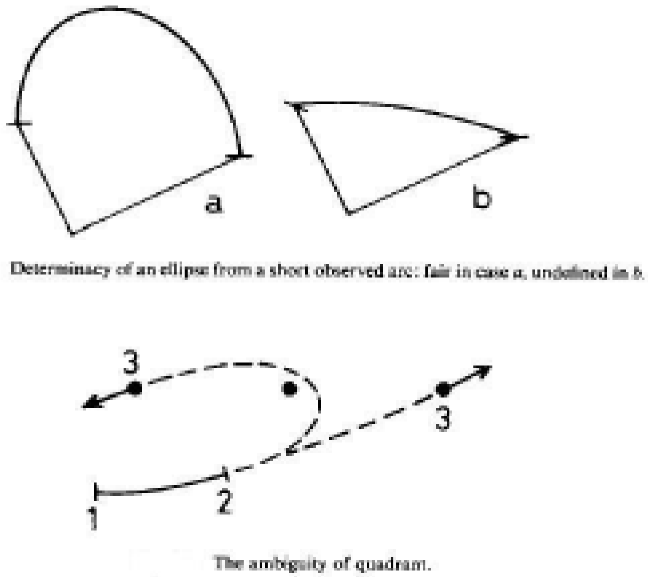

Figure 1: Ambiguities

set and such data must be removed. The graphical methods are therefore to be preferred. But to avoid the use of graphical procedures it is also possible to make the numerical solution more accurate calculating the constant of areal velocity by *finite differences approximation* and keeping the overall precision at a good level for a *preliminary orbit.*

Even though adopting such calculation procedure, serious restrictions are encountered because no account is taken of the law of area in the determination of the conic coefficients by the LSM. This restriction can be made with much less significance since one do the calculation with a reasonable data set of observation, as for example, a short period orbit. In this case two or more orbital periods are known, or more accurate observations are available.

2.1 Formulation of the Least Square Method

The general equation of a conic which is to be fit in the set of observational data is given by

$$By^2 + 2Hyx + Ax^2 + 2Fy + 2Gx = 1 \tag{1}$$

the values of the constants B, H, A, F and G, may be computed by the LSM. In order to obtain a real ellipse one should have observed the following conditions:

$$\begin{aligned} A &> 0 \\ B &> 0 \\ AB - H^2 &> 0 \end{aligned} \tag{2}$$

If one assume the position of the primary star as origin, one may calculate the five constants of the eq(1), since that the (x_i, y_i) are known by the following relations:

$$x_i = \rho_i \cos(\theta_i) \tag{3}$$
$$y_i = \rho_i \sin(\theta_i) \tag{4}$$
$$i = 1, \ldots n \tag{5}$$

where n corresponds the number of pair (ρ_i, θ_i) in different times of observation.

There is a confusing situation in relation to the eq(1) and in consequence the relations of Kowalsky method present some signal mistakes. For example, in Aitken [1] the eq(1) is given as

$$By^2 + 2Hyx + Ax^2 + 2Fy + 2Gx = -1 \tag{6}$$

which is the same notation as used by Smart [4].

But in Heintz [2] he writes as in eq(1) and the Kowalsky relations are quite different from the former ones. The present author implemented both ways but only the Heintz notation [2] was able to produce the correct values for the orbital elements.

The LSM procedure require to minimize the residual function given by

$$\Phi = \sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right]^2 \tag{7}$$

where w_i is the weight. Now, consider observations of different weight. It is normal, therefore, to take into account different weights in any series of observation, which is done simply by multiplying each conditional equation by its weight w_i. The integer value of w_i is taken equal the number of observation nights [7].

Φ is a function of the parameters B, H, A, F and G and is known that a minimum of the square sum of the residuals defined by Φ happen when the partial differential of $\Phi(B, H, A, F, G)$ in relation to B, H, A, F, G are all identically to zero. Writing in formal way, one have the following relations:

$$\begin{array}{lcll}
\frac{\partial\Phi}{\partial B} & = & 2\sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right] y_i^2 & = 0 \\
\frac{\partial\Phi}{\partial H} & = & 2\sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right] 2y_ix_i & = 0 \\
\frac{\partial\Phi}{\partial A} & = & 2\sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right] x_i^2 & = 0 \\
\frac{\partial\Phi}{\partial F} & = & 2\sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right] 2y_i & = 0 \\
\frac{\partial\Phi}{\partial G} & = & 2\sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right] 2x_i & = 0
\end{array} \tag{8}$$

The set of eq(8) state the necessary limitations which are need for the system. It yields a set of five algebraic equations involving the sum of (x_i, y_i) given by eq(5) and can be solved in order to obtain the geometric parameters B, H, A, F and G. Such a system of equations are known as the *normal equations* and can be written in a more direct form. Defining the partial differential related to the parameters as:

$$\frac{\partial \sum_i w_i \left[By_i^2 + 2Hy_ix_i + Ax_i^2 + 2Fy_i + 2Gx_i - 1\right]}{\partial(B, H, A, F, G)} = \sum_i f_j \tag{9}$$

One can write the final form of the system given by eq(8) as:

$$\sum_i w_i \left[Bf_1 + 2Hf_2 + Af_3 + 2Ff_4 + 2Gf_5 - 1\right] f_j = 0 \tag{10}$$

$\bigvee$ i and $j = 1, \ldots, 5$. Doing the multiplication indicated in eq(10) the following result is obtained

$$B\sum_i w_if_jf_1 + 2H\sum_i w_if_jf_2 + A\sum_i w_if_jf_3 + 2F\sum_i w_if_jf_4 + 2G\sum_i w_if_jf_5 = \sum_i w_if_j \tag{11}$$

where $j = 1, \ldots, 5$.

Eq(11) can be written in matrix form, where the sum on $i = 1, \ldots, n$ represents the number of observation points and $j = 1, \ldots, 5$ indicate the five parameter of the conic. So, eq(11) in matrix form is written as:

$$\begin{bmatrix} \sum_i w_i f_1^2 & \sum_i w_i f_1 f_2 & \sum_i w_i f_1 f_3 & \sum_i w_i f_1 f_4 & \sum_i w_i f_1 f_5 \\ \sum_i w_i f_2 f_1 & \sum_i w_i f_i^2 & \sum_i w_i f_2 f_3 & \sum_i w_i f_2 f_4 & \sum_i w_i f_2 f_5 \\ \sum_i w_i f_3 f_1 & \sum_i w_i f_3 f_2 & \sum_i w_i f_3^2 & \sum_i w_i f_3 f_4 & \sum_i w_i f_3 f_5 \\ \sum_i w_i f_4 f_1 & \sum_i w_i f_4 f_2 & \sum_i w_i f_4 f_3 & \sum_i w_i f_4^2 & \sum_i w_i f_4 f_5 \\ \sum_i w_i f_5 f_1 & \sum_i w_i f_5 f_2 & \sum_i w_i f_5 f_3 & \sum_i w_i f_5 f_4 & \sum_i w_i f_5^2 \end{bmatrix} \begin{bmatrix} B \\ H \\ A \\ F \\ G \end{bmatrix} = \begin{bmatrix} \sum_i w_i f_1 \\ \sum_i w_i f_2 \\ \sum_i w_i f_3 \\ \sum_i w_i f_4 \\ \sum_i w_i f_5 \end{bmatrix} \tag{12}$$

The solution of eq(12) give the parameters B, H, A, F, G which minimize the residual function Φ or make the residuals minimum for the data set of observation. So, the ellipse obtained by this procedure is said to fit, in the sense of *"least squares"* to the ellipse curve of the apparent orbit.

In the present work the chosen method to solve eq(12) is the *Gauss Elimination* which is efficient for a (5x5) matrix. Is relevant to say about the particular characteristic of this matrix which is symmetric and much less work is need to generate all its elements, actually only half the matrix need to be generated.

2.2 The Conic Classification

As stated in the beginning of this section sometimes there is no enough observations data and the one which are available do not cover more than 270° in θ. Depending on the quality and number of points available, the LSM can fail, giving a odd result. In order to restrict such behavior a test providing a classification of the conic obtained is derived based on the five parameters of the conic equation. Based on the analytical geometry one can state the following invariants of a general conic as given by eq(1).

$$\begin{aligned} \triangle &= \begin{bmatrix} A & H & G \\ H & B & F \\ G & F & -1 \end{bmatrix} \\ \delta &= \begin{bmatrix} A & H \\ H & B \end{bmatrix} \\ S &= A + B \end{aligned} \tag{13}$$

The quantities $\triangle, \delta, S$ do not modify when the frame of the coordinate system is translated or even rotated. They are called invariants of the curve. Based on this, one can state the following conditions:

$$\text{Central Curve}\,\delta \neq 0 \begin{cases} \delta > 0 & \begin{cases} \triangle \neq 0 & \text{Ellipse} & \triangle . S < 0 \quad \text{Real} \\ \triangle = 0 & \text{Pair of Imag. S. Lines} & \triangle . S > 0 \quad \text{Imaginary} \end{cases} \\ \delta < 0 & \begin{cases} \triangle \neq 0 & \text{Hyperbole} \\ \triangle = 0 & \text{Pair of Conv. S. Lines} \end{cases} \end{cases}$$

$$\delta = 0 \begin{cases} \triangle \neq 0 & \text{Parabola} \\ \triangle = 0 & \text{Pair of S. Lines} \begin{cases} G^2 - A > 0 & \text{Parallel} \\ G^2 - A = 0 & \text{Coincident} \\ G^2 - A < 0 & \text{Imaginary} \end{cases} \end{cases}$$

Where $S.$ and $Conv.$ stands for $Straight$ and $Convergent$ respectively. These conditions are implemented just as the solution of the system given by eq(12) is obtained, such implementation make capable the procedure to follow or stop depending on the classification given above.

3 The Kowalsky Method

Kowalsky's method is essentially an analytical method and a detailed description of this method originally proposed by Kowalsky, for deriving the orbital elements of a visual binary, is given in [5] and also in [1]. However the mathematical analysis is presented by Smart [4]. The derivation of the orbital elements from the five conic parameters, which are need to define the general equation of the apparent ellipse, and which is the orthogonal projection of the true orbit, is some what presented in the literature in a confused way. The authors in [1] and [4] represent the eq(1) in a complete different way as in [3] which the derivation of Kowalsky's relations present signal mistakes. Due this discrepancy the present author could not obtain correct solution for the orbital elements implementing the set of relations given in [1] and [4]. But, the set of relations presented in [3] are consistent and show to be the correct relations and are used [10] in the present calculation.

In order to determine the orbital elements, the following relations involving the five conic parameters are established and are written as:

$$\frac{\tan^2(i)}{p^2} = -\frac{2(H + GF)}{\sin(2\Omega)} \tag{14}$$

$$\frac{tan^2(i)}{p^2} = \frac{(F^2 + B) - (G^2 + A)}{\cos(2\Omega)} \tag{15}$$

$$\frac{\tan^2(i)}{p^2} = (F^2 + B) + (G^2 + A) - \frac{2}{p^2} \tag{16}$$

$$e \sin(\omega) = (F\cos(\Omega) - G\sin(\Omega))\, p \cos(i) \tag{17}$$

$$e \cos(\omega) = (F\sin(\Omega) + G\cos(\Omega))\, p \tag{18}$$

$$a = \frac{p}{1 - e^2} \tag{19}$$

Once the apparent orbit has been defined by the LSM still remains to derive the elements which define form and size of the true orbit, the position of the orbit plane, the position of the orbit within that plane, and the position of the companion star in the orbit at any specified time. The six relations given above involving the five conic parameters of the apparent ellipse determine three angles and two geometric parameters of the orbit. Some of these elements are independent of the spatial location of the binary system and others are used to relate the binary system to the earth's orbit.

The following four elements are independent of the angular ones, and are defined as *dynamical elements* of the orbit

- P $\rightarrow$ the orbital period of revolution (years)
- T $\rightarrow$ the time of periastron passage (year)
- e $\rightarrow$ the eccentricity
- a $\rightarrow$ the semi axis major (second of arc)

The remains elements are the *Campbell Elements* of the orbit

- $\Omega \rightarrow$ the position of the nodal point which lies between $0°$ and $180°$. Measurements of the position angle and separation provide information only about the apparent orbit, which lies in the plane perpendicular to the line of sight. One can not distinguish between the ascending and descending node, or between direct and retrograde motion in the ordinary sense. In some systems the observed position angles increase with the time, in others they decrease. Ω is measured with respect to the north pole at a specified epoch, so, it suffer from the effect of precession, that is to say it will change slowly with time. It is conventional to select a value for Ω less than $180°$, unless radial velocity measurements of the companion give an indication of the true inclination of the orbit.
- $\omega \rightarrow$ the angle in the plane of true orbit between the line of nodes and the major axis, measured from the nodal point Ω to the point of periastron passage in the direction of the companion's motion and may have any value from $0°$ to $360°$.

- $i \rightarrow$ the inclination of the orbit plane; the value lies between 0° and 180°, direct motion of the companion (increasing θ) is indicated by $0° \leq i \leq 90°$, retrograde motion (decreasing θ) by $90° \leq i \leq 180°$. The computed value of i is often shown as $\pm$ until the indetermination of i and Ω is removed by such radial velocity measures. When these are available i is taken to be positive if the orbital motion at the nodal point is taking the companion away from the observer, or negative if the motion is toward the observer at this point of the orbit. See the true and apparent orbital elements depicted in the Figures(2 and 3) respectively.

The Kowalsky relations can be worked out in order to give all elements with exception of the period and the periastron passage. To calculate the period one need to know the *constant of areal velocity* Γ which will be described in the following section.

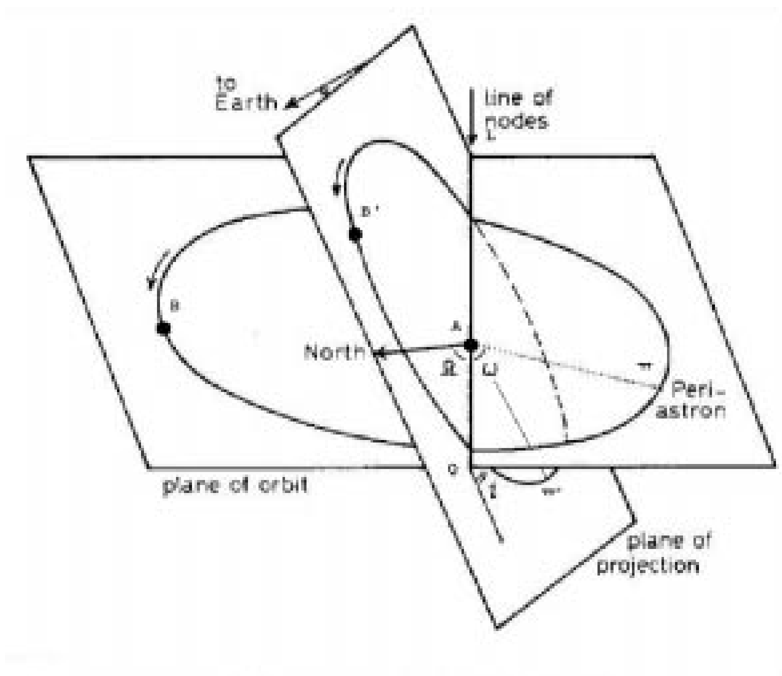

Figure 2: True and Project Orbit Elements

3.1 The Constant of Areal Velocity

The classical study to determine the *constant of areal velocity* Γ is by means of graphical trials, and by analytical adjustment using the (ρ, θ) pairs of observation until the areas law is satisfied. There is a small problem in order to carry out such an adjustment, the "computer" need to have a good practice and feeling and all the procedures depend on human ability. What is needed at this point is no "human" intervention of any kind leaving it as a totally numerical technique. So, the numerical procedure will evaluate the value of Γ as much as precisely as possible. Based on this, the *finite difference scheme* designed here is adopted to calculate the *law of areas* which it is stated as:

$$\Gamma = \rho^2 \frac{d\theta}{dt} (\text{rad/year}) \tag{20}$$

where $(d\theta/dt)$ may be expressed in radian per year, and ρ is in seconds of arc.

The reliability of the observations is shown since the constancy of Γ is kept. Interpolation curves may be adjusted to some extent in order to yield more nearly equal values of Γ for different times. This interpolation

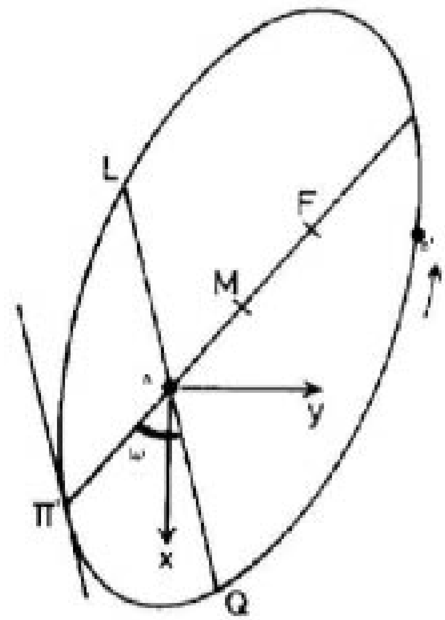

Figure 3: Apparent Orbit Elements

curves are used in the graphical process, the *finite difference* stated as following can be used to uncouple this need. Such an area relation comes from *Green's theorem* as:

$$\Gamma = \frac{\frac{\sum_i^{n-1}(x_i \triangle y_i - y_i \triangle x_i)}{\triangle t}}{(n-1)} \tag{21}$$

Without any adjustment the value of Γ given by eq(21) most nearly satisfies the law of areas. The value of Γ is the *double of the areal constant.*

3.2 The Period Calculation

The period can be calculated by the definition of

$$P = \frac{area}{\mathcal{C}}(\text{year}) \tag{22}$$

The ellipse area is known since the semi axis major a is already determined by the relations of Kowalsky's method. The value of $\mathcal{C}$ corresponds the *constant of areal velocity* of the true orbit. This value is related to Γ corresponding the apparent orbit by [6] the following relation:

$$\mathcal{C} = \frac{\Gamma}{\cos(i)} \tag{23}$$

and the period as given by eq(22) can be rewritten as:

$$P = \frac{2\,\pi\,a^2\sqrt{1-e^2}}{\Gamma}\cos(i) \tag{24}$$

Bear in mind that Γ is the *double areal constant.*

3.3 The Periastron Passage Calculation

One way to determine the time of periastron passage T, is by means of anomalies M(mean anomaly), computed from the observations by taking the ephemeris formulae in reverse order. Every pair of (ρ_i, θ_i) will give a value for M_i and an equation for T_i. The following classical relations of the ephemeris calculation are:

$$\tan(v+\omega) = \arctan\left[\frac{\tan(\theta-\Omega)}{\cos(i)}\right] \to v \tag{25}$$

$$\tan(\frac{E}{2}) = \frac{1}{\sqrt{\frac{1+e}{1-e}}}\left[\tan(\frac{1}{v})\right] \to E \tag{26}$$

$$M = \mu(t-T) = E - e\sin(E) \tag{27}$$

$$T = \frac{1}{\mu}\left[\mu t - (E - e\sin(E))\right] \tag{28}$$

In order to avoid odd results as for example: T being be less than the value of the first time of observation or T being greater than the last time, a criterion must be implemented in such way that states the following condition:

$$t_1 < T < t_n \tag{29}$$

The value of T is calculated as the arithmetical mean of T_i values which satisfy this criterion. In many observational problems the arithmetical mean is the preferred value and is considered the most accurate. It is clear that this criterion states indirectly that one needs to know a sufficient portion of the apparent orbit and presume that the companion has passed through the periastron at least once in the time of the observations. With all the parameters calculated as described before, some numerical experiments are carried out in order to test the procedure.

4 Application

– The orbit of ADS 9982 –

The data set for the ADS 9982 is obtained from [3] and is written as follow:

```
 -------------------------
 DATE  |  rho |  theta | w
 -------------------------
1853.8   1.90   333.7    1
1870.9   1.36   322.0    1
1880.5   1.04   311.0    1
1890.5   0.71   290.9    1
1899.5   0.50   245.0    1
1907.1   0.43   181.1    1
1912.8   0.40   120.6    1
1920.4   0.66   70.6     1
1926.1   0.92   55.0     1
1931.8   1.22   47.1     1
```

These data was tried by the authors in [3] using the Russell graphical method [9] and the differential corrections producing what they classify as orbit II which have the following Innes elements [1]:

```
A = -2.977                e = 0.86
B = -0.277                T = 1906.8
F = +0.238                mi= 0.00923 rad/year
P = 680 years             a = 3.04 (second of arc)
i = +/- 130.6
```

Using the designed procedure, presented above, the following orbital elements are calculated as:

```
              ORBITAL ELEMENTS
       CALCULATED BY THE PRESENT METHOD

PERIOD (YEARS) ==>  629.576519
AREAL CONST.   ==>     .027383
MAJOR AXIS a   ==>    2.803509
ECCENTRICITY e ==>     .843848
INCLINATION i  ==>  130.586372
OMEGA          ==>   14.723172
OMEGAP         ==>  195.185154

VALUE OF PERIASTRON PASSAGE ==> 1896.524873
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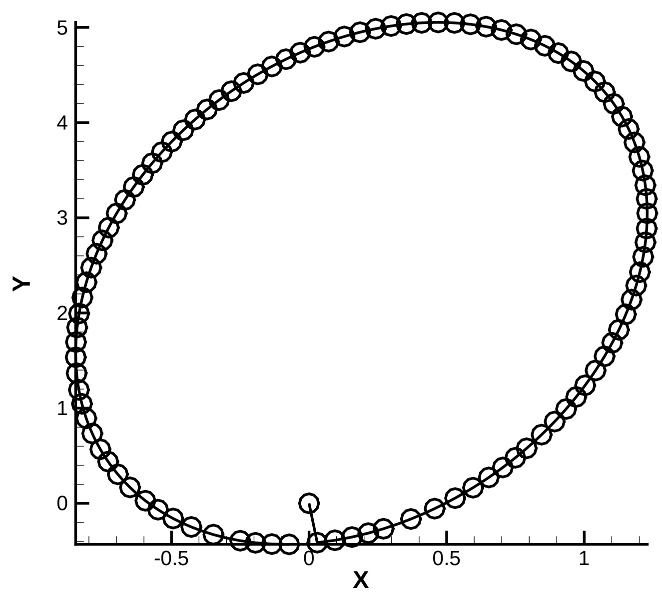

Figure 4: Orbit for ADS 9982

– The orbit of ADS 11520 –

The data set for the ADS 11520 is obtained from [1]. The orbit of this binary system was calculated by the Glasenapp–Kowalsky method and the method of Zwiers by Aitken [1]. The data set for this pair is given as follow:

DATE	rho	theta	w
1900.46	0.14	353.2	3
1901.56	0.14	338.3	3
1902.66	0.12	318.1	3
1903.40	0.11	293.6	3
1904.52	0.14	278.4	4
1905.53	0.12	224.8	4
1906.48	0.13	199.1	4
1907.30	0.14	193.5	1
1908.39	0.15	178.1	3
1909.67	0.10	150.4	2
1910.56	0.11	47.0	2
1911.55	0.15	18.7	1
1912.57	0.15	356.1	3
1914.55	0.14	331.2	4
1915.52	0.15	306.4	3
1916.24	0.13	277.2	1
1916.63	0.16	243.0	2
1916.76	0.14	248.8	2
1917.62	0.10	222.5	1
1917.64	0.14	228.1	2
1918.52	0.14	200.4	3
1918.76	0.14	196.9	1
1919.62	0.15	188.4	3
1920.37	0.16	173.6	2
1920.67	0.16	172.6	5
1921.52	0.15	143.5	1
1921.53	0.12	144.4	4
1923.57	0.14	10.9	4
1923.76	0.18	11.8	4
1924.51	0.15	354.9	1
1924.65	0.12	344.2	1
1925.61	0.15	340.6	3
1928.63	0.14	272.2	2
1931.66	0.14	187.5	2
1932.78	0.15	177.7	2
1933.60	0.11	159.3	4

Glasenapp's Method

P = 12.12 years
T = 1910.10
a = 0.176
e = 0.276
i = 117.6
w1= 269.9
w2= 2.4

Zwiers' Method

P = 12.12 years
T = 1910.10
a = 0.175
e = 0.273
i = 117.75
w1= 270.6
w2= 3.9

w1 = ω and w2 = Ω.

The orbital elements calculated by the present procedure are as follow:

```
                    ORBITAL ELEMENTS
              CALCULATED BY THE PRESENT METHOD

         PERIOD (YEARS) ==>   11.640049
         AREAL CONST.   ==>     .010537
         MAJOR AXIS a   ==>     .150021
         ECCENTRICITY e ==>     .030356
         INCLINATION i  ==>  150.200084
         OMEGA          ==>   15.495030
         OMEGAP         ==>  223.211238

VALUE OF PERIASTRON PASSAGE ==> 1917.019808
```

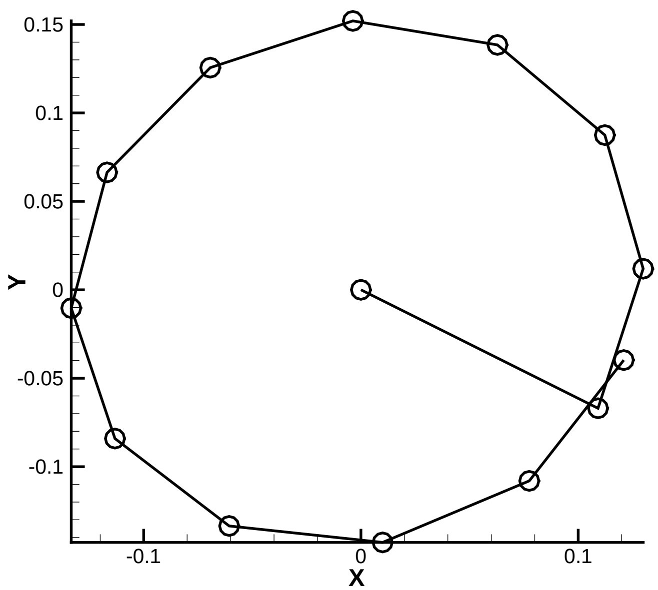

Figure 5: Orbit for ADS-11520

– The orbit of ADS 10786 –

The data set for the ADS 10786 is obtained from [8]. The orbit elements calculation was by Couteau and was based on the method of the opposite points. The data set for the ADS 10786 as given in [8] is written as follow:

DATE	rho	theta	w
1857.50	1.82	59.2	2
1865.79	1.17	84.1	7

1878.08	1.03	233.5	13
1885.56	0.67	286.7	5
1894.74	1.23	41.4	30
1902.94	1.63	63.6	32
1906.28	1.37	73.0	18
1908.46	1.22	80.1	22
1911.59	0.72	102.4	8
1915.66	0.49	175.4	7
1921.11	1.00	233.0	14
1928.59	0.66	283.3	3
1933.53	0.64	8.7	7
1940.00	1.37	47.6	36
1946.43	1.60	63.7	26
1951.10	1.26	79.5	8
1954.54	0.77	99.9	7
1956.57	0.55	126.8	9
1960.65	0.70	204.0	4
1964.68	1.07	234.2	4
1974.54	0.51	330.8	7

The orbital elements calculate by Couteau are

```
P = 43.20 years
T = 1965.40
a = 1.360
e = 0.178
i = 66.2
w1= 174.0
w2= 60.7
```

The orbital elements calculated by the present procedure are as follow:

```
                    ORBITAL ELEMENTS
             CALCULATED BY THE PRESENT METHOD

       PERIOD (YEARS) ==>   50.752358
       AREAL CONST.   ==>     .087931
       MAJOR AXIS a   ==>    1.380174
       ECCENTRICITY e ==>     .201759
       INCLINATION i  ==>   67.623392
       OMEGA          ==>   61.968940
       OMEGAP         ==>  170.319104

VALUE OF PERIASTRON PASSAGE ==> 1924.701467
```

5 Conclusion and Comments

As was discussed before, all the effort applied in develop the present calculation procedure is based on the fact that the graphical methods may be avoided, since one can be able to have a reliable numerical result on hand any

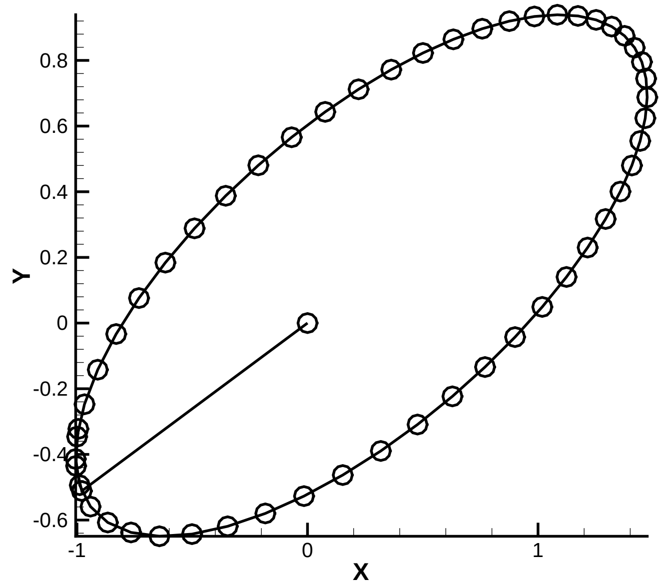

Figure 6: Orbit for ADS-10786

time. However, should be clear, that the graphical methods, still have their purpose and will not be discarded even because in some situations the numerical procedure, as the present one, is not able to compute small orbit arcs.

The results obtained show that the present numerical procedure is able to deal with two different situations. The first one is when the θ decrease with time and the second one s when the θ increase with time. It is well known that both of these cases are common in the orbital elements calculations and depending of the adopted method quite different behavior of solution may certainly happens.

In the three cases presented above, a very good agreement is obtained. Should be said at once that no adjustment of any kind at all was applied to all three data set of observations and even though good results for the orbital elements were obtained. Seems that if some kind of adjustment is applied to the ρ and θ values before the LSM be applied a much better improvement can be obtained not only in the angular but also for the dynamical elements and also to the constant of areal velocity. Such adjustments are applied in order to keep the law of areas satisfied for all data points. It is important to say that such adjustments should only be tried by who have some experience in the subject another wise meaningless results are obtained. With the present results it is possible to carry out a study of accuracy based on the C–O differences. The numerical calculation underlying the presented results were carried out in a PC computer and did not take more than 1/2 second for each orbit calculation.

6 Acknowledgments

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