An Evaluation of the Nominal Stress Method for Life Prediction of Cylindrical Circumferential V-notched Specimens Tested Under Variable Amplitude Loading

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Abstract. An experimental device was constructed to test various cylindrical V-notched specimens until fracture and under variable amplitude torsional loads. The specimens had different notch depths resulting in different values of the stress concentration factor. Strain gages directly bonded at the specimens’ surface and using a slip ring system for their communication with the conditioner, allowed the measurement of the actual applied loads. The well-known rain flow cycle counting procedure was then applied on the scaled signal to identify the frequency of 64 classes of stress amplitudes and means. The traditional nominal stress-based approach was then evaluated as the most widely used tool for fatigue lifetime calculations. As the occurrence of stress amplitudes above the endurance fatigue limit tends to lower it, the Miner elementary method was used. The results show that the damage sum \( \sum N_i / N_f \) ranges between 0.5 and 6.4 with a mean value of 2.0. Despite the small size of the sample used in the present paper (only 13 tests), these significant deviations, from the most widely used unit value, are in agreement with previous results reported by different researchers.

Introduction

Fatigue problems are normally addressed by the stress-based approach. This is essentially an empirical method based on the relation between the completely reversed stress amplitude \( \sigma_{ar} \) and the number of cycles \( N_f \) for complete failure of small specimens. Two fitting constants or material properties (\( \sigma'_f \) and \( b \)) are needed for relating these parameters and the resulting equation is considered the fatigue strength under zero mean stress:

\[
\sigma_{ar} = \sigma'_f (2N_f)^b
\]  

Typical values for an SAE 1015 steel are \( \sigma'_f = 1020 \) MPa and \( b = -0.138 \) [1]. Machine components, however, are seldom exposed to the above conditions and new functions accounting for stress concentration at notches, multiaxial stresses, mean stresses, variable amplitude loading and damage accumulation must be used. These issues will be addressed below.

Stress Concentration

The nucleation of fatigue cracks is the result of a highly localized process of cyclic plasticity. This process is favored by stress raisers (notches), which are unavoidable in machine elements. The stress analysis used in the stress-based approach is purely elastic. Consequently, peak stresses are calculated by multiplying the nominal net stresses \( S \) by the corresponding stress concentration factor SCF \( K_t \). Solutions for \( K_t \) are available in the form of equations or charts that have been derived from the elasticity theory. The drop on the local elastic stresses as a function of the distance from the notch tip, according to \( K_t \) solutions, is inversely proportional to the notch tip radius \( \rho \). Therefore, a high stress gradient exists, mainly for sharp notches. For these cases, only a very small volume of material is
under the influence of peak stress. As a result, the fatigue strength of notched specimens is above the expected $S_{ar}$ value based on $K_t$. This is more evident in the near-threshold fatigue region and a fatigue notch reduction factor $K_f$ has been introduced. On classical formulations, the ratio $K_f/K_t$ depends on a material parameter $c_N(\sigma_u)$ [2] which has units of length, and where $\sigma_u$ is the ultimate monotonic strength of the material and quantifies the so-called notch sensitivity.

More recent formulations suggest the use of a stress field criterion based on the Linear Elastic Fracture Mechanics LEFM for dealing with the problem of stress concentration in sharp notches [3]. Blunt notches, on the other hand, continue to be treated by the stress field criterion but it is suggested to use, conservatively, the traditional SCF instead of the notch strength fatigue reduction factor $K_f$. By doing so, the small differences between these quantities for blunt notches are neglected. The separation between sharp and blunt notches is made through a relevant value of notch size $a_N$, which is believed to be completely sensitive to the peak stress [4]:

$$a_N = \frac{K_t^2 \cdot a_0}{F^2} \quad (2)$$

In Eq. 2, $a_0$ is a material parameter characterizing the defect sensitivity [5] and $F$ is the geometry factor used in the characterization of crack tip stress fields by the stress intensity factor SIF. The factor $F$ should be calculated considering the same depth “a” for both, the crack and the notch. If the notch under consideration has a depth greater than $a_N$, the severity of the notch is relevant and the SCF should be used. If not, the notch reduction factor $K_f$, whose new definition according to the LEFM is presented below, should be used instead:

$$K_f = \sqrt{\frac{F^2 \cdot a}{a_0} + 1} \quad (3)$$

### Combined Effect of Mean and Multiaxial Stresses

Different methods exist for reducing multiaxial stress states to a uniaxial equivalent. Among the most popular are those based on the invariants of the stress tensor. They consider that, since yielding and fatigue are both mechanisms controlled by plastic deformation, static failure theories can therefore be applied to fatigue [6]. For the most usual case of combined bending and torsion in ductile materials, the equivalent stress amplitude (based on the deviatoric part of the strain energy density) and equivalent mean stress are defined as:

$$\tilde{\sigma}_a = \sqrt{\sigma_a^2 + 3 \cdot \tau_a^2}$$
$$\tilde{\sigma}_m = \sigma_m \quad (4)$$

The effect of mean load is introduced through the concept of equivalent completely reversed stress amplitude $\sigma_{ar}$ which, in turn, should be a function of $\tilde{\sigma}_a$ and $\tilde{\sigma}_m$. In 1970 Smith et. al. [7] proposed that, for any mean load, a certain function of fatigue life $g(N_f)$ is in agreement with the product between the maximum stress and the strain amplitude:

$$\sigma_{max} \cdot \varepsilon_a = g(N_f) \quad (5)$$

For the particular case of $\sigma_m = 0$ the product in Eq. 5 can be written as $\sigma_{max} \cdot \varepsilon_a = \sigma_{ar} \cdot \varepsilon_{ar}$ which, for uniaxial case $\varepsilon_{ar} = \sigma_{ar}/E$, becomes $\sigma_{max} \cdot \varepsilon_a = \sigma_{ar}^2/E$. Then the life function $g(N_f)$ is related to Eq. 1 as follows:

$$g(N_f) = \left[\frac{\sigma_f}{(2N_f)^{1/2}}\right]^2 \quad (6)$$
This approach is known as SWT, which stands for the authors’ initials [7]. The success of the approach can be visualized by plotting experimental fatigue strength data obtained under various mean stress levels (in terms of the product $\sigma_{\text{max}}\sigma_a = \sigma_m\sigma_a + \sigma_a^2$) against fatigue life $N_f$. If data points plotted this way fall near the curve described by Eq. 6, it means that the SWT correction is able to eliminate the mean load dependency in fatigue resistance curves and that the function:

$$\sigma_{ar} = \sqrt{\sigma_a^2 + \sigma_a \cdot \sigma_m}$$

is suitable for design purposes. It is worth noting that only constants obtained under the zero mean stress condition ($\sigma'_f$ and b) are needed for plotting Eq. 6.

**Variable Amplitude Loading and Damage Accumulation**

Fatigue load histories consist simply of a sequence of peaks and valleys. Such histories need to be manipulated by statistical means in order to be useful in fatigue analysis models. The result of this process is the so-called “load spectrum”. Among the various procedures developed in the past with this purpose, the rain-flow cycle counting RFCC approach [8] is still the most widely used. Special purpose algorithms are applied to the (almost always) digitalized quantity that represents the load history. As a result, the number of occurrences of such quantity $N_i$ for a given range (or amplitude) and mean, are stored in the form of a matrix. For this reason the RFCC is considered a two-parameter approach. The present paper uses an implementation of the RFCC as a MatLab® function developed by Nieslony [9].

The damaging contribution of each cycle counted (or each element of the matrix) is calculated as the fraction $N_i / N_{fi}$, where $N_{fi}$ calculations are based on the reference curve (Eq. 1) after the introduction of corrections for multiaxiality (Eq. 4) and non-zero mean stresses (Eq. 7). At the moment of fatigue failure, the fatigue resistance of the material is believed to be totally consumed and the total damage is simply:

$$D = \sum_i \frac{N_i}{N_{fi}}$$

This is the well-known Miner’s rule [10]. Some experiments done by this author with joints and also with unnotched specimens gave values of $D$ using Eq. 8 that varied between 0.61 and 1.45. The average value was, however, 1.0. Schutz [11] obtained similar results for $D$, again on average, in tests with a constant mean stress. For cases where a representative portion of the loading history (in the form of load spectrum) is available, in units of time $t$, Eq. 8 provides a relative damage $D/t$. The predicted life for these situations is obtained by the following equation (which assumes that $D = 1$):

$$L_{\text{pred}} = \frac{t}{D} = \frac{t}{\sum_i \frac{N_i}{N_{fi}}}$$

In the fatigue life assessment performed in this paper, neither the existence of a nominal endurance limit $\sigma_e$ nor changes in the reference curve slope $b$ below $\sigma_e$ were considered. The explanation for this is simple: cycles with high stress amplitude are able to develop small fatigue cracks that can grow later under the action of low amplitude stress cycles. In the literature this providence has been designated as Miner elementary [14].

**Experimental Setup and Data Processing**

The experimental setup consisted of an alternating current motor that supported the specimen by one of its sides, the other being attached to a wheel (Fig. 1, a). At a radial distance from the center of the wheel, a spring was attached to provide the specimen with a reaction torque. The amount of resistive torque applied was controlled by the spring stiffness. Whichever the real strain history the
specimen suffered, the strain was measured through gages bonded at the surface of the mandrel that was holding the specimen (Fig. 1, b). The same figure shows how the system of slip rings transmitted the analogical signal to the conditioner and the controlling software. The latter was responsible for storing the experimental data.

The specimens used in the present investigation were cylindrical bars with circumferential V-notches (Fig. 2 a). They were machined from bars of SAE 1015 normalized steel (\(E = 200 \text{ GPa}, \sigma_0 = 228 \text{ MPa}\)) whose fatigue properties are: \(\sigma_{f} = 1020 \text{ MPa}\) and \(b = -0.138\) [12]. The material parameter and geometry factor were \(a_0 = 296 \text{ µm}\) and \(F = 2\), respectively [4]. The depth of the notch \(h\) varied between 1.95 mm and 2.35, which resulted in a SCF for torsion and dependent on the notch angle \(\alpha\) of \(1.62 \leq K_{t\alpha} \leq 1.55\) (Fig. 2 b). The analytical \(K_{t\alpha}\) solutions were extracted from Peterson’s classical manual [13].

The tests were conducted until the total fracture of the specimen. This happened, in some tests, in a few minutes due to the combined effects of applied load and notch sensitivity. The total strain analysis was recorded and processed after the tests. Data processing consisted in transforming each point of vector strain \(\varepsilon_i\) into a net nominal shear stress vector \(\tau_{ni}\) using elementary material strength formulae. In practical terms this means scaling the strain readings proportionally to elastic constants (\(E, \nu\)) and to the relation between the diameters of the mandrel and the specimen (\(D/d\)) (Eq. 10, below).

\[
\tau_{ni} = \frac{E}{1 + \nu} \left(\frac{D}{d}\right)^3 \varepsilon_i
\]  

Once the strain history was converted into a net nominal shear stress history, cycles of the same amplitude and mean (according to yet to be defined stress classes) needed to be counted. The RFCC function performs this task. This function only works for a stress history composed of peaks and valleys, so another function, also provided in the MatLab toolbox [9], needed be used before. The objective was leaving only turning points in the signal. Obviously, this action significantly reduces the number of points, mainly if high sample rates were used in the original signal.

In order to have a rainflow matrix of manageable size, the stress range was divided into classes and the individual values were then allocated in the corresponding group. The number of stress classes used in this paper was 64. Since this is the statistical representation of the measured strain history (load spectrum), the distribution had reflexes in the damage and in the associated predicted life, as will be discussed below.

Fig. 1 A general view of the experimental setup used in the present investigation (a) and details of the slip ring system responsible for the communication between the conditioner (not shown) and the strain gage SGs sensors (b).
Results and Discussion

Damage calculations in each cell of the rainflow matrix were based on the reference curve (Eq. 1). The influence of stress concentration factors, multiaxial stress states and mean loads should be addressed before. The input values are the material properties and the nominal shear stress $\tau_{ni}(\varepsilon_i)$ vector (Eq. 10) for the representative portion of the load history (block) being analyzed. The first step consists of applying the RFCC procedure to the vector $\tau_{ni}$. As a result, two vectors of 64 elements (or classes) each and a 64 square matrix $R F_{ij}$ are obtained, corresponding to the extracted amplitude and mean shear stress values $\tau_{ani}$ and $\tau_{mnj}$ and their respective frequencies $N_{ij}$.

The substitution of the material’s properties $a_0 = 296 \, \mu m$ and $F = 2$ in Eq. 2 for every element of the $K_{ts\alpha i}$ vector allowed classifying the notches of all specimens as blunt notches ($a_{ni} >> h$). Therefore, for each specimen, a scalar expansion was performed on the $\tau_{ani}$ and $\tau_{mnj}$ vectors according to the simple equation:

$$
\tau_{ani} = K_{ts\alpha i} \cdot \tau_{ani}
$$

$$
\tau_{mnj} = K_{ts\alpha j} \cdot \tau_{mnj}
$$

(12)

Torsional loads induce a proportional multiaxial stress state in every point at the specimen surface. Mean loads are also present and the combined effect of these variables is treated by means of Eq. 4 and Eq. 7 resulting in $\sigma_{ar ij} = \tilde{\sigma}_{ai}$. Then, only shear stress amplitudes are significant for life predictions. To each cell of the $R F_{ij}$ matrix a $\sigma_{ar ij}$ value is associated. Fatigue lives $N_{ij}$ are then calculated (Eq. 13) using the Miner elementary approach and for each completely reversed stress level through the reference curve (Eq. 1). The damage increments $D_{ij}$ are the quotient between the frequency $N_{ij}$ and the $N_{fij}$ (see also Eq. 13). They are obviously concentrated at the high stress amplitudes levels as shown the map color of this matrix (Fig. 3).

$$
N_{fij} = \frac{1}{2} \left( \frac{\sigma_f}{\sigma_{ar ij}} \right)
$$

$$
D_{ij} = \frac{N_{ij}}{N_{fij}}
$$

(13)
The best way of evaluating the accuracy of life predictions using the Miner elementary method described in this paper is by comparison with the experimental results. With this aim, let the parameter $D_{\text{eff}}$ [15] be defined as the quotient between the experimental life and the predicted life, in seconds:

$$D_{\text{eff}} = \frac{L_{\text{exp}}}{L_{\text{pred}}}$$  \hfill (14)

After inserting Eq. 8 into Eq. 14 we see that $D_{\text{eff}}$ is nothing more than the product of the number of time periods $t$ of the representative portion of the loading spectrum that compound the total load spectrum $L_{\text{exp}}/t$ (or number of blocks $B$), times the damage in that period. Then, the damage parameter $D_{\text{eff}}$ can be viewed as a failure criterion for the specific component under consideration. It is plotted for each test of the present investigation in Fig. 4.

$$D_{\text{eff}} = \frac{L_{\text{exp}}}{L_{\text{pred}}} = \frac{L_{\exp}}{t} \cdot \sum \frac{N_{ij}}{N_{ij}} = B \cdot \sum D_{ij}$$  \hfill (15)

On average ($D_{\text{eff, ave}} = 1.9$) the predictions were on the safe side, i.e. the experimental lives were higher than the predicted ones. There was however, a large scatter on the results. No clear correlation was found between the SCFs used and the damage parameter $D_{\text{eff}}$ that should be explained by the small variations in the former (Fig. 2).
Conclusions

The aim of the present investigation was testing the accuracy of the nominal stress approach for fatigue lifetime prediction. A Miner elementary modification was used for dealing with completely reversed stress amplitudes below the fatigue limit. The essential information related to the damage process was registered by the stress spectrum obtained through a rainflow cycle counting procedure. Although most of the results were conservative, a large scatter was observed. The results in terms of accuracy and scatter for other modifications and methods will be tested in future papers.

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