Evaluation of three current methods for including the mean stress effect in fatigue crack growth rate prediction

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ABSTRACT

The fatigue crack growth rates curves of engineering materials depend on two parameters. In addition to the dependence on the classical stress intensity factor (SIF) range $\Delta K$, there is a dependence on the mean load (or mean SIF), mainly in the near-threshold region. The present paper provides some useful suggestions and good practices for using three of the current available methods to reduce this second dependence through the use of tuning constants. The methods considered here are the Elber, Walker and Vasudevan (or unified approach). For each approach, multiple regression analyses are performed on experimental data from the literature, and the correlations in two and three dimensions are graphically analyzed. Numerical examples of crack growth analysis for cracks growing under nominal stresses of constant amplitude in single-edge and notch/hole geometries are performed, assuming an identical material component to that of the available experimental data. The resulting curves of crack size versus number of cycles ($a$ versus $N$) are then compared. All three models gave approximately the same ($a$ versus $N$) curves in both geometries. Differences between the behaviors of the ($a$ versus $N$) curves in both geometries are highlighted, and the reasons for these particular behaviors are discussed.

Keywords  

 crack propagation modeling; life prediction; mean stress effects.

NOMENCLATURE

- $a$ = crack size
- $C_m \gamma$ = material parameters
- $da/dN$ = fatigue crack growth rate
- $F(s)$ = geometric factor as a function of normalized crack size
- $K_e$ or $K_n$ = stress intensity factors (SIFs) for single-edge crack and crack from notch/hole geometries, respectively.
- $N_{if}$ = number of cycles between the initial and final crack size
- $P$ = load
- $R$ = ratio of the minimum to maximum load
- $R_o$ = radius of the hole
- $s = a/b$ or $a/(a+R_o)$
- $S$ = nominal stress
- $S_y$ = yield strength
- $\Delta K$ = stress intensity factor range
- $\delta$ = displacement

INTRODUCTION

Damage-tolerant design methods aim for the survival of engineering components and structures, even in the presence of cracks. Brittle fracture occurs when the stress intensity factor (SIF) (variable $K$) of a cracked body is equal to the fracture toughness of its composing material. Critically sized cracks rarely exist in new components. Most frequently, small cracks grow by subcritical mechanisms, including fatigue or environment-assisted cracking. To
analyze fatigue crack growth (FCG) during the design and structural assessment phases, FCG rates curves are used. The aim of FCG analysis is to determine safety factors in stress or life, wherein the crack size is considered to be the independent variable.

The FCG rates of an engineering material are a two-parameter phenomenon. Data of the FCG rates \( \frac{da}{dN} \), where \( a \) is the crack size and \( N \) is the number of cycles, tend to align in a single curve when plotted in log-log coordinates against the SIF range \( \Delta K \). At low rates, however, many experimental data show a strong dependency on the mean load. For constant amplitude loading, the mean load or the load ratio \( R (=K_{\min}/K_{\max}) \) has the same effect on the displacement of the loading sinusoids. The load ratio is almost always used as a second parameter. Therefore, a general relation for the FCG rates curves is \( \frac{da}{dN}=f(\Delta K, R) \). The number of cycles \( N_f \) between the initial and final crack sizes \( a_i \) and \( a_f \), respectively, can be calculated by the simple integration of this differential relation, or

\[
\frac{N_f}{N_i} = \frac{a_f}{a_i} = \frac{\int da}{df(\Delta K, R)} = N_f = \int \frac{da}{df(\Delta K, R)}
\]

The Paris relation, \( \frac{da}{dN}=C \Delta K^{mp} \), where \( C \) and \( mp \) are material parameters, is an approximation that fits the empirical data well in a particular region of the FCG rate curve for a given load ratio. The Paris relation can be easily introduced into Eq. (1) and used for designing purposes. For other load ratios, however, the fitting constants \( C \) and \( mp \) should not be the same. Indeed, it is not possible to obtain empirical data for the whole load-ratio range.

Much effort has been expended to reduce the dependence of Eq. (1) to just one variable, such as by replacing \( \Delta K \) with another FCG driving force that includes the load-ratio effect. The dependence can also be reduced by using a set of fitting constants for all load ratios; in Paris form, this means obtaining a \( C(R) \) or \( \Delta K(R) \) function. The most widely accepted mechanism to account for the load-ratio effect is the crack closure concept,\(^1\) which is based on the effective SIF range \( \Delta K_{eff} \). The closure concept will be treated in detail in the succeeding text. For engineering applications, \( \Delta K_{eff} \) should collapse FCG data at any load ratio to a single curve. Other well-known methods for dealing with the load-ratio effects on FCG curves are the Walker approach\(^2\) and the more recent unified approach (UA),\(^3,4\) which are briefly described in the following sections.

The aim of the present paper is to evaluate the Elber, Walker and UAs, while giving numerical examples of the complete process for calculating the residual life of two cracked engineering components loaded in fatigue at any load ratio. First, for each approach, the steps for obtaining the material properties or calibration constants from a series of experimental FCG rate versus \( \Delta K \) data at different mean loads are described for a 2024-T3 aluminum alloy. The growth of cracks from a notch/hole geometry and for a single-edge crack geometry is simulated under a range of constant stresses. The number of cycles needed to propagate these cracks between two specified sizes is calculated, assuming that the component is made from the same 2024-T3 aluminum alloy. No attempt is made to predict the classical crack nucleation resistance (SN) curves with these linear elastic fracture mechanics (LEFM) approaches, because the models do not include the necessary corrections for short cracks.

**REVIEW OF THREE METHODS**

Plasticity-induced crack closure PICC and the effective stress intensity factor range \( \Delta K_{eff} \): The Elber approach

The influence of the load ratio on \( \frac{da}{dN} \) curves is generally credited to the well-known PICC phenomenon,\(^1\) which has been the object of many studies.\(^5\) Consider a cracked component under constant amplitude loading. At the maximum load of each cycle, a ‘monotonic plastic zone’ at the crack tip will undergo plastic deformation. The size of this region is proportional to \( (K_{\max}/S_y)^2 \), where \( S_y \) is the material yield strength. Most FCG occurs at low \( \Delta K \), which implies a small plastic zone. The residual ligament has a much larger stiffness than the plastic zone, which is squeezed during unloading. Assuming a kinematic hardening model,\(^6\) the yield during compression requires a stress increment on the order of twice \( S_y \), and the size of the cyclic plastic zone is about 1/4 that of the monotonic plastic zone. A small, but finite, positive plastic strain remains at the crack tip. In the next cycle, the crack passes through this region and is permanently elongated at its lips. After many fatigue cycles, a thin layer of plastically extended material forms in the wake of the crack. This layer prevents the crack from being opened unless the tensile stress exceeds an opening value \( S_{op} \). Consequently, the crack remains closed for stresses between the minimum value \( (S_{min}) \) and \( S_{op} \).

Stress singularity at the crack tip is only valid for cracks that are completely opened. Thus, the whole SIF range, \( \Delta K \), cannot be used to describe the FCG. Instead, a portion of it, defined as the effective SIF range, \( \Delta K_{eff} \) represents the net crack driving force. The closure factor can then be defined as

\[
U = \frac{\Delta K_{eff}}{\Delta K} = \frac{(S_{max} - S_{op})}{(S_{max} - S_{min})} \frac{\sqrt{\pi a} F}{\sqrt{\pi a} F} = \frac{\Delta S_{eff}}{\Delta S}
\]
In Eq. (2), the net or effective SIF range should be constant during FCG, which implies that $S_{op}$ does not vary with the size of the crack or the residual ligament, as was empirically concluded by Elber. The cyclic plastic zone depends on $S_{min}$. Consequently, the closure factor should depend on the load ratio. For example, Schijve\(^7\) proposed the following empirical relation for $U(R)$:

$$U(R) = 0.55 + 0.33R + 0.12R^2 \quad (3)$$

This expression covers the whole range of values for the load ratio and has been successfully applied to the 2024-T3 aluminum alloy. Thus, the Paris relation can be rewritten as in Eq. (4):

$$\frac{da}{dN} = C(\Delta K_{eff})_{mp} = C(U(R) \Delta K)_{mp} \quad (4)$$

When many experimental data for different load ratios are condensed into a single scatter band according to Eq. (4), the FCG dependence is reduced to only one variable, $\Delta K(a)$. Thus, in design, only a few tests at one load ratio are needed to obtain the material parameters ($C$ and $mp$) and to use Eq. (1). The same analysis can be extended to other materials and to joints, provided that a relation of the type of Eq. (3) is available. In the present paper, this methodology will be called the Elber approach, as it is based on the PICC mechanism.

However, significant problems arise during the practical implementation of PICC for design. The most important of these problems is related to the determination of the opening load, and this explains the development of empirical equations based on the $R$ ratios, as Eq. (3). Other approaches used to manage the two-variable problem in FCG are addressed in the succeeding text.

The Walker approach

Dealing with the mean stress effects in SN curves, Walker\(^2\) introduced the concept of an equivalent zero to maximum equivalent stress, $S_{eq} = S_{max}^{\gamma} = \Delta S^\gamma$, where $\gamma$ was assumed to be a material constant. The same idea can be extended to the SIF range, $\Delta K_{eq}$, which would generate the same FCG rate $da/dN$ as the actual ($K_{max}$, $R$) combination for $R > 0$.

$$\Delta K_{eq} = K_{max}(1 - R)^\gamma \quad (5)$$

The material parameter $\gamma$ varies between zero and 1.0 (maximum and minimum sensitivity to load ratio, respectively). For example, for $\gamma = 1$, $\Delta K_{eq} = \Delta K$, which implies that the load ratio $R$ has no effect. Thus, $\gamma$ can be considered to be an inverse measure of the sensitivity of FCG to the mean load or load ratio. One way, but not the only way, to find $\gamma$ will be described later. Because $\Delta K = K_{max}(1 - R)$, a relation between the equivalent and applied SIF range is

$$\Delta K_{eq} = \Delta K(1 - R)^{\gamma - 1} \quad (6)$$

Let one of the Paris parameters for the particular case of $R = 0$ be denoted as $C_0$ and assume that propagation occurs in a stable region of the FCG curve. Then

$$\frac{da}{dN}(R = 0) = C_0 \cdot \Delta K_{mw} \quad (7)$$

According to Eq. (6), for $R = 0$, $\Delta K_{eq} = \Delta K$, and the following relation holds true:

$$\frac{da}{dN} = C_0 \cdot [\Delta K(1 - R)^{\gamma - 1}]_{mw} \quad (8)$$

Equation (8) represents dependence for all load ratios. The surface that best fits the $(da/dN, \Delta K, R)$ experimental data can be obtained through a regression analysis that returns ($C_0$, $\gamma$, $mw$). The number of variables that describe FCG is reduced to two, either $(da/dN, \Delta K)$ or $(da/dN, \Delta K_{eq})$, depending on which of the following forms of Eq. (9) is used:

$$C = C_0(1 - R)^{\gamma - 1} \quad \Rightarrow \quad \frac{da}{dN} = C \Delta K_{mw}$$

$$\Delta K_{eq} = \Delta K(1 - R)^{\gamma - 1} \quad \Rightarrow \quad \frac{da}{dN} = C_0 \Delta K_{eq}$$

The integration in Eq. (1) can now proceed for any load ratio, provided that the material constants are available. If the function $\Delta K(a)$ (or $\Delta K_{eq}(a)$) is not trivial, then numerical integration will be needed. As aforementioned, the Walker relationship is valid for $R \geq 0$. Dinda and Kujawski\(^8\) proposed a similar relation for positive and negative load ratios. However, their proposal used the $\Delta K^*$ parameter that represents the positive part of the applied SIF and assumed that the negative part of $\Delta K$ does not contribute to FCG.

The unified approach

To account for discrepancies in relation to PICC (see in the preceding text), Vasudevan and colleagues\(^3,4\) proposed a new concept requiring two driving forces for an advancing fatigue crack: $\Delta K$ and $K_{max}$. Two physical reasons motivated their choice: (1) the total damage ahead of a crack tip is the sum of monotonic and cyclic damages due to $K_{max}$ and $\Delta K$, respectively, and (2) FCG can only occur if tensile stresses exist in the monotonic plastic zone (i.e. for $K_{max} > 0$).
Vasudevan et al. collected extensive raw (without crack closure correction) da/dN – ΔK data for a wide range of load ratios and plotted them in the ΔK – K\text{max} coordinates. Regardless of the test method, all of the data for a given FCG rate fell on a single curve, demonstrating that the behavior is unique for a particular material. The authors determined that an intrinsic threshold exists for each variable ΔK\text{th} and K\text{max}*, corresponding to a given da/dN. These thresholds increase with increasing FCG rate. For an FCG test at a constant load ratio, the intersection of both thresholds forms the so-called FCG trajectory, which corresponds to different crack growth mechanisms. For a specific material and environment, the FCG mechanism is controlled by K\text{max} at low load ratios and by ΔK at high FCG rates.\textsuperscript{9}

According to the UA, the load ratio R is not a driving force and does not have a threshold. Consequently, it cannot be used as a second parameter for the FCG. Figure 1 shows a schematic representation of the constant da/dN curves in the ΔK versus K\text{max} coordinates and their respective intrinsic thresholds. In this approach, there is no need for an extrinsic factor (e.g. crack closure). The constant da/dN curves fully describe the material behavior, and laboratory data can be directly used in design. Both ΔK and K\text{max} are LEFM parameters that, by definition, include the load level, geometry and crack size, thereby preserving the independence of the measured da/dN values of these parameters.\textsuperscript{10}

In terms of life prediction, the constant da/dN curves according to the UA can be described by a power law, of the form

\[
\frac{da}{dN} = A \cdot (ΔK – ΔK\text{th})^p \cdot (K\text{max} – K\text{max}^*)^q \tag{10}
\]

As in the Walker approach, the constants in Eq. (10) can be determined by a multiple linear regression (MLR) of the experimental data for various load ratios.

### NUMERICAL FITTING OF EXPERIMENTAL DATA

Linear regression is a powerful tool that is widely used by engineers to find the curve (or surface) that best fits experimental data. The process assumes that a linear relation exists between the variables. Linear forms of Eqs. (4), (8) and (10) can be obtained by taking the logarithms of both sides. For the Elber approach, this procedure transforms Eq. (4) into an expression for a straight line in the xy-plane:

\[
\log \frac{da}{dN} = mp \cdot \log ΔK\text{eff} + \log C
\]

For the UA approach (Eq. (10)), the result is

\[
\log \frac{da}{dN} = \log A + p \cdot \log (ΔK – ΔK\text{th}) + q \cdot \log (K\text{max} – K\text{max}^*)
\]

which represents the equation of a plane in a three-dimensional coordinate system, xyz:

\[
y = a_0 + a_1 \cdot x + a_2 \cdot z
\]

\[
y = \log \frac{da}{dN} = \log (ΔK – ΔK\text{th})z = \log (K\text{max} – K\text{max}^*)
\]

\[
a_0 = \log A \quad a_1 = p \quad a_2 = q
\]

The same procedure allows Walker’s parameters to be determined, as follows:

\[
y = a_0 + a_1 \cdot x + a_2 \cdot z
\]

\[
y = \log \frac{da}{dN} \quad x = \log (1 – R) \quad z = \log ΔK
\]

\[
a_0 = \log C_0 \quad a_1 = m_c(y – 1) \quad a_2 = m_c
\]

Although intuitive, the idea of including the mean stress corrections into the FCG equations and performing a single fitting procedure was only proposed recently.\textsuperscript{11} To determine the coefficients of the line and planes in Eqs. (11), (13) and (14) (and, consequently, the parameters for each approach), a complete set of experimental data (da/dN versus ΔK) at different load ratios is needed. Over 300 experimental data points (ΔK, da/dN) for the 2024-T3 aluminum alloy at positive load ratios from 0.1 to 0.8 were obtained from the literature.\textsuperscript{12} Material parameters corresponding to the Elber approach were obtained by calculating ΔK\text{eff} according to Eqs. (2) and (3), inserting ΔK\text{eff} into Eq. (4) and performing a simple linear regression analysis to fit a
straight line in log–log coordinates to the \((\Delta K_{eq} \text{ da/dN})\) data points, as shown in Fig. 2. The calculated material parameters \((C\text{ and } m_p)\) are listed in Table 1.

For the Walker and UAs, MLR analyses are needed. MLR is an easy and well-established process, the mathematical details of which can be found elsewhere.\(^{13}\) To accomplish this task, code was written in the MAPLE\textsuperscript{TM} software environment. The vectors \(x, y\) and \(z\) were initially obtained from experimental data, according to Eqs. (13) and (14). For the UA, the thresholds were \(\Delta K^*_{th} = 1.0 \text{ MPa m}^{1/2}\) and \(K^*_\text{max} = 3 \text{ MPa m}^{1/2}\), corresponding to the minimum values of the experimental data. These thresholds are in the expected range for aluminum alloys.\(^{14}\) Figure 3 shows, for both approaches, the plane that best fitted the set of points \((x_i, y_i, z_i)\), where \(i\) is between 1 and the length of the vectors \(x, y\) and \(z\), together with these points. The visual correlation was quite good, and the mathematical coefficient exceeded 99.99\% in both cases.

Coefficients of these planes allowed the parameters for each approach to be determined, in accordance with Eqs. (13) and (14). These results are summarized in Table 1.

Through the fitting parameter \(\gamma\), the Walker approach reduces the number of variables that describe the FCG. In addition to using Fig. 3, the success of applying Eq. (9) to eliminate the \(R\)-dependence can be visualized by whether the corrected experimental data align in a single scatter band around the relation \(\text{da/dN} = f(C_0 \times \Delta K_{eq}(R))\) (or \(\text{da/dN} = f(C(R) \times \Delta K)\)) in \(\text{da/dN}\) versus \(\Delta K_{eq}\) (or \(\Delta K\)) coordinates. Figure 4 shows the experimental data before and after the correction, along with the Paris-type relation \(\text{da/dN} = C_0 \Delta K_{eq}^{mp}\), which is a straight line in log–log coordinates. Good agreement was found for this set of experimental data.

The graphical success of the correlation in UA can also be visualized in \(K_{\text{max}}\) versus \(\Delta K\) coordinates, by plotting the raw experimental FCG data, along with some constant FCG rate curves (Eq. (10)) with parameters from Table 1 (see results in Fig. 5). As expected from the analysis of Fig. 3, good agreement was found between the experimental data and the fitted curve.

Fig. 2 Experimental fatigue crack growth curves (left) for the 2024-T3 aluminum alloy.\(^ {12}\) The correlation \(U(R)\) proposed by Schijve\(^ {7}\) (Eq. (3)) was used to reduce the dependence to only one variable \(\Delta K_{eq}\) (right) and works fine mainly in the mid regime.

### Table 1

<table>
<thead>
<tr>
<th>Approach</th>
<th>Coefficients</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_0)</td>
<td>(a_1)</td>
</tr>
<tr>
<td>Elber</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Walker</td>
<td>–10.97</td>
<td>–1.24</td>
</tr>
<tr>
<td>Unified approach</td>
<td>–9.47</td>
<td>1.97 = (p)</td>
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</table>
Once the parameters of both approaches have been defined, and considering that these parameters already include the load-ratio effects, the designer can calculate the residual life of any component, given a specific material and SIF for the particular geometry and type of loading. This issue is addressed in the succeeding text.

**LIFE PREDICTION**

Cracks growing from notches are among the most common situations encountered by designers. Here, the case of a pair of cracks emanating from a circular hole in a wide plate is used as a numerical example. The SIF-solution (Fig. 6) for this particular case can be found in the classical handbook of Tada et al.15 The solution includes modes I and III SIFs, but only mode I (loading normal to the crack surface) is analyzed in this paper. The $F_s(s)$ parameter has a pronounced variation for $0.2 < s < 0.8$. The residual life $N_f$ can be determined with Eq. (1) through numerical integration. Details of the numerical integration process are not described here but can be found in previously published papers. A code was developed with this aim.

In the numerical example, the circular hole had a radius $R_0 = 10$ mm (Fig. 6), and the initial crack size was
2.5 mm (or \( s = 0.2 \)). The residual life \( N_{if} \) was calculated in the range \( 0.2 < s < 0.8 \) (i.e. final crack size = 40 mm). The SIF range (in MPa m\(^{1/2}\)) varied in the interval 3.4 < \( \Delta K \) < 6.6, and the load ratio was maintained at \( R = 0.4 \). Three methods to handle the \( R \)-dependence in FCG were simulated. The material was assumed to be the same (2024-T3 aluminum alloy) for all methods, the FCG experimental data of which are shown in Fig. 4. Parameters from Table 1 were used in connection with each approach. Figure 7 shows the results in \( N_{if} \) versus \( s_f \)-coordinates, where \( s_f \) is the final value of the \( s \)-parameter for each calculation.

**DISCUSSION**

Two of the three methodologies used for the numerical examples – namely, the Walker and UAs – overcome the closure problem (which is controversial due to the inherent difficulties related with the measuring of the opening loads) and its derive \( U(R) \) relations. These methods are two-parameter approaches (\( \Delta K \) and \( K_{max} \) instead of \( \Delta K \) and \( R \)) and use surface fitting to find the adjustable constants. Once these constants are found for a given material, the similitude principle can be used to design any component or structure, given only the expression of \( \Delta K \) and for any \( R \). History effects are not considered because the simulations were made for a constant stress range and not for a real load-time history.

By virtue of the classical form of FCG curves, cracks should spend most of their lives in the near-threshold region of low \( da/dN \). In curves like those in Fig. 7, \( N_{if} \) is expected to be independent of the final crack size. However, the situation depicted in Fig. 7 is the result of the particular behavior of cracks emanating from notches. For small crack sizes, their SIF-solution approximately follows that of a surface crack in an infinite body, except for the use of the notch stress concentration factor for increasing the nominal gross stress. Once these cracks have grown far from the hole, the SIF-solution follows that of a single long crack in a wide plate, and falls well below that for the single-edge crack member. Only a small part of the total life is spent with the crack under the influence of the notch stress fields. In accordance with Fig. 7, the residual life of these types of cracks does not depend primarily on the initial crack size, a fact that facilitates the use of LEFM approaches for stress-life predictions.

A simulation made in a plate (width \( b = 40 \) mm, thickness \( t \)) with a single-edge crack and under a nominal stress \( S \) showed the most common trend (Fig. 7)—namely, the greater the final crack size was, the less influence it had on the residual life calculations. This trend appears because these types of cracks have a geometric factor that grows rapidly with the parameter \( s_f \). A final value \( s_f = 0.5 \) was considered sufficient for illustration purposes. This simulation was made under similar load ratio and material conditions as those used for the crack emanating from a hole, with the main difference being the initial (8 mm) and final (20 mm) crack sizes. The SIF varied from 3.6 to 11.8 MPa m\(^{1/2}\). Table 2 summarizes the loading conditions used for the FCG simulations in two geometries: geometry 1, a pair of cracks emanating from a notch/hole in a wide plate (Fig. 6); and geometry 2, a single-edge crack in a plate (Fig. 8).

A more formal analysis can be carried out to explain the differences between the (\( a \) versus \( N_{if} \)) curves for cracks departing from notches (Fig. 7) or growing from initially smooth surfaces (Fig. 8). First, a dimensionless SIF expression for both types of cracks can be obtained as follows:

![Fig. 6 Stress intensity factor for a pair of cracks emanating from a circular hole in a wide plate under a nominal stress S, and variation of the \( F_\lambda(s) \) parameter.](image)

![Fig. 7 Calculated residual life (Eq. (1)) versus the final value of the \( s \)-parameter for a crack growing between 2.5 and 40 mm from the circular hole of Fig. 6.](image)
Table 2 Loading conditions for the fatigue crack growth simulations in geometries 1 and 2, with constant load ratio ($R = 0.4$)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$S_{\text{max}}$, MPa</th>
<th>Initial SIF, $K_{\text{SIF}}$, MPa $m^{1/2}$</th>
<th>Final SIF, $K_{\text{SIF}}$, MPa $m^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>3.4</td>
<td>6.6</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>3.6</td>
<td>11.8</td>
</tr>
</tbody>
</table>

SIF, stress intensity factor.

Fig. 8 Residual life $N_f$ calculated for a single-edge crack geometry (with width $b$, thickness $t$, under nominal stress $S$). Stress intensity factor varied between 3.6 and 11.8 MPa $m^{1/2}$. The material and load ratio were the same as in the preceding text.

\[
K = S\sqrt{\pi aF(s)} = S\sqrt{\frac{R_0}{Ro}}F(s) \tag{15}
\]

\[
\frac{K}{S\sqrt{R_0}} = \sqrt{\frac{a}{R_0}}F(s)
\]

Geometric factors for both geometries can be expressed as functions of the dimensionless parameter $X = a/R_0$, where geometry 2 is also dependent on the ratio $Ro/b$:

\[
F(X) = \frac{3 - X/(X + 1)}{2}[1 + 1.243(1 - X/(X + 1))]^3
\]

\[
F(X, Ro/b) = 0.265(1 - X \cdot Ro/b)^2 + 0.857 + 0.265X \cdot Ro/b \left(1 - X \cdot Ro/b\right)^{1/2} \tag{16}
\]

Equations (15) and (16) are plotted in Fig. 9 for small and large $b$, including $Ro/b = 1/4$ (as used in this paper). The single-edge crack component has no hole, so the graphs should be interpreted only on a comparative basis. For small $Ro/b$, the crack driving force in single-edge crack components ($K_e$) asymptotically approximates to the SIF in cracked from notch/hole components ($K_n$) when the crack size is much larger than the hole radius. For large values of $a/R_0$, $K_e$ will exceed $K_n$, and the crack will grow quicker in the single-edge cracked component than in the cracked from notch/hole. The same outcome occurs for smaller values of $a/R_0$ (near 1.0) and for $Ro/b = 1/4$ but with a steeper curve.

The opposite trends in the slope of each ($a$ versus $N_f$) curve (Figs. 7 and 8) can now be easily understood. Because of the stress concentration and the relative proximity of the free surface in the single-edge crack component (high $Ro/b$), $K_e$ is greater than $K_n$ in the short crack regime, but $K(a)$ and, consequently, the FCG rates are always growing in the single-edge crack component. The opposite situation exists in the cracked from notch/hole component, where $K(a)$ and $da/dN$ both decrease. As a result, the pair of cracks emanating from the hole grows continuously but at a decreasing rate in the analyzed $s$-range (Fig. 7). Cracks growing from the surface spend most of their lives in the near-threshold (low $\Delta K$) region (Fig. 8). The relative position of the curves in Fig. 9 is purely geometric, as can be deduced from the generating equations (Eqs. (15) and (16)).

For the particular case of a wide plate of 2024-T3 aluminum alloy with a pair of growing cracks at the edge of a circular central hole, the residual lives predicted by the Walker and Elber approaches were always greater than the life predicted by the UA (Fig. 7), especially when the final crack size differed substantially from its initial value. At first sight, this is a non-conservative error that cannot be generalized unless experimental (or real component) data under the same conditions become available for comparison.

The relative positions of the Walker and Elber approaches in the curves of Figs. 7 and 8 can be anticipated by a simple analysis of their respective regression coefficients (Table 1). The slopes of Eqs. (4) and (9) ($mp$ and $mw$, respectively) are nearly the same (~3.9).
The ratio between the equivalent SIFs according to the Walker and Elber approaches for $R=0.4$ and $\gamma=0.68$ is constant and equal to 1.67. Therefore, the FCG rates by the Walker approach are approximately 80% of those calculated by the Elber approach $(1.06/9.74 \times (1.67)^{3.9} \approx 0.8)$. Life predictions based on closure are therefore, below those predicted by the Walker approach based on the two-parameter driving force concept.

An even higher rate was predicted by the UA in both simulations (Figs. 7 and 8). However, the thresholds for this approach were chosen as the minimum values among the available experimental data ($\Delta K_{\text{EF}} = 1.0 \text{ MPa m}^{1/2}$; $K_{\text{eff}} = 3 \text{ MPa m}^{1/2}$) and were not measured by specific tests. It is worth noting that, in spite of the fact that the Elber and Walker models overestimate the FCG rates in the low $\Delta K$ regime, their predicted lives are almost always above those predicted by the UA.

It can be argued that the $\Delta K_{\text{EF}}$ versus $N_{\text{th}}$ curves for both simulations (Figs. 7 and 8) may exhibit accumulated errors because of the integration process. The same trends can be found, however, when the predicted $da/dN$ versus $\sigma_f$ curves are plotted (Fig. 10). The number of cycles between two normalized crack sizes is the area under the inverse of these curves. It means that for showing the same behavior, the relative positions of each approach should be inverted in Figs. 7 and 8 in relation to Fig. 10. This inversion is more evident for geometry 1 than for geometry 2. In the latter case, the curves of the three approaches are crossing each other, and a visual evaluation of the areas under the curves is not so evident. The differences between the three approaches, however, become more pronounced at longer crack sizes, as in Fig. 8.

CONCLUSIONS

In this report, three methods that used to reduce the dependence of the FCG problem on pure LEFM parameters were numerically tested in the whole sense, namely, from the process of fitting parameters to the residual life calculations. All of the methods provided similar results, with the UA returning more conservative results. Designing against the FCG based only in LEFM parameters has some advantages: no consideration about closure is needed, and the load level, geometry and crack size are all included, thus facilitating the application of the similarity principle.

REFERENCES


