uff Universidade Federal Fluminense

EGM - Instituto de Matemática

GMA - Departamento de Matemática Aplicada

LISTA 3 - 2011-2

Integração por partes

Integrais de potências de

funções trigonométricas

Nos exercícios 1 a 18, calcule a integral indicada, utilizando a técnica de integração por partes.

1. $\int \operatorname{arcsen} x \, dx$

2. $\int x \, \operatorname{sen} x \, dx$

3. $\int x^2 \ln x \, dx$

4. $\int x^2 \cos x \, dx$

5. $\int x \arctan x \, dx$

6. $\int \sec^3 x \, dx$

7. $\int (\ln x)^3 \, dx$

8. $\int \sqrt{x} \, \ln x \, dx$

9. $\int x(\ln x)^2 \, dx$

10. $\int x \, 3^x \, dx$

11. $\int x \sec^2 x \, dx$

12. $\int \operatorname{sen}(\ln x) \, dx$

13. $\int \operatorname{arcsec} \sqrt{x} \, dx$

14. $\int \frac{\ln(\ln x)}{x} \, dx$

15. $\int \frac{\arctan \sqrt{x}}{\sqrt{x}} \, dx$

16. $\int \tan^2 x \sec^3 x \, dx$

17. $\int \csc^5 x \, dx$

18. $\int \operatorname{sen} 3x \cos 2x \, dx$

Nos exercícios 19 a 30 calcule a integral do produto ou quociente de potências de funções trigonométricas.

19. $\int \operatorname{sen}^4 x \, dx$

20. $\int \operatorname{sen}^4 x \cos^2 x \, dx$

21. $\int \operatorname{sen}^3 x \cos^2 x \, dx$

22. $\int \cos^5 x \, dx$

23. $\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \, \operatorname{sen}^3 x \, dx$

24. $\int \frac{\operatorname{sen}^3 x}{\cos^4 x} \, dx$

25. $\int_0^{\frac{1}{2}} \cos(\pi x) \cos\left(\frac{\pi x}{2}\right) \, dx$

26. $\int \tan^2 x \sec^3 x \, dx$

27. $\int \tan^5 x \sec^3 x \, dx$

28. $\int \tan^3 x \sqrt{\sec x} \, dx$

29. $\int \frac{\tan^5 x}{\sec^3 x} \, dx$

30. $\int \cot^4 x \, dx$

Lista de algumas fórmulas exponenciais e trigonométricas que eventualmente serão usadas nas resoluções de algumas integrais:

(I) $a^x = e^{x \ln a}, \ a \in \mathbb{R}, a > 0$

(II) $\operatorname{sen}^2 x + \cos^2 x = 1$ (III) $1 + \tan^2 x = \sec^2 x$ (IV) $1 + \cot^2 x = \csc^2 x$

(V) $\cos^2 x = \frac{1 + \cos 2x}{2}$ (VI) $\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$

(VII) $\operatorname{sen} a \cos b = \frac{\operatorname{sen}(a - b) + \operatorname{sen}(a + b)}{2}$ (VIII) $\operatorname{sen} a \operatorname{sen} b = \frac{\cos(a - b) - \cos(a + b)}{2}$

(IX) $\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$

RESPOSTAS DA LISTA 3

1. $x \arcsin x + \sqrt{1-x^2} + C$

2. $\operatorname{sen} x - x \cos x + C$

3. $\dfrac{1}{3} x^3 \ln x - \dfrac{1}{9} x^3 + C$

4. $x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C$

5. $\dfrac{1}{2} \left(x^2 \arctan x - x + \arctan x \right) + C$

6. $\dfrac{1}{2} \left(\sec x \tan x + \ln |\sec x + \tan x| \right) + C$

7. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$

8. $\dfrac{2}{3} x^{\frac{3}{2}} \ln x - \dfrac{4}{9} x^{\frac{3}{2}} + C$

9. $\dfrac{1}{2} x^2 (\ln x)^2 - \dfrac{1}{2} x^2 (\ln x) + \dfrac{1}{4} x^2 + C$

10. $\dfrac{1}{\ln 3} \; x \; 3^x - \dfrac{1}{(\ln 3)^2} \; 3^x + C$

11. $x \tan x - \ln |\sec x| + C$

12. $\dfrac{1}{2} x \left(\operatorname{sen}(\ln x) - \cos(\ln x) \right) + C$

13. $x \operatorname{arcsec} \sqrt{x} - (x-1)^{\frac{1}{2}} + C$

14. $\ln x \ln(\ln x) - \ln x + C$

15. $2\sqrt{x} \; \arctan(\sqrt{x}) - \ln(1+x) + C$

16. $\dfrac{1}{4} \sec^3 x \tan x - \dfrac{1}{8} \sec x \tan x -$
$\dfrac{1}{8} \ln |\sec x + \tan x| + C$

17. $-\dfrac{1}{4} \csc^3 x \cot x - \dfrac{3}{8} \cot x \csc x -$
$\dfrac{3}{8} \ln |\csc x - \cot x| + C$

18. $-\dfrac{1}{10} \cos 5x - \dfrac{1}{2} \cos x + C$

19. $\dfrac{3}{8} x + \dfrac{1}{32} \operatorname{sen} 4x - \dfrac{1}{4} \operatorname{sen} 2x + C$

20. $\dfrac{1}{16} x - \dfrac{1}{64} \operatorname{sen} 4x - \dfrac{1}{48} \operatorname{sen}^3 2x + C$

21. $-\dfrac{1}{5} \cos^3 x \operatorname{sen}^2 x - \dfrac{2}{15} \cos^3 x + C$

22. $\dfrac{1}{5} \operatorname{sen}^5 x - \dfrac{2}{3} \operatorname{sen}^3 x + \operatorname{sen} x + C$

23. $\dfrac{2}{7} \cos^{\frac{7}{2}} x - \dfrac{2}{3} \cos^{\frac{3}{2}} x \Big]_0^{\frac{\pi}{2}} = \dfrac{8}{21}$

24. $\dfrac{1}{3} \cos^{-3} x - \cos^{-1} x + C$

25. $\left(\dfrac{1}{3\pi} \operatorname{sen} \dfrac{3\pi x}{2} + \dfrac{1}{\pi} \operatorname{sen} \dfrac{\pi x}{2} \right) \Big]_0^{\frac{1}{2}} = \dfrac{2\sqrt{2}}{3\pi}$

26. $\dfrac{1}{4} \sec^3 x \tan x - \dfrac{1}{8} \sec x \tan x -$
$\dfrac{1}{8} \ln |\sec x + \tan x| + C$

27. $\dfrac{1}{7} \sec^7 x - \dfrac{2}{5} \sec^5 x + \dfrac{1}{3} \sec^3 x + C$

28. $\dfrac{2}{5} \sec^{\frac{5}{2}} x - 2 \sec^{\frac{1}{2}} x + C$

29. $\sec x + 2 \sec^{-1} x - \dfrac{1}{3} \sec^{-3} x + C$

30. $\cot x - \dfrac{1}{3} \cot^3 x + x + C$