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# A discontinuous finite element formulation for Helmholtz problems

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*Abstract: The Helmholtz equation is the linear mathematical model that describes time-harmonic acoustic, elastic and electromagnetic waves. The finite element method is often used to obtain numerical solutions of the Helmholtz problem. The oscillatory behavior of the exact solution and the quality of the approximate numerical solution depend on the wave number* $\boldsymbol{k}$*. The resolution of the mesh* $\boldsymbol{n_{res}}$ *should be adjusted to the wave number follows a "rule of the thumb"* $\boldsymbol{n_{res}=\lambda/h=2\pi/kh}$*, where* $\lambda$ *is the wave-length and* $\boldsymbol{h}$ *is the element diameter of the mesh [1].*

*The rule of thumb controls the interpolation error and for low waves number the approximate solution of the classic finite element method guarantee a reasonable results [2]. However, the performance of the Galerkin finite element method is deteriorated when* $\boldsymbol{k}$ *is increased. The finite element error grows with the wave number also on meshes where the rule of thumb is satisfied. Today, this is known as the pollution of the finite element solution. The errors in* $H^1$*-norm are bounded only if the mesh resolution is appropriately increased:* $\boldsymbol{kh<<1}$ *in the preasymptotic range [3] and* $\boldsymbol{hk^2<<1}$ *in the asymptotic range of convergence [4,5,6]. The pollution effect can only be avoided after a drastic refinement of the mesh. This obviously impede the numerical analyses of the Helmholtz equation by the finite element method in the mid and high frequency.*

*Numerous approaches are proposed to handle this misbehavior of the standard finite element formulation. In this work we presented a discontinuous finite element method for the Helmholtz equation, in which the standard finite element space is enriched by discontinuity of the shape functions across interelement boundaries [7]. The proposed formulation introduces two parameters* $\beta$ *and* $\lambda$ *that should be chosen for each problem. The* $\beta$ *and* $\lambda$ *functions are determined numericaly by solving a one dimension homogeneus Helmholtz equation with constant coefficient and Dirichlet boundary conditions [8]. The numerical results obtained indicate a good accuracy of the approximate solution of the discontinuous finite element method in one dimension. In this case, the error is controlled by the magnitude of* $\boldsymbol{kh}$*. The accurate of the approximate solution is maintained when the mesh is coarse. For the two dimensional case, a numerical study the dispersion properties demostrate the good performance of the discontiuous finite element method. The results presented through the paper are just being valid for linear (one-dimensional case) and bilinear (two-dimensional case) shape function. In the next work, results will be presented for the h-p version of the proposed discontinuous finite element method.*

*Keywords: Discontinuous Galerkin, Helmholtz Equation, Stabilization, Discontinuous FEM*

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