	UFF – Universidade Federal Fluminense **Escola de Engenharia Industrial e Metalúrgica de Volta Redonda** **Disciplina: Cálculo I** **Prof. Gustavo Benitez Alvarez** **Nome do Aluno (letra forma): _________________________** **Assinatura do Aluno: ______________________________** **Prova Escrita Nº 2 Turma V1 01/2008**

Observações:

- **Desligue os aparelhos celulares;**
- **Não rasure esta folha, pois cálculos realizados nesta, não serão considerados. Use a folha de Respostas;**
- **Não existem dúvidas a serem esclarecidas. A interpretação de cada questão faz parte da Avaliação;**
- **Provas respondidas à lápis não terão direito a recorreção. Logo, faça a prova com caneta azul ou preta;**
- **Não é permitido compartilhar materiais didáticos;**
- **É permitido o uso de calculadoras científicas;**

Questão 1: (Valor 2,5) Encontre as primitivas da função $f(x) = (x+2)^2 e^{2x}$. Isto é, calcule $\int (x+2)^2 e^{2x}\, dx$.

Questão 2: (Valor 2,5) Determine $\int_3^4 \frac{x+1}{x^3 - 4x^2 + 4x} dx$.

Questão 3: (Valor 2,5) Calcule o volume do corpo de revolução gerado pela rotação da curva hiperbólica $y = sh(x) = \frac{e^x - e^{-x}}{2}$ no intervalo $x \in [\ln(1), \ln(2)]$ ao redor do eixo OX.

Questão 4: (Valor 2,5) Determine a área da figura plana limitada pelas curvas $y = 2x - x^2$ e $y = x^2 - 6x + 6$.

① $\int (x+2)^2 e^{2x}\,dx$ | mudança de variavel $z = x+2 \Rightarrow x = z-2$, $dz = dx$

$= \int z^2 e^{2(z-2)}\,dz = \int z^2 e^{(2z-4)}\,dz = \int z^2 e^{2z}\cdot e^{-4}\,dz = e^{-4}\int z^2 e^{2z}\,dz$

$= \frac{1}{e^4}\int z^2 e^{2z}\,dz$ | Integração por parte $u = z^2 \Rightarrow du = 2z\,dz$; $dv = e^{2z}dz \Rightarrow v = \int e^{2z}dz$; $v = \int \frac{1}{2}e^{2z}\,d(2z) = \frac{1}{2}e^{2z}$.

$= \frac{1}{e^4}\left[\frac{1}{2}z^2e^{2z} - \int \frac{1}{2}e^{2z}\,2z\,dz\right] =$

$= \frac{1}{e^4}\left[\frac{1}{2}z^2e^{2z} - \int z e^{2z}\,dz\right]$ | Integração por parte $u = z \Rightarrow du = dz$; $dv = e^{2z}dz \Rightarrow v = \frac{1}{2}e^{2z}$

$= \frac{1}{e^4}\left[\frac{1}{2}z^2e^{2z} - \left(\frac{1}{2}z e^{2z} - \int \frac{1}{2}e^{2z}\,dz\right)\right] =$

$= \frac{1}{e^4}\left[\frac{1}{2}z^2e^{2z} - \frac{1}{2}z e^{2z} + \frac{1}{2}\int e^{2z}\,dz\right] =$

$= \frac{1}{e^4}\left[\frac{1}{2}z^2e^{2z} - \frac{1}{2}z e^{2z} + \frac{1}{2}\cdot\frac{1}{2}e^{2z}\right] + C$

$= \frac{1}{2}e^{(2z-4)}\left[z^2 - z + \frac{1}{2}\right] + C = \frac{1}{2}e^{[2(x+2)-4]}\left[(x+2)^2 - (x+2) + \frac{1}{2}\right] + C$

$= \frac{1}{2}e^{2x}\left[x^2+4x+4 - x - 2 + \frac{1}{2}\right] + C = \frac{1}{2}e^{2x}\left[x^2+3x+\frac{5}{2}\right] + C.$

2) $\frac{P(x)}{Q(x)} = \frac{x+1}{x^3-4x^2+4x} = \frac{x+1}{x(x^2-4x+4)} = \frac{x+1}{x(x-2)^2}$, ou seja, o polinômio $Q(x)$ tem como raizes $x=0$ e $x=2$, sendo que $x=2$ tem multiplicidade 2. Transformando em frações simples temos:

$$\frac{P(x)}{Q(x)} = \frac{x+1}{x(x-2)^2} = \frac{A}{x} + \frac{B_1}{(x-2)} + \frac{B_2}{(x-2)^2} = \frac{A(x-2)^2 + B_1 x(x-2) + B_2 x}{x(x-2)^2}$$

$$x+1 = A(x-2)^2 + B_1 x(x-2) + B_2 x$$ ⊛ atribuindo valores obtemos os coeficientes indeterminados

se $x=0 \Rightarrow 1 = A(-2)^2 \Rightarrow 1 = A4 \Rightarrow A = \frac{1}{4}$

se $x=2 \Rightarrow 3 = B_2 \cdot 2 \Rightarrow B_2 = \frac{3}{2}$

se $x=1 \Rightarrow 2 = A(-1)^2 + B_1(-1) + B_2 \Rightarrow 2 = A - B_1 + B_2$

$B_1 = A + B_2 - 2 = \frac{1}{4} + \frac{3}{2} - 2 = \frac{1+6-8}{4} = -\frac{1}{4}$ logo

$$\frac{x+1}{x(x-2)^2} = \frac{1}{4}\frac{1}{x} - \frac{1}{4}\frac{1}{(x-2)} + \frac{3}{2}\frac{1}{(x-2)^2}$$ e a integral fica

$$\int_3^4 \frac{x+1}{x^3-4x^2+4x}\,dx = \int_3^4 \left[\frac{1}{4}\frac{1}{x} - \frac{1}{4}\frac{1}{(x-2)} + \frac{3}{2}\frac{1}{(x-2)^2}\right] dx =$$

$$= \frac{1}{4}\underbrace{\int_3^4 \frac{1}{x}\,dx}_{\mathrm{I}} - \frac{1}{4}\underbrace{\int_3^4 \frac{1}{(x-2)}\,dx}_{\mathrm{II}} + \frac{3}{2}\underbrace{\int_3^4 \frac{1}{(x-2)^2}\,dx}_{\mathrm{III}} = \frac{1}{4}\ln\left(\frac{4}{3}\right) - \frac{1}{4}\ln 2 + \frac{3}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{4}\ln\left(\frac{2}{3}\right) + \frac{3}{4} = \frac{1}{4}\left[\ln\left(\frac{2}{3}\right) + 3\right]$$

$$\mathrm{II} = \int_3^4 \frac{dx}{(x-2)} = \int_3^4 \frac{d(x-2)}{(x-2)} = \ln(x-2)\Big|_3^4 = \ln(4-2) - \ln(3-2)$$

$$= \ln 2 - \ln 1 = \ln 2$$

$$\mathrm{I} = \int_3^4 \frac{dx}{x} = \ln x \Big|_3^4 = \ln 4 - \ln 3 = \ln\left(\frac{4}{3}\right)$$

$$\mathrm{III} = \int_3^4 \frac{dx}{(x-2)^2} = \int_3^4 \frac{d(x-2)}{(x-2)^2} = \int_3 (x-2)^{-2}\,d(x-2) = \frac{(x-2)^{-2+1}}{-2+1}\Big|_3^4 =$$

$$= -\frac{1}{(x-2)}\Big|_3^4 = -\left[\frac{1}{(4-2)} - \frac{1}{(3-2)}\right] = -\left[\frac{1}{2} - 1\right] = \frac{1}{2}$$

(3) $V_x = \pi \int_a^b f^2(x)\, dx$ $\qquad y = sh(x) = \dfrac{e^x - e^{-x}}{2}$ $\qquad x \in [\ln(1), \ln(2)]$ (2)

$$V_x = \pi \int_{\ln(1)}^{\ln(2)} \left(\frac{e^x - e^{-x}}{2}\right)^2 dx = \pi \int_{\ln(1)}^{\ln(2)} \frac{1}{4}\left[e^{2x} - 2e^x e^{-x} + e^{-2x}\right] dx$$

$$= \frac{\pi}{4}\left[\int_{\ln(1)}^{\ln(2)} e^{2x}\, dx - 2\int_{\ln(1)}^{\ln(2)} dx + \int_{\ln(1)}^{\ln(2)} e^{-2x}\, dx\right] =$$

$$= \frac{\pi}{4}\left(\int_{\ln(1)}^{\ln(2)} \frac{1}{2} e^{2x}\, d(2x) - 2\, x\Big|_{\ln(1)}^{\ln(2)} + \int_{\ln(1)}^{\ln(2)} -\frac{1}{2} e^{-2x}\, d(-2x)\right) =$$

$$= \frac{\pi}{4}\left[\frac{1}{2} e^{2x}\Big|_{\ln(1)}^{\ln(2)} - 2\left(\ln(2) - \ln(1)\right) - \frac{1}{2} e^{-2x}\Big|_{\ln(1)}^{\ln(2)}\right] =$$

$$= \frac{\pi}{4}\left[\frac{1}{2}\left(e^{2\ln(2)} - e^{2\ln(1)}\right) - 2\ln(2) - \frac{1}{2}\left(e^{-2\ln(2)} - e^{-2\ln(1)}\right)\right] =$$

$$= \frac{\pi}{4}\left[\frac{1}{2}\left(2^2 - 1^2\right) - \ln 2^2 - \frac{1}{2}\left(2^{-2} - 1^{-2}\right)\right] =$$

$$= \frac{\pi}{4}\left[\frac{3}{2} - \ln 4 - \frac{1}{2}\left(-\frac{3}{4}\right)\right] = \frac{\pi}{4}\left[\frac{15}{8} - \ln 4\right].$$

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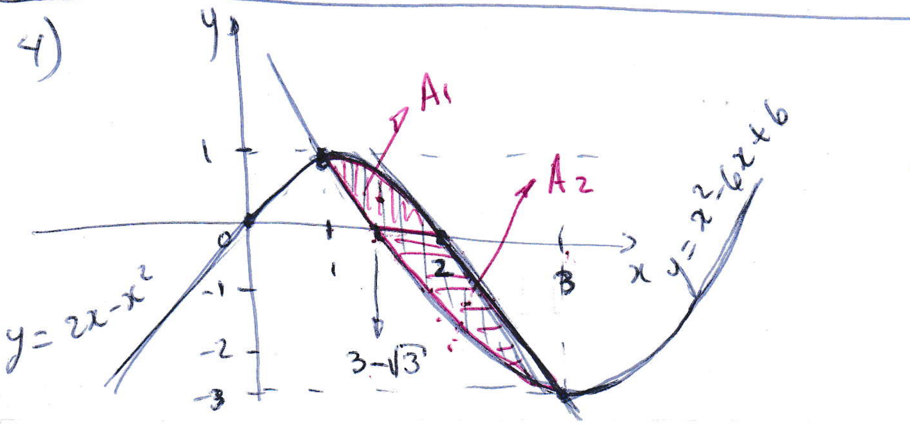
4)



$2x - x^2 = x^2 - 6x + 6$

$2x^2 - 8x + 6 = 0$

$x^2 - 4x + 3 = 0$

$x = 1$ e $x = 3$

$$A_1 = \int_1^2 (2x - x^2)\,dx - \int_1^{3-\sqrt{3}} (x^2-6x+6)\,dx$$

$$= \left(2\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_1^2 - \left(\frac{x^3}{3} - 6\frac{x^2}{2} + 6x\right)\Big|_1^{3-\sqrt3} = \left(x^2 - \frac{x^3}{3}\right)\Big|_1^2 - \left(\frac{x^3}{3} - 3x^2 + 6x\right)\Big|_1^{3-\sqrt3}$$

$$= \left[\left(2^2 - \frac{2^3}{3}\right) - \left(1^2 - \frac{1^3}{3}\right)\right] - \left[\left(\frac{(3-\sqrt3)^3}{3} - 3(3-\sqrt3)^2 + 6(3-\sqrt3)\right) - \left(\frac{1^3}{3} - 3\cdot 1^2 + 6\right)\right] =$$

$$= \left[\left(4-\frac{8}{3}\right) - \left(1-\frac13\right)\right] - \left[\left(\frac{(3-\sqrt3)^3}{3} - 3(3-\sqrt3)^2 + 6(3-\sqrt3)\right) - \left(\frac13+3\right)\right] =$$

$$= \left[\frac43 - \frac23\right] - \left[\frac{(3-\sqrt3)^3 - 9(3-\sqrt3)^2 + 18(3-\sqrt3)}{3} - \frac{10}{3}\right]$$

$$= \frac{12 - (3-\sqrt3)^3 + 9(3-\sqrt3)^2 - 18(3-\sqrt3)}{3}$$

$$\boxed{(3-\sqrt3)^2 = 9 - 6\sqrt3 + 3 = 3(3-2\sqrt3+1)}$$

$$= \frac{12 - 3(3-2\sqrt3+1)(3-\sqrt3) + 9\cdot 3(3-2\sqrt3+1) - 18(3-\sqrt3)}{3}$$

$$= 4 - (3-2\sqrt3+1)(3-\sqrt3) + 9(3-2\sqrt3+1) - 6(3-\sqrt3) = 40 - 22\sqrt3 \geq 0.$$

$$A_2 = \int_{3-\sqrt3}^3 -(x^2-6x+6)\,dx - \int_2^3 -(2x-x^2)\,dx = \left(2\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_2^3 - \left(\frac{x^3}{3} - 6\frac{x^2}{2}+6x\right)\Big|_{3-\sqrt3}^3 =$$

$$= \left(\frac{3x^2 - x^3}{3}\right)\Big|_2^3 - \left(\frac{x^3-9x^2+18x}{3}\right)\Big|_{3-\sqrt3}^3$$

$$= \left[\left(\cancel{\frac{3\cdot3^2-3^3}{3}}^{\,0}\right) - \left(\frac{3\cdot2^2-2^3}{3}\right)\right] - \left[\left(\cancel{\frac{3^3-9\cdot3^2+18\cdot3}{3}}^{\,0}\right) - \left(\frac{(3-\sqrt3)^3 - 9(3-\sqrt3)^2+18(3-\sqrt3)}{3}\right)\right] =$$

$$= -\frac43 + \frac{3(3-2\sqrt3+1)(3-\sqrt3) - 9\cdot 3(3-2\sqrt3+1)+18(3-\sqrt3)}{3}$$

$= \frac13\left[6\sqrt3 - 4\right] \geq 0.$ Portanto a área é a soma:

$$A = A_1 + A_2 = (40-22\sqrt3) + \left(\frac{6\sqrt3-4}{3}\right) = \frac{116}{3} - 20\sqrt3$$