	UFF – Universidade Federal Fluminense **Escola de Engenharia Industrial e Metalúrgica de Volta Redonda** **Disciplina: Cálculo I** **Prof. Gustavo Benitez Alvarez** **Nome do Aluno (letra forma): ____________________** **Assinatura do Aluno: ____________________** **Prova Escrita Nº 2 Turma V3 01/2010**

Observações:

- **Desligue os aparelhos celulares;**
- **Não rasure esta folha, pois cálculos realizados nesta, não serão considerados. Use a folha de Respostas;**
- **Não existem dúvidas a serem esclarecidas. A interpretação faz parte da Avaliação;**
- **Faça a prova com caneta azul ou preta. Respostas à lápis não terão direito a recorreção;**
- **Não é permitido compartilhar materiais didáticos;**
- **É permitido o uso de calculadoras científicas;**
- ***Seja o mais explicito possível para responder as questões;***

Questão 1: (Valor 2,0) Encontre as primitivas da função $f(x)=x\sqrt{x-1}$. Isto é, calcule $\int x\sqrt{x-1}\,dx$.

Questão 2: (Valor 2,0) Determine $\int_0^1 \sqrt{e^x+1}\,dx$.

Questão 3: (Valor 2,0) Calcule o volume do corpo de revolução gerado pela rotação da curva trigonométrica $y=\cos(3x-\frac{\pi}{2})$ no intervalo $x\in[0,\frac{\pi}{3}]$ ao redor do eixo OX.

Questão 4: (Valor 2,0) Calcule a área da figura plana limitada pelas curvas $f(x)=x^3-5x$ e $g(x)=-x$.

Questão 5: (Valor 2,0) Determine o comprimento da ciclóide dada em forma paramétrica por $x(t)=a(t-sen(t))$, $y(t)=a(1-\cos(t))$, onde $a>0$.

Fórmulas: $L=\int_\alpha^\beta\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$; $L=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$; $V_x=\pi\int_a^b[f(x)]^2\,dx$;

$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C,\quad a\neq 0$; $\int\frac{dx}{x^2+a^2}=\frac{1}{a}arctg\left(\frac{x}{a}\right)+C,\quad a\neq 0$;

$\int\frac{dx}{\cos(x)}=\ln|\sec(x)+tg(x)|+C,\quad a\neq 0$; $\frac{1}{\cos(x)}=\sec(x)$

Prova No 2 V3 1/2010

Q1) $\int x\sqrt{x-1}\,dx$

mudança de variáveis

$t=\sqrt{x-1} \Rightarrow x=t^2+1$

$dx=2t\,dt$

$$= \int (t^2+1)t\cdot 2t\,dt$$

$$= 2\int (t^4+t^2)dt = 2\left\{\int t^4 dt + \int t^2 dt\right\} = 2\left\{\frac{t^5}{5}+\frac{t^3}{3}\right\}+C$$

$$= 2\left\{\frac{(x-1)^{\frac{5}{2}}}{5}+\frac{(x-1)^{\frac{3}{2}}}{3}\right\}+C$$

Q2) $\int_0^1 \sqrt{e^x+1}\,dx$

mudança de variáveis.

$x=\ln(u^2-1)$

$$\sqrt{e^x+1} = \sqrt{e^{\ln(u^2-1)}+1} = \sqrt{u^2-1+1} = \sqrt{u^2} = |u|$$

$u^2=e^x+1$ e $x\in[0,1] \Rightarrow u\in[\sqrt{2},\sqrt{e+1}]$.

$dx = \frac{dx}{du}du = \frac{1}{u^2-1}\cdot 2u\,du$. Logo, segue

$$\int_0^1 \sqrt{e^x+1}\,dx = \int_{\sqrt{2}}^{\sqrt{e+1}} u\cdot\frac{2u}{u^2-1}du = \int_{\sqrt{2}}^{\sqrt{e+1}} \frac{2u^2}{u^2-1}du = 2\int_{\sqrt{2}}^{\sqrt{e+1}} \frac{(u^2+1-1)}{u^2-1}du$$

$$= 2\left\{\int_{\sqrt{2}}^{\sqrt{e+1}}\left[\frac{u^2-1}{u^2-1}+\frac{1}{u^2-1}\right]du\right\} = 2\left\{\int_{\sqrt{2}}^{\sqrt{e+1}} du + \int_{\sqrt{2}}^{\sqrt{e+1}}\frac{du}{u^2-1}\right\} =$$

$$= 2\left\{u\Big|_{\sqrt{2}}^{\sqrt{e+1}} + \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\Big|_{\sqrt{2}}^{\sqrt{e+1}}\right\}$$

$$= 2\left\{(\sqrt{e+1}-\sqrt{2}) + \frac{1}{2}\left[\ln\left(\frac{\sqrt{e+1}-1}{\sqrt{e+1}+1}\right) - \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]\right\}$$

$$= 2\left\{(\sqrt{e+1}-\sqrt{2}) + \frac{1}{2}\left[\ln\left(\frac{\sqrt{e+1}-1}{\sqrt{e+1}+1}\right) - \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]\right\}.$$

43) $V_x = \pi \int_0^{\frac{\pi}{3}} \cos^2\left(3x - \frac{\pi}{2}\right) dx$

$\cos\left(3x - \frac{\pi}{2}\right) = \cos(3x)\cancelto{0}{\cos\left(\frac{\pi}{2}\right)} + \text{sen}(3x)\cancelto{1}{\text{sen}\left(\frac{\pi}{2}\right)} = +\,\text{sen}(3x)$

$V_x = \pi \int_0^{\frac{\pi}{3}} \left[-\text{sen}(3x)\right]^2 dx = \pi \int_0^{\frac{\pi}{3}} \text{sen}^2(3x)\, dx$

$\text{sen}^2 x = \frac{1}{2}\left[1 - \cos(2x)\right] \Rightarrow \text{sen}^2(3x) = \frac{1}{2}\left[1 - \cos(2 \cdot 3x)\right]$

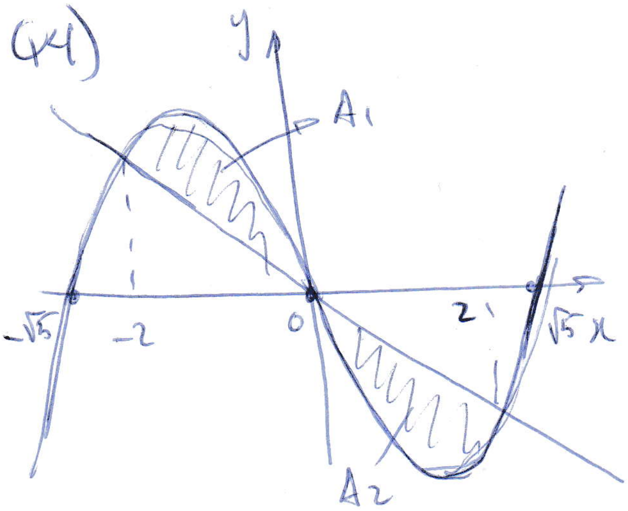
$V_x = \pi \int_0^{\frac{\pi}{3}} \frac{1}{2}\left[1 - \cos(6x)\right] dx = \frac{\pi}{2} \int_0^{\frac{\pi}{3}} \left[1 - \cos(6x)\right] dx$

$= \frac{\pi}{2} \left\{ x \Big|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \cos(6x) \frac{6}{6} dx \right\} = \frac{\pi}{2} \left\{ \left(\frac{\pi}{3} - 0\right) - \frac{1}{6}\, \text{sen}(6x) \Big|_0^{\frac{\pi}{3}} \right\}$

$= \frac{\pi}{2} \left\{ \frac{\pi}{3} - \frac{1}{6}\left[\cancelto{0}{\text{sen}\left(6 \cdot \frac{\pi}{3}\right)} - \cancelto{0}{\text{sen}(6 \cdot 0)}\right] \right\} = \frac{\pi^2}{6}$

---

44)



$x^3 - 5x = -x \Rightarrow x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x_1 = 0 \quad x_2 = 2\,, \quad x_3 = -2$

$A_1 = \int_{-2}^{0} \left[(x^3 - 5x) - (-x)\right] dx$

$= \int_{-2}^{0} (x^3 - 4x)\, dx = \left(\frac{x^4}{4} - \frac{4x^2}{2}\right)\Big|_{-2}^{0} = 0 - \left[\frac{(-2)^4}{4} - 4\frac{(-2)^2}{2}\right]$

$= -\left[4 - 8\right] = 4$

$$A_2 = \int_0^2 \left| (x^3-5x) - (-x) \right| dx = \int_0^2 \left[-(x^3-5x) - (x) \right] dx$$

$$= \int_0^2 (-x^3+4x)\, dx = \left(-\frac{x^4}{4} + 4\frac{x^2}{2} \right) \Big|_0^2$$

$$= \left[-\frac{2^4}{4} + 4 \cdot \frac{2^2}{2} \right] - \left[0 \right] = \left[-4+8 \right] = 4$$

Logo, $A = A_1 + A_2 = 4+4 = 8.$

---

(45) $x(t) = a(t - \operatorname{sen} t) \Rightarrow \frac{dx}{dt} = a(1-\cos t)$ $\quad t \in [0, 2\pi]$

$y(t) = a(1-\cos t) \rightarrow \frac{dy}{dt} = a(\operatorname{sen} t)$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{a^2(1-\cos t)^2 + a^2 \operatorname{sen}^2 t}$$

$$= \sqrt{a^2\left[1 - 2\cos t + \cos^2 t + \operatorname{sen}^2 t\right]}$$

$$= \sqrt{a^2\left[2 - 2\cos t\right]} = \sqrt{2a^2(1-\cos t)}$$

$$= \sqrt{2}\,|a|\sqrt{1-\cos t} = \sqrt{2}\, a \sqrt{2\operatorname{sen}^2\left(\frac{t}{2}\right)}$$

$\operatorname{sen}^2 t = \frac{1}{2}\left[1 - \cos 2t\right] \Rightarrow \operatorname{sen}^2\left(\frac{t}{2}\right) = \frac{1}{2}\left[1 - \cos t\right]$. Logo.

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{2\pi} 2a \left| \operatorname{sen}\left(\frac{t}{2}\right) \right| dt$$

$u = \frac{t}{2}$
$du = \frac{1}{2} dt$
$u \in [0, \pi]$.

$$= 2a \int_0^{\pi} |\operatorname{sen} u|\, 2\, du = 4a \int_0^{\pi} \operatorname{sen} u\, du$$

$$= 4a(-\cos u)\Big|_0^{\pi} = -4a\left[\underbrace{\cos\pi}_{-1} - \underbrace{\cos 0}_{1}\right] = -4a(-2)$$

$$= 8a.$$