

uff Universidade Federal Fluminense
 EGM - Instituto de Matemática
 GMA - Departamento de Matemática Aplicada

LISTA 3 - 2018-1
 Integração por partes
 Integrais de potências de
 funções trigonométricas

Nos exercícios 1 a 18, calcule a integral indicada, utilizando a técnica de integração por partes.

- | | | |
|---|--|--|
| 1. $\int \arcsen x \, dx$ | 7. $\int (\ln x)^3 \, dx$ | 13. $\int \operatorname{arcsec} \sqrt{x} \, dx$ |
| 2. $\int x \, \operatorname{sen} x \, dx$ | 8. $\int \sqrt{x} \, \ln x \, dx$ | 14. $\int \frac{\ln(\ln x)}{x} \, dx$ |
| 3. $\int x^2 \ln x \, dx$ | 9. $\int x(\ln x)^2 \, dx$ | 15. $\int \frac{\arctan \sqrt{x}}{\sqrt{x}} \, dx$ |
| 4. $\int x^2 \cos x \, dx$ | 10. $\int x \, 3^x \, dx$ | 16. $\int \tan^2 x \sec^3 x \, dx$ |
| 5. $\int x \arctan x \, dx$ | 11. $\int x \sec^2 x \, dx$ | 17. $\int \csc^5 x \, dx$ |
| 6. $\int \sec^3 x \, dx$ | 12. $\int \operatorname{sen}(\ln x) \, dx$ | 18. $\int \operatorname{sen} 3x \cos 2x \, dx$ |

Nos exercícios 19 a 30 calcule a integral do produto ou quociente de potências de funções trigonométricas.

- | | | |
|--|---|--|
| 19. $\int \operatorname{sen}^4 x \, dx$ | 23. $\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \operatorname{sen}^3 x \, dx$ | 27. $\int \tan^5 x \sec^3 x \, dx$ |
| 20. $\int \operatorname{sen}^4 x \cos^2 x \, dx$ | 24. $\int \frac{\operatorname{sen}^3 x}{\cos^4 x} \, dx$ | 28. $\int \tan^3 x \sqrt{\sec x} \, dx$ |
| 21. $\int \operatorname{sen}^3 x \cos^2 x \, dx$ | 25. $\int_0^{\frac{1}{2}} \cos(\pi x) \cos\left(\frac{\pi x}{2}\right) \, dx$ | 29. $\int \frac{\tan^5 x}{\sec^3 x} \, dx$ |
| 22. $\int \cos^5 x \, dx$ | 26. $\int \tan^2 x \sec^3 x \, dx$ | 30. $\int \cot^4 x \, dx$ |

Lista de algumas fórmulas exponenciais e trigonométricas que eventualmente serão usadas nas resoluções de algumas integrais:

(I) $a^x = e^{x \ln a}$, $a \in \mathbb{R}$, $a > 0$

(II) $\operatorname{sen}^2 x + \cos^2 x = 1$ (III) $1 + \tan^2 x = \sec^2 x$ (IV) $1 + \cot^2 x = \csc^2 x$

(V) $\cos^2 x = \frac{1 + \cos 2x}{2}$ (VI) $\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$

(VII) $\operatorname{sen} a \cos b = \frac{\operatorname{sen}(a - b) + \operatorname{sen}(a + b)}{2}$ (VIII) $\operatorname{sen} a \operatorname{sen} b = \frac{\cos(a - b) - \cos(a + b)}{2}$

(IX) $\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$

RESPOSTAS DA LISTA 3

1. $x \arcsen x + \sqrt{1 - x^2} + C$
2. $\sen x - x \cos x + C$
3. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$
4. $x^2 \sen x + 2x \cos x - 2 \sen x + C$
5. $\frac{1}{2} (x^2 \arctan x - x + \arctan x) + C$
6. $\frac{1}{2} (\sec x \tan x + \ln |\sec x + \tan x|) + C$
7. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$
8. $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{4}{9}x^{\frac{3}{2}} + C$
9. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2(\ln x) + \frac{1}{4}x^2 + C$
10. $\frac{1}{\ln 3} x 3^x - \frac{1}{(\ln 3)^2} 3^x + C$
11. $x \tan x - \ln |\sec x| + C$
12. $\frac{1}{2}x (\sen(\ln x) - \cos(\ln x)) + C$
13. $x \operatorname{arcsec} \sqrt{x} - (x - 1)^{\frac{1}{2}} + C$
14. $\ln x \ln(\ln x) - \ln x + C$
15. $2\sqrt{x} \arctan(\sqrt{x}) - \ln(1 + x) + C$
16. $\frac{1}{4} \sec^3 x \tan x - \frac{1}{8} \sec x \tan x - \frac{1}{8} \ln |\sec x + \tan x| + C$
17. $-\frac{1}{4} \csc^3 x \cot x - \frac{3}{8} \cot x \csc x - \frac{3}{8} \ln |\csc x - \cot x| + C$
18. $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$
19. $\frac{3}{8}x + \frac{1}{32} \sen 4x - \frac{1}{4} \sen 2x + C$
20. $\frac{1}{16}x - \frac{1}{64} \sen 4x - \frac{1}{48} \sen^3 2x + C$
21. $-\frac{1}{5} \cos^3 x \sen^2 x - \frac{2}{15} \cos^3 x + C$
22. $\frac{1}{5} \sen^5 x - \frac{2}{3} \sen^3 x + \sen x + C$
23. $\left. \frac{2}{7} \cos^{\frac{7}{2}} x - \frac{2}{3} \cos^{\frac{3}{2}} x \right]_0^{\frac{\pi}{2}} = \frac{8}{21}$
24. $\frac{1}{3} \cos^{-3} x - \cos^{-1} x + C$
25. $\left. \left(\frac{1}{3\pi} \sen \frac{3\pi x}{2} + \frac{1}{\pi} \sen \frac{\pi x}{2} \right) \right]_0^{\frac{1}{2}} = \frac{2\sqrt{2}}{3\pi}$
26. $\frac{1}{4} \sec^3 x \tan x - \frac{1}{8} \sec x \tan x - \frac{1}{8} \ln |\sec x + \tan x| + C$
27. $\frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$
28. $\frac{2}{5} \sec^{\frac{5}{2}} x - 2 \sec^{\frac{1}{2}} x + C$
29. $\sec x + 2 \sec^{-1} x - \frac{1}{3} \sec^{-3} x + C$
30. $\cot x - \frac{1}{3} \cot^3 x + x + C$