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**LISTA 3 - 2012-1**  
 Integração por partes  
 Integrais de potências de  
 funções trigonométricas

Nos exercícios 1 a 18, calcule a integral indicada, utilizando a técnica de integração por partes.

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|---|--|--|
| 1. $\int \arcsen x \, dx$                 | 7. $\int (\ln x)^3 \, dx$                  | 13. $\int \operatorname{arcsec} \sqrt{x} \, dx$    |
| 2. $\int x \, \operatorname{sen} x \, dx$ | 8. $\int \sqrt{x} \, \ln x \, dx$          | 14. $\int \frac{\ln(\ln x)}{x} \, dx$              |
| 3. $\int x^2 \ln x \, dx$                 | 9. $\int x(\ln x)^2 \, dx$                 | 15. $\int \frac{\arctan \sqrt{x}}{\sqrt{x}} \, dx$ |
| 4. $\int x^2 \cos x \, dx$                | 10. $\int x \, 3^x \, dx$                  | 16. $\int \tan^2 x \sec^3 x \, dx$                 |
| 5. $\int x \arctan x \, dx$               | 11. $\int x \sec^2 x \, dx$                | 17. $\int \csc^5 x \, dx$                          |
| 6. $\int \sec^3 x \, dx$                  | 12. $\int \operatorname{sen}(\ln x) \, dx$ | 18. $\int \operatorname{sen} 3x \cos 2x \, dx$     |

Nos exercícios 19 a 30 calcule a integral do produto ou quociente de potências de funções trigonométricas.

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|--|---|--|
| 19. $\int \operatorname{sen}^4 x \, dx$          | 23. $\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \operatorname{sen}^3 x \, dx$       | 27. $\int \tan^5 x \sec^3 x \, dx$         |
| 20. $\int \operatorname{sen}^4 x \cos^2 x \, dx$ | 24. $\int \frac{\operatorname{sen}^3 x}{\cos^4 x} \, dx$                      | 28. $\int \tan^3 x \sqrt{\sec x} \, dx$    |
| 21. $\int \operatorname{sen}^3 x \cos^2 x \, dx$ | 25. $\int_0^{\frac{1}{2}} \cos(\pi x) \cos\left(\frac{\pi x}{2}\right) \, dx$ | 29. $\int \frac{\tan^5 x}{\sec^3 x} \, dx$ |
| 22. $\int \cos^5 x \, dx$                        | 26. $\int \tan^2 x \sec^3 x \, dx$  | 30. $\int \cot^4 x \, dx$                  |

Lista de algumas fórmulas exponenciais e trigonométricas que eventualmente serão usadas nas resoluções de algumas integrais:

(I)  $a^x = e^{x \ln a}$ ,  $a \in \mathbb{R}$ ,  $a > 0$

(II)  $\operatorname{sen}^2 x + \cos^2 x = 1$       (III)  $1 + \tan^2 x = \sec^2 x$       (IV)  $1 + \cot^2 x = \csc^2 x$

(V)  $\cos^2 x = \frac{1 + \cos 2x}{2}$       (VI)  $\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$

(VII)  $\operatorname{sen} a \cos b = \frac{\operatorname{sen}(a - b) + \operatorname{sen}(a + b)}{2}$       (VIII)  $\operatorname{sen} a \operatorname{sen} b = \frac{\cos(a - b) - \cos(a + b)}{2}$

(IX)  $\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$

RESPOSTAS DA LISTA 3

1.  $x \arcsen x + \sqrt{1 - x^2} + C$
2.  $\sen x - x \cos x + C$
3.  $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$
4.  $x^2 \sen x + 2x \cos x - 2 \sen x + C$
5.  $\frac{1}{2} (x^2 \arctan x - x + \arctan x) + C$
6.  $\frac{1}{2} (\sec x \tan x + \ln |\sec x + \tan x|) + C$
7.  $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$
8.  $\frac{2}{3}x^{\frac{3}{2}} \ln x - \frac{4}{9}x^{\frac{3}{2}} + C$
9.  $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2(\ln x) + \frac{1}{4}x^2 + C$
10.  $\frac{1}{\ln 3} x 3^x - \frac{1}{(\ln 3)^2} 3^x + C$
11.  $x \tan x - \ln |\sec x| + C$
12.  $\frac{1}{2}x (\sen(\ln x) - \cos(\ln x)) + C$
13.  $x \operatorname{arcsec} \sqrt{x} - (x - 1)^{\frac{1}{2}} + C$
14.  $\ln x \ln(\ln x) - \ln x + C$
15.  $2\sqrt{x} \arctan(\sqrt{x}) - \ln(1 + x) + C$
16.  $\frac{1}{4} \sec^3 x \tan x - \frac{1}{8} \sec x \tan x - \frac{1}{8} \ln |\sec x + \tan x| + C$
17.  $-\frac{1}{4} \csc^3 x \cot x - \frac{3}{8} \cot x \csc x - \frac{3}{8} \ln |\csc x - \cot x| + C$
18.  $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$
19.  $\frac{3}{8}x + \frac{1}{32} \sen 4x - \frac{1}{4} \sen 2x + C$
20.  $\frac{1}{16}x - \frac{1}{64} \sen 4x - \frac{1}{48} \sen^3 2x + C$
21.  $-\frac{1}{5} \cos^3 x \sen^2 x - \frac{2}{15} \cos^3 x + C$
22.  $\frac{1}{5} \sen^5 x - \frac{2}{3} \sen^3 x + \sen x + C$
23.  $\left. \frac{2}{7} \cos^{\frac{7}{2}} x - \frac{2}{3} \cos^{\frac{3}{2}} x \right]_0^{\frac{\pi}{2}} = \frac{8}{21}$
24.  $\frac{1}{3} \cos^{-3} x - \cos^{-1} x + C$
25.  $\left. \left( \frac{1}{3\pi} \sen \frac{3\pi x}{2} + \frac{1}{\pi} \sen \frac{\pi x}{2} \right) \right]_0^{\frac{1}{2}} = \frac{2\sqrt{2}}{3\pi}$
26.  $\frac{1}{4} \sec^3 x \tan x - \frac{1}{8} \sec x \tan x - \frac{1}{8} \ln |\sec x + \tan x| + C$
27.  $\frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$
28.  $\frac{2}{5} \sec^{\frac{5}{2}} x - 2 \sec^{\frac{1}{2}} x + C$
29.  $\sec x + 2 \sec^{-1} x - \frac{1}{3} \sec^{-3} x + C$
30.  $\cot x - \frac{1}{3} \cot^3 x + x + C$