Universidade Federal Fluminense
EGM - Instituto de Matemática
GMA - Departamento de Matemática Aplicada

LISTA 12 - 2006-2
Função logarítmica
Função exponencial
Funções hiperbólicas

1. Seja $f(x) = \dfrac{\ln\left(x^2-3\right)}{\sqrt{(x-1)(x+3)}}$. Determine o domínio de f, os valores de x onde a f se anula e os intervalos onde a f é positiva e onde a f é negativa.

Nos exercícios 2. a 5. esboce o gráfico da função.

2. $f(x) = \ln|x-4|$
3. $y = |\ln|x+1||$
4. $F(x) = e^{|x+2|}$
5. $g(t) = \frac{1}{2} - e^{-t}$

Derive as funções dos exercícos 6. a 16. (se for conveniente, use derivação logarítmica)

6. $f(x) = \dfrac{e^{\operatorname{sen} 2x}\sqrt{x}}{e^{\cos 3x}}$
7. $f(x) = e^{\sqrt{x}} \ln\sqrt{x}$
8. $f(x) = \ln\left(x\sqrt{x^2+1}\right)$
9. $f(x) = (e^x)^x$
10. $f(x) = e^{x^x}$
11. $f(x) = (x^x)^x$
12. $f(x) = \log_2 x^2$
13. $f(x) = (\operatorname{sen} x)^{\operatorname{arcsen} x}$
14. $f(x) = x^\pi + \pi^x$
15. $f(x) = (\ln x)^x \; x^{\ln x}$
16. $\ln \dfrac{\sqrt{x+1}}{(x-1)^3}$

Calcule y' nos exercícios 17. a 19.

17. $\ln\left(\dfrac{x}{y} + \dfrac{y}{x}\right) = 5$
18. $\operatorname{sen} e^{xy} = x$
19. $\dfrac{y^2\cos x}{e^x} = 2^{\ln y}$, para $x = 0$ e $y = 1$
20. Se $\operatorname{senh} x = -\dfrac{1}{4}$, encontre: (a) $\cosh x$ (b) $\tanh x$ (c) $\operatorname{senh} 2x$
21. Determine x tal que $\tanh x = -\dfrac{1}{4}$.

Nos exercícios 22. a 24. mostre que as igualdades se verificam.

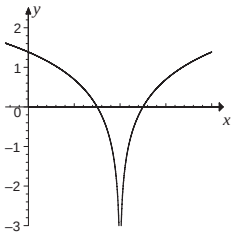
22. $\tanh(\ln x) = \dfrac{x^2-1}{x^2+1}$
23. $\cosh x + \operatorname{senh} x = e^x$
24. $(\cosh x + \operatorname{senh} x)^n = \cosh nx + \operatorname{senh} nx$ (sugestâo: use o exercício 23.)

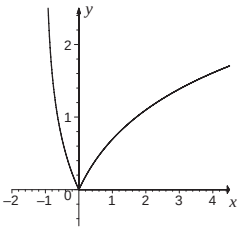
Derive as funções dos exercícios 25. a 27.

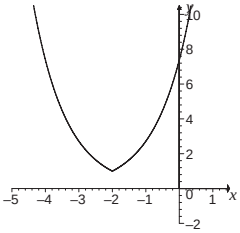
25. $f(x) = \tanh(\operatorname{sen} x)$
26. $f(x) = \operatorname{senh}(\ln 2x) + \cosh(\ln 2x)$
27. $f(x) = x^{\cosh x}$
28. Mostre que $\cot(\pi\cosh(\ln 3)) - \operatorname{arcsen}(\tan(\pi\operatorname{senh}(\ln 2))) = \dfrac{\pi}{2} - \dfrac{\sqrt{3}}{3}$.

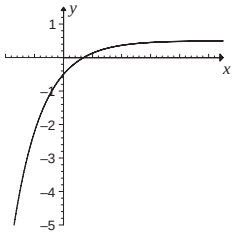
RESPOSTAS

1. Domínio $= (-\infty, -3) \cup \left(\sqrt{3}, \infty\right)$;
   $f = 0$ em $x = 2$
   $f > 0$ para $x < -3$ ou $x > 2$;
   $f < 0$ para $\sqrt{3} < x < 2$

2. 

3. 

4. 

5. 

6. $f'(x) = \dfrac{(1 + 4x\cos 2x + 6x \operatorname{sen} 3x)e^{\operatorname{sen} 2x}}{2e^{\cos 3x}\sqrt{x}}$

7. $f'(x) == \dfrac{e^{\sqrt{x}}\left(1 + \sqrt{x}\ \ln\sqrt{x}\right)}{2x}$

8. $f'(x) = \dfrac{2x^2 + 1}{x\left(x^2 + 1\right)}$

9. $f'(x) = 2xe^{x^2}$

10. $f'(x) = x^x e^{x^x}(1 + \ln x)$

11. $f'(x) = (x^x)^x\,(x + 2x\ln x)$

12. $f'(x) = \dfrac{2}{x\ln 2}$

13. $f'(x) = (\operatorname{sen} x)^{\operatorname{arcsen} x}\left(\cot x\ \ \operatorname{arcsen} x + \dfrac{\ln(\operatorname{sen} x)}{\sqrt{1 - x^2}}\right)$

14. $f'(x) = \pi x^{\pi - 1} + (\ln\pi)\pi^x$

15. $f'(x) = (\ln x)^x\left(x^{\ln x}\right)\left(\dfrac{1}{\ln x} + \ln(\ln x) + \dfrac{2\ln x}{x}\right)$

16. $f'(x) = \dfrac{-(5x + 7)}{2\left(x^2 - 1\right)}$

17. $y' = \dfrac{y}{x}$

18. $y' = \dfrac{1 - ye^{xy}\cos e^{xy}}{xe^{xy}\cos e^{xy}}$

19. $y' = \dfrac{1}{2 - \ln 2}$

20. (a) $\dfrac{\sqrt{17}}{4}$ (b) $-\dfrac{\sqrt{17}}{17}$ (c) $-\dfrac{\sqrt{17}}{8}$

21. $\dfrac{1}{2}\ln\dfrac{3}{5}$

22. $\tanh(\ln x) = \dfrac{\frac{1}{2}\left(e^{\ln x} - e^{-\ln x}\right)}{\frac{1}{2}\left(e^{\ln x} + e^{-\ln x}\right)} = \dfrac{x - \frac{1}{x}}{x + \frac{1}{x}} = \dfrac{x^2 - 1}{x^2 + 1}$

23. $\cosh x + \operatorname{senh} x = \dfrac{e^x - e^{-x}}{2} + \dfrac{e^x + e^{-x}}{2} = \dfrac{2e^x}{2} = e^x$

24. $\cosh nx + \operatorname{sen} nx = e^{nx} = (e^x)^n = (\cosh x + \operatorname{senh} x)^n$

25. $f'(x) = \cos x \operatorname{sech}^2(\operatorname{sen} x)$

26. $f'(x) = 2$

27. $x^{\cosh x}\left(\operatorname{senh} x\ln x + \dfrac{\cosh x}{x}\right)$