

RESPOSTAS DA LISTA 4 - Trigonometria

1. $9\sqrt{3} m$

2. $h = 6\sqrt{7} km$

3. $25,34 m$

4. (a) $8 cm$ (b) $\frac{5\sqrt{3}}{3} cm$

5. $\frac{3}{2}$

6. $-\frac{2}{3}$

7. $\frac{\sqrt{6}}{2}$

8. (a) $-3 \leq m \leq -\sqrt{7}$

ou $\sqrt{7} \leq m \leq 7$

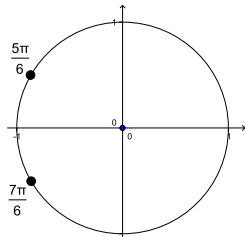
(b) $-\frac{1}{7} \leq m \leq 1$

(c) $-1 \leq m \leq 3$

10. (a) $\cot x$ (b) $\tan x$

11. (a) $x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$

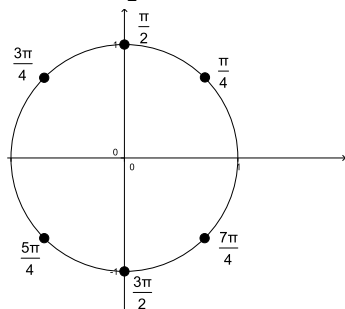
ou $x = \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$



(b) $x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

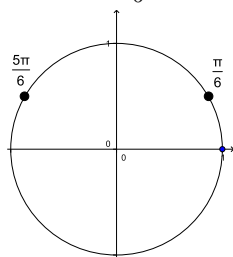
ou $x = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}$

ou $x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$



(c) $x = \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

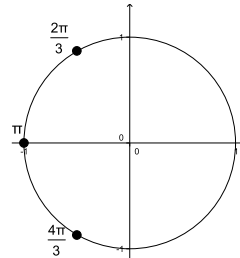
ou $x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$



(d) $x = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$

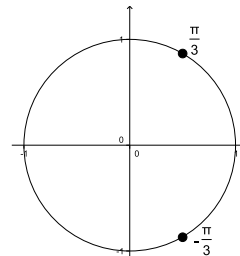
ou $x = \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z}$

ou $x = \pi + 2k\pi, k \in \mathbb{Z}$



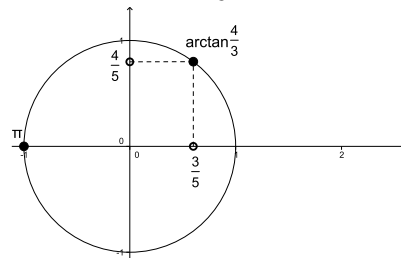
(e) $x = \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

ou $x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$



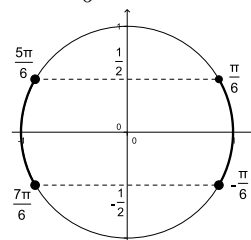
(f) $x = \pi + 2k\pi, k \in \mathbb{Z}$

ou $x = \arctan \frac{4}{3} + 2k\pi, k \in \mathbb{Z}$



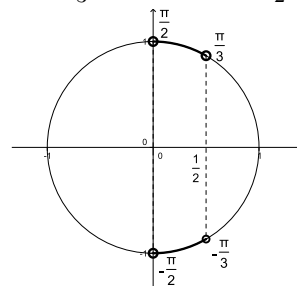
(g) $-\frac{\pi}{6}\pi + 2k\pi < x < \frac{\pi}{6}\pi + 2k\pi, k \in \mathbb{Z}$

ou $\frac{5\pi}{6} + 2k\pi < x < \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$

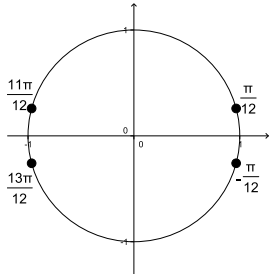


(h) $-\frac{\pi}{2} + 2k\pi < x < -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

ou $\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$



- (i) $x = \frac{\pi}{12} + k\pi, k \in \mathbb{Z}$
 ou $x = -\frac{\pi}{12} + k\pi, k \in \mathbb{Z}$



- (j) Se desenvolver a expressão usando as identidades

$$\text{sen}(4x) = 2 \text{sen}(2x) \cos(2x)$$

$$\text{sen}(2x) = 2 \text{sen}(x) \cos(x)$$

$$\cos(2x) = \cos^2 x - \text{sen}^2 x$$

encontra-se a solução na seguinte forma:

$$x = k\pi, k \in \mathbb{Z},$$

$$\text{ou } x = \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$\text{ou } x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

$$\text{ou } x = \theta_1 + 2k\pi, k \in \mathbb{Z},$$

onde θ_1 é o ângulo tal que:

$$\cos(\theta_1) = \frac{-1+\sqrt{5}}{4} \quad \text{e} \quad \text{sen}(\theta_1) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\text{isto é, } \theta_1 = \arctan \frac{\sqrt{10+2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\text{ou } x = \theta_2 + 2k\pi, k \in \mathbb{Z},$$

onde θ_2 é o ângulo tal que:

$$\cos(\theta_2) = \frac{-1+\sqrt{5}}{4} \quad \text{e} \quad \text{sen}(\theta_2) = -\frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\text{isto é, } \theta_2 = -\arctan \frac{\sqrt{10+2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\text{ou } x = \theta_3 + 2k\pi, k \in \mathbb{Z},$$

onde θ_3 é o ângulo tal que:

$$\cos(\theta_3) = \frac{-1-\sqrt{5}}{4} \quad \text{e} \quad \text{sen}(\theta_3) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

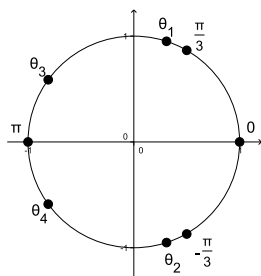
$$\text{isto é, } \theta_3 = \pi + \arctan \frac{\sqrt{10-2\sqrt{5}}}{-1-\sqrt{5}}$$

$$\text{ou } x = \theta_4 + 2k\pi, k \in \mathbb{Z},$$

onde θ_4 é o ângulo tal que:

$$\cos(\theta_4) = \frac{-1-\sqrt{5}}{4} \quad \text{e} \quad \text{sen}(\theta_4) = -\frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\text{isto é, } \theta_4 = \pi - \arctan \frac{\sqrt{10-2\sqrt{5}}}{-1-\sqrt{5}}$$



Se desenvolver a expressão usando a identidade:

$$\text{sen } p + \text{sen } q = 2 \text{sen} \left(\frac{p+q}{2} \right) \cos \left(\frac{p-q}{2} \right)$$

encontra-se a expressão

$$\text{sen } x + \text{sen}(4x) = 2 \text{sen} \left(\frac{5x}{2} \right) \cos \left(\frac{3x}{2} \right)$$

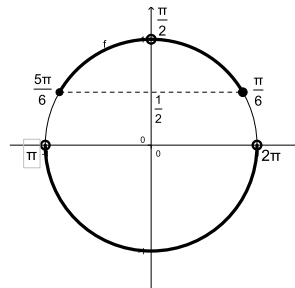
e a mesma solução anterior, escrita de outra forma:

$$x = \frac{2k\pi}{5} \quad \text{ou} \quad x = \frac{\pi}{3} + \frac{2k\pi}{3}.$$

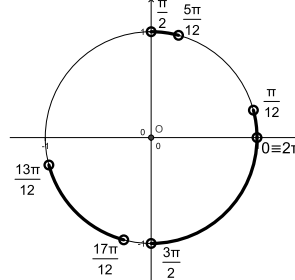
Observe que:

$$\theta_1 = \frac{2\pi}{5}; \quad \theta_2 = \frac{4\pi}{5}; \quad \theta_3 = -\frac{4\pi}{5}; \quad \theta_4 = -\frac{2\pi}{5}$$

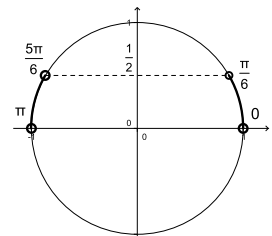
- (k) $\left[\frac{\pi}{6}, \frac{\pi}{2} \right] \cup \left(\frac{\pi}{2}, \frac{5\pi}{6} \right) \cup (\pi, 2\pi)$



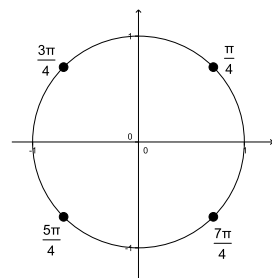
- (l) $\left(0, \frac{\pi}{12} \right) \cup \left(\frac{5\pi}{12}, \frac{\pi}{2} \right) \cup \left(\frac{13\pi}{12}, \frac{17\pi}{12} \right) \cup \left(\frac{3\pi}{2}, 2\pi \right)$



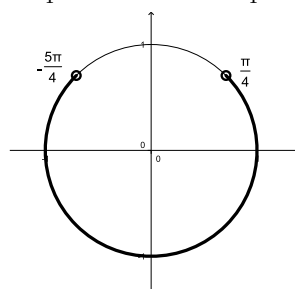
- (m) $\left[0, \frac{\pi}{6} \right] \cup \left(\frac{5\pi}{6}, \pi \right)$



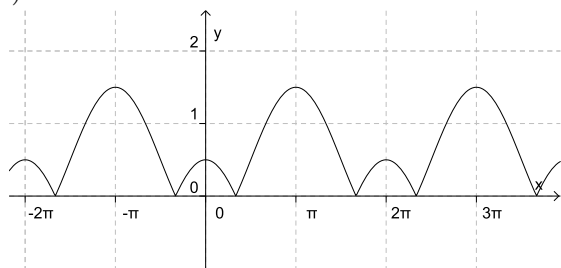
- (n) $x = \frac{k\pi}{4}, k \in \mathbb{Z}$



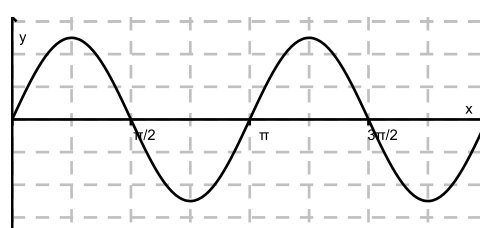
- (o) $-\frac{5\pi}{4} + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$



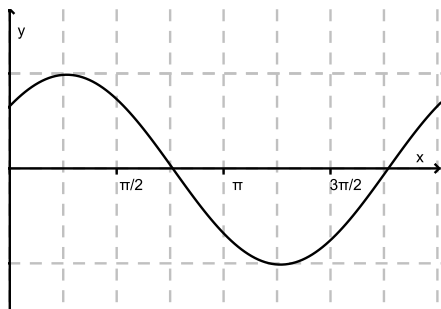
12. (a)



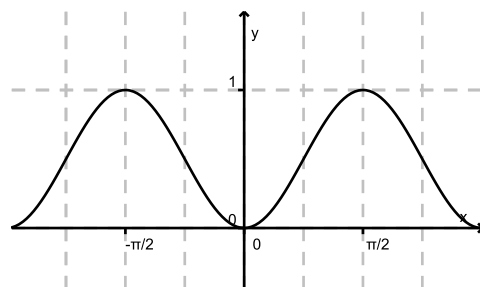
(g)



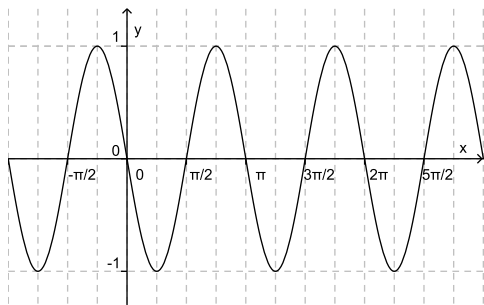
(b)



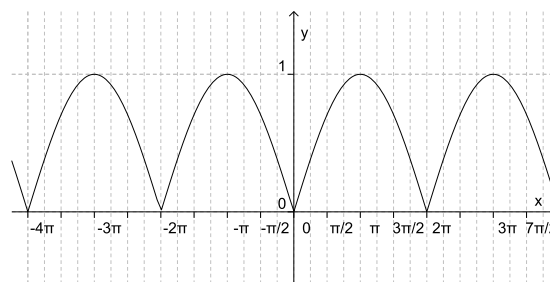
(h)



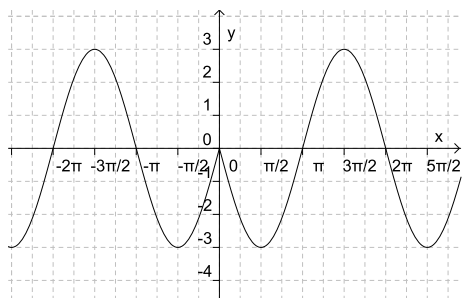
(c)



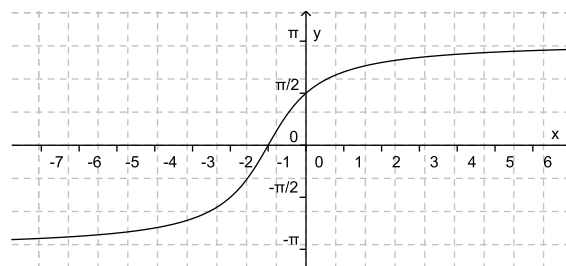
(i)



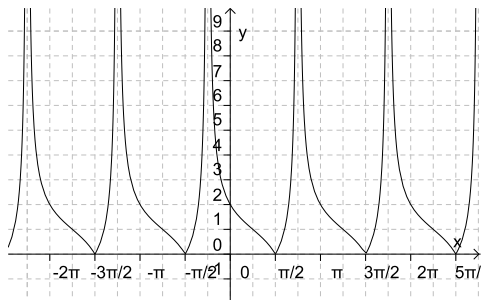
(d)



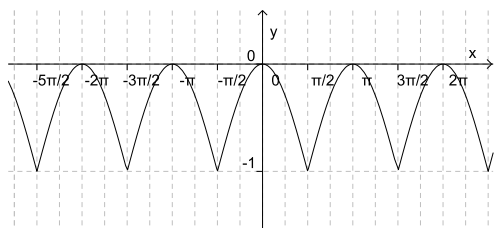
(j)



(e)



(f)



13. (a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{4}$ (c) π

14. Queremos calcular $\cos(\arcsen x)$.

Considere $\theta = \arcsen x$.

Nesse caso, sabemos que

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad \cos \theta \geq 0, \quad x = \sen \theta.$$

Queremos calcular $\cos \theta$. Mas,

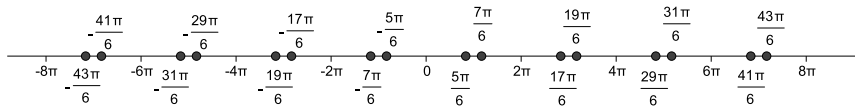
$$\cos^2 \theta = 1 - \sen^2 \theta \implies \cos \theta = \pm \sqrt{1 - \sen^2 \theta}.$$

$$\text{Como } \cos \theta \geq 0, \quad \cos \theta = \sqrt{1 - \sen^2 \theta}$$

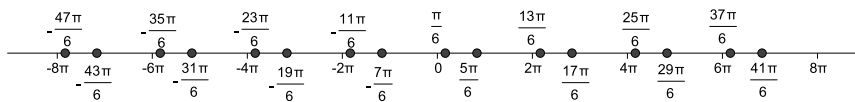
$$\text{Como } x = \sen \theta, \quad \cos \theta = \sqrt{1 - x^2},$$

$$\text{Como } \theta = \arcsen x, \quad \cos(\arcsen x) = \sqrt{1 - x^2}.$$

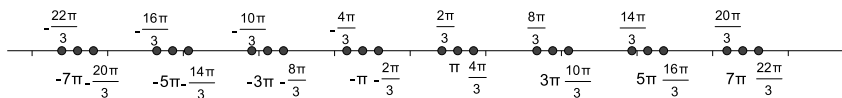
15. (a)



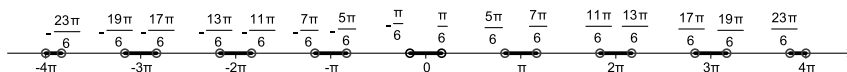
(b)



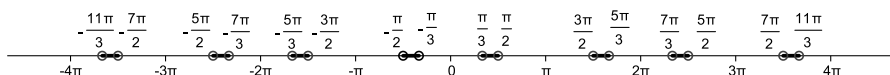
(c)



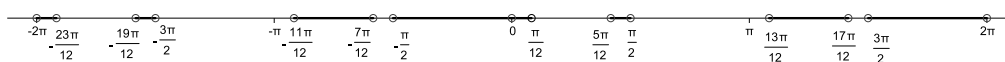
(d)



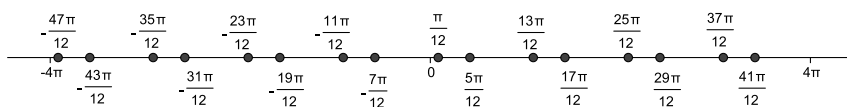
(e)



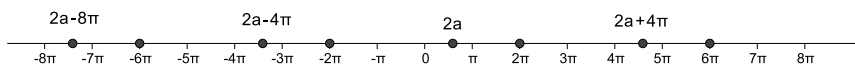
(f)



(g)



(h) $a = \arctan\left(\frac{4}{3}\right) \simeq 0,93$



(i)

