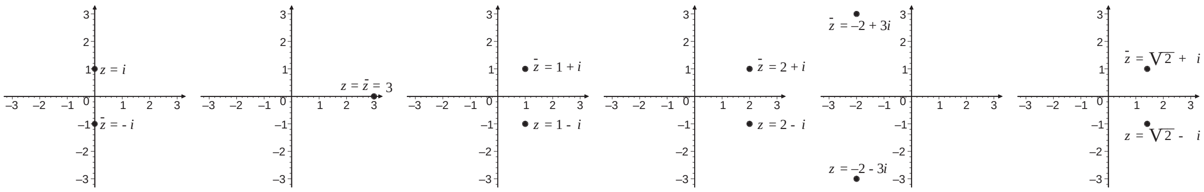
RESPOSTAS DA LISTA 9 (alguns estão com a resolução ou o resumo da resolução):

1. (a)



(b) $|z| = |i| = \sqrt{0+1} = 1;$ $|\bar{z}| = |-i| = \sqrt{0+1} = 1.$ $|z| = |3| = \sqrt{9+0} = 3;$ $|\bar{z}| = |3| = 3.$

$|z| = |1-i| = \sqrt{1+1} = \sqrt{2};$ $|\bar{z}| = |1+i| = \sqrt{1+1} = \sqrt{2}.$ $|z| = |2-i| = \sqrt{4+1} = \sqrt{5};$ $|\bar{z}| = |2+i| = \sqrt{4+1} = \sqrt{5}.$

$|z| = |-2-3i| = \sqrt{4+9} = \sqrt{13};$ $|\bar{z}| = |-2+3i| = \sqrt{4+9} = \sqrt{13}.$ $|z| = |\sqrt{2}-i| = \sqrt{2+1} = \sqrt{3};$ $|\bar{z}| = |\sqrt{2}+i| = \sqrt{2+1} = \sqrt{3}.$

(c) $w_1 = \dfrac{z}{|z|} = \dfrac{i}{|i|} = \dfrac{i}{1} = i \Longrightarrow |w_1| = |i| = 1$

$w_2 = \dfrac{z}{|z|} = \dfrac{3}{|3|} = \dfrac{3}{3} = 1 \Longrightarrow |w_2| = |1| = 1$

$w_3 = \dfrac{z}{|z|} = \dfrac{1-i}{|1-i|} = \dfrac{1-i}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\, i$

$\Longrightarrow |w_3| = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\, i = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1.$

$w_4 = \dfrac{z}{|z|} = \dfrac{2-i}{|2-i|} = \dfrac{2-i}{\sqrt{5}} = \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\, i$

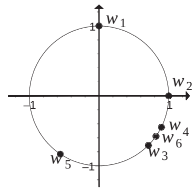
$\Longrightarrow |w_4| = \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\, i = \sqrt{\frac{4}{5} + \frac{1}{5}} = \sqrt{1} = 1.$

$w_5 = \dfrac{z}{|z|} = \dfrac{-2-3i}{|-2-3i|} = \dfrac{-2-3i}{\sqrt{13}} = \frac{-2}{\sqrt{13}} + \frac{-3}{\sqrt{13}}\, i$

$\Longrightarrow |w_5| = \frac{-2}{\sqrt{13}} + \frac{-3}{\sqrt{13}}\, i = \sqrt{\frac{4}{13} + \frac{9}{13}} = \sqrt{1} = 1.$

$w_6 = \dfrac{z}{|z|} = \dfrac{\sqrt{2}-i}{|\sqrt{2}-i|} = \dfrac{\sqrt{2}-i}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}\, i$

$\Longrightarrow |w_6| = \frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}\, i = \sqrt{\frac{2}{3} + \frac{1}{3}} = \sqrt{1} = 1.$



2. Considerando $z \in \mathbb{C}, \quad z = a + bi, \quad a, b \in \mathbb{R}, \quad z \neq 0.$

Para $z \neq 0$, temos que $|z| \neq 0$ e $\left|\dfrac{z}{|z|}\right| = \left|\dfrac{a+bi}{\sqrt{a^2+b^2}}\right| = \left|\dfrac{a}{\sqrt{a^2+b^2}} + \dfrac{b}{\sqrt{a^2+b^2}}\, i\right| = \sqrt{\dfrac{a^2}{a^2+b^2} + \dfrac{b^2}{a^2+b^2}} = \sqrt{\dfrac{a^2+b^2}{a^2+b^2}} = 1.$

3. (a) $z = \dfrac{3i+2+4i}{2} = \dfrac{2+7i}{2} = 1 + \frac{7}{2}\, i.$

(b) $z = \dfrac{9}{z_1} = \dfrac{9}{3i} = \dfrac{3\cdot(-i)}{i\cdot(-i)} = \dfrac{-3i}{-i^2} = \dfrac{-3i}{-(-1)} = \dfrac{-3i}{1} = -3i.$

(c) $z = (3i)(2+4i) = 6i + 12i^2 = 6i - 12 = -12 + 6i.$

(d) $z = \dfrac{4-i}{3i} = \dfrac{(4-i)(-i)}{3(i)(-i)} = \dfrac{-4i+i^2}{-3i^2} = \dfrac{-4i-1}{3} = -\frac{1}{3} - \frac{4}{3}\, i.$

(e) $z = \dfrac{3i}{2+4i} = \dfrac{3i(2-4i)}{(2+4i)(2-4i)} = \dfrac{6i-12i^2}{4-8i+8i-16i^2} = \dfrac{6i-12(-1)}{4-16(-1)} = \dfrac{12+6i}{20} = \frac{3}{5} + \frac{3}{10}\, i.$

4. (a) Sabemos que $\forall z \in \mathbb{C}$, a representação na forma polar é $z = |z|(\cos\theta + i \,\text{sen}\,\theta)$.
Logo, para $z_1 = 4\,(\cos(\pi/3) + i\,\text{sen}\,(\pi/3))$, temos que $|z_1| = 4$ e $\theta = \pi/3$.
Daí, $\dfrac{z_1}{|z_1|} = \dfrac{4\,(\cos(\pi/3) + i\,\text{sen}\,(\pi/3))}{4} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\, i.$

(b) Sabemos que $\forall z_1, z_2 \in \mathbb{C}$, tal que $z_1 = |z_1|(\cos\theta_1 + i\,\text{sen}\,\theta_1)$ e $z_2 = |z_2|(\cos\theta_2 + i\,\text{sen}\,\theta_2)$,
produto é calculado por $z_1 \cdot z_2 = |z_1| \cdot |z_2|(\cos(\theta_1+\theta_2) + i\,\text{sen}\,(\theta_1+\theta_2))$.
Para $z_1 = 4\,\left(\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\,\left(\frac{\pi}{3}\right)\right)$, temos que $|z_1| = 4$ e $\theta = \frac{\pi}{3}$
Para $z_2 = (1/2)\,\left(\cos\left(\frac{2\pi}{9}\right) + i\,\text{sen}\,\left(\frac{2\pi}{9}\right)\right)$ temos que $|z_2| = 1/2$ e $\theta = \frac{2\pi}{9}$.
Daí, $z_1 z_2 = 4\dfrac{1}{2}\left(\cos\left(\frac{\pi}{3} + \frac{2\pi}{9}\right) + i\,\text{sen}\,\left(\frac{\pi}{3} + \frac{2\pi}{9}\right)\right) = 2\left(\cos\left(\frac{5\pi}{9}\right) + i\,\text{sen}\,\left(\frac{5\pi}{9}\right)\right).$
Sabemos que $\dfrac{5\pi}{9}$ corresponde a 100°, não há fórmula simples para calcular o valor exato de $\cos\left(\frac{5\pi}{9}\right)$ e $\text{sen}\,\left(\frac{5\pi}{9}\right)$. Neste caso deixamos indicado ou usamos calculadora para calcular aproximadamente esses valores.

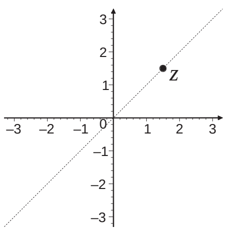
(c) Sabemos que $\left|\dfrac{z}{|z|}\right| = 1, \forall z \neq 0$, logo $\left|\dfrac{z_2 z_3}{|z_2 z_3|}\right| = 1$ e $\dfrac{z_2 z_3}{|z_2 z_3|} = \cos\left(\frac{2\pi}{9} + \frac{7\pi}{6}\right) + i\,\text{sen}\,\left(\frac{2\pi}{9} + \frac{7\pi}{6}\right) = \cos\left(\frac{25\pi}{18}\right) + i\,\text{sen}\,\left(\frac{25\pi}{18}\right).$
Sabemos que $\dfrac{25\pi}{18}$ corresponde a 250°, não há fórmula simples para calcular o valor exato de $\cos\left(\frac{25\pi}{18}\right)$ e $\text{sen}\,\left(\frac{25\pi}{18}\right)$. Neste caso deixamos indicado ou usamos calculadora para calcular aproximadamente esses valores.

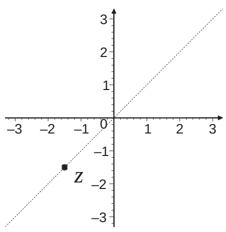
(d) $z_1 z_2 z_4 = (z_1 z_2)(z_4) = (|z_1| \cdot |z_2|(\cos(\theta_1+\theta_2) + i\,\text{sen}\,(\theta_1+\theta_2)\ )\ ) \cdot (|z_4|(\cos(\theta_4) + i\,\text{sen}\,(\theta_4)\ )\ ) =$
$|z_1| \cdot |z_2| \cdot |z_4|(\cos(\theta_1+\theta_2+\theta_4) + i\,\text{sen}\,(\theta_1+\theta_2+\theta_4)\ ).$
Pelos dados do exercício, $|z_1| = 4; \quad |z_2| = 1/2; \quad |z_4| = \sqrt{2}; \quad \theta_1 = \pi/3; \quad \theta_2 = 2\pi/9; \quad \theta_4 = 7\pi/4.$
Daí, $z_1 z_2 z_4 = 4\dfrac{1}{2}\sqrt{2}\left(\cos\left(\frac{\pi}{3} + \frac{2\pi}{9} + \frac{7\pi}{4}\right) + i\,\text{sen}\,\left(\frac{\pi}{3} + \frac{2\pi}{9} + \frac{7\pi}{4}\right)\ \right) = 2\sqrt{2}\left(\cos\left(\frac{83\pi}{36}\right) + i\,\text{sen}\,\left(\frac{83\pi}{36}\right)\ \right) =$
$2\sqrt{2}\left(\cos\left(\frac{83\pi}{36} - 2\pi\right) + i\,\text{sen}\,\left(\frac{83\pi}{36} - 2\pi\right)\ \right) = 2\sqrt{2}\left(\cos\left(\frac{11\pi}{36}\right) + i\,\text{sen}\,\left(\frac{11\pi}{36}\right)\ \right).$

(e) Sabemos que $\dfrac{z_1}{z_4} = \dfrac{z_1\bar{z}_4}{z_4\bar{z}_4} = \dfrac{z_1\bar{z}_4}{|\bar{z}_4|^2} = \dfrac{z_1\bar{z}_4}{|z_4|^2}$. A justificativa da última igualdade é que $|z| = |\bar{z}|, \quad \forall z \in \mathbb{C}$.
Para $z_4 = |z_4|(\cos\theta_4 + i\,\mathrm{sen}\,\theta_4))$, temos $\bar{z}_4 = |\bar{z}_4|(\cos\theta_4 - i\,\mathrm{sen}\,\theta_4) = |z_4|(\cos(-\theta_4) + i\,\mathrm{sen}\,(-\theta_4)\ )$.
Logo, $\dfrac{z_1}{z_4} = \dfrac{|z_1||z_4|}{|z_4|^2}(\cos(\theta_1 + (-\theta_4)\ ) + i\,\mathrm{sen}\,(\theta_1 + (-\theta_4)\ )\ ) = \dfrac{|z_1|}{|z_4|}(\cos(\theta_1 - \theta_4) + i\,\mathrm{sen}\,(\theta_1 - \theta_4)\ )$
Assim, pelos dados do exercício, $|z_1| = 4$; $|z_4| = \sqrt{2}$; $\theta_1 - \theta_4 = \dfrac{\pi}{3} - \dfrac{7\pi}{4} = \dfrac{-17\pi}{12} \equiv \dfrac{-17\pi}{12} + 2\pi = \dfrac{7\pi}{12}$.
Daí, $\dfrac{z_1}{z_4} = \frac{4}{\sqrt{2}}\left(\cos\left(\frac{7\pi}{12}\right) + i\left(\frac{7\pi}{12}\right)\ \right) = 2\sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) + i\left(\frac{7\pi}{12}\right)\ \right)$.
Sabemos que $\dfrac{7\pi}{12}$ corresponde a $105° = 135° - 30°$, logo
$\cos(105°) = \cos(135° - 30°) = \cos(135°)\cos(30°) + \mathrm{sen}\,(135°)\,\mathrm{sen}\,(30°) = -\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{-\sqrt{6}+\sqrt{2}}{4}$.
$\mathrm{sen}\,(105°) = \mathrm{sen}\,(135° - 30°) = \mathrm{sen}\,(135°)\cos(30°) - \mathrm{sen}\,(30°)\cos(135°) = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$.
Logo, $\dfrac{z_1}{z_4} = 2\sqrt{2}\left(\frac{-\sqrt{6}+\sqrt{2}}{4} + i\frac{\sqrt{6}+\sqrt{2}}{4}\right) = \left(-\sqrt{3}+1\right) + \left(\sqrt{3}+1\right)i$

(f) Sabemos que $\left|\dfrac{z}{|\bar{z}|}\right| = 1, \quad \forall z \in \mathbb{C}$, logo $\left|\dfrac{z_1^2 z_3^6}{|z_1^2 z_3^6|}\right| = 1$. Assim, para calcular $\dfrac{z_1^2 z_3^6}{|z_1^2 z_3^6|}$, basta calcular os ângulos que z_1^2 e z_3^6 fazem com o eixo real para aplicar a fórmula polar do produto de complexos.
Sabemos que para $z = |z|(\cos\theta + i\,\mathrm{sen}\,\theta)$, $n \in \mathbb{N}$, vale a fórmula de De Moivre: $z^n = |z|^n(\cos(n\theta) + i\,\mathrm{sen}\,(n\theta)\ )$.
Assim, pelos dados do exercício, $z_1^2 = |z_1|^2\left(\cos\left(2\dfrac{\pi}{3}\right) + i\,\mathrm{sen}\,\left(2\dfrac{\pi}{3}\right)\right)$; $z_3^2 = |z_3|^2\left(\cos\left(6\dfrac{7\pi}{6}\right) + i\,\mathrm{sen}\,\left(6\dfrac{7\pi}{6}\right)\right)$.
Logo, $\dfrac{z_1^2 z_3^6}{|z_1^2 z_3^6|} = 1\left(\cos\left(\dfrac{2\pi}{3} + 7\pi\right) + i\,\mathrm{sen}\,\left(\dfrac{2\pi}{3} + 7\pi\right)\right) = \left(\cos\left(\dfrac{2\pi}{3} + \pi\right) + i\,\mathrm{sen}\,\left(\dfrac{2\pi}{3} + \pi\right)\right) =$
$\left(\cos\left(\dfrac{5\pi}{3}\right) + i\,\mathrm{sen}\,\left(\dfrac{5\pi}{3}\right)\right) = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

5. (a) $z = a + ai, \quad a > 0$ $\qquad$ $z = a + ai, \quad a < 0$





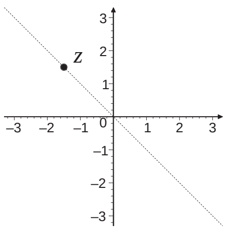
(b) Para $a > 0$, $\arg(z) = \pi/4$
Para $a < 0$, $\arg(z) = 5\pi/4$

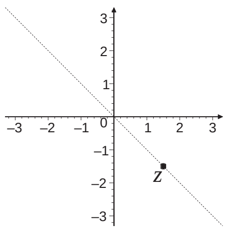
(c) Para $a > 0$, $\arg\left(z^2\right) = \pi/2$.

(d) Observando a figura, vemos que é preciso multiplicar $\pi/4$ por 8 até chegar novamente em 2π. Logo,
$\arg(z^n) = r\pi/4$, onde r é o resto da divisão de n por 8.

(e) Observando a figura, na primeira multiplicação, z por z, temos $arg(z^2) = \pi/2$, na segunda multiplicação, z^2 por z, temos $arg(z^3) = 2\pi - \pi/4 = 7\pi/4$.

6. (a) $z = -a + ai, \quad a > 0$ $\qquad$ $z = -a + ai, \quad a < 0$





(b) Para $a > 0$, $\arg(z) = 3\pi/4$
Para $a < 0$, $\arg(z) = 7\pi/4$

(c) Para $a > 0$, $\arg\left(z^2\right) = 3\pi/2$.

(d) Observando a figura, vemos que é preciso multiplicar $3\pi/4$ por 8 até chegar novamente em um múltiplo de 2π. Logo,
$\arg(z^n) = r \cdot 3\pi/4$, r é o resto da divisão de n por 8.

(e) Observando a figura, na primeira multiplicação, z por z, temos $arg(z^2) = 3\pi/2$, na segunda multiplicação, z^2 por z, temos $arg(z^3) = 5\pi/4$.

7. (a) Sabemos que $z = a + bi = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}(a + bi) = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + i\frac{b}{\sqrt{a^2+b^2}}\right) = |z|(\cos\theta + i\,\mathrm{sen}\,\theta)$.
Logo $|z| = \sqrt{a^2+b^2}$, $\cos\theta = \dfrac{a}{\sqrt{a^2+b^2}}$, $\mathrm{sen}\,\theta = \dfrac{b}{\sqrt{a^2+b^2}}$.
Assim, pelos dados do exercício, $|z| = |4\sqrt{3} + 4i| = \sqrt{16 \cdot 3 + 16} = 8$; $\cos(\theta) = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$; $\mathrm{sen}\,(\theta) = \frac{4}{8} = \frac{1}{2} \Longrightarrow \theta = \pi/6$.
Logo, $4\sqrt{3} + 4i = 8(\cos(\pi/6) + i\,\mathrm{sen}\,(\pi/6)\ )$.

(b) $\left|-\frac{1}{3} + \frac{\sqrt{3}}{3}i\right| = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(\frac{\sqrt{3}}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{3}{9}} = \dfrac{2}{3}$; $\cos(\theta) = \dfrac{\frac{-1}{3}}{\frac{2}{3}} = -\frac{1}{2}$; $\mathrm{sen}\,(\theta) = \dfrac{\frac{\sqrt{3}}{3}}{\frac{2}{3}} = \frac{\sqrt{3}}{2} \Longrightarrow \theta = 2\pi/3$.
Logo, $-\frac{1}{3} + \frac{\sqrt{3}}{3}i = \frac{2}{3}(\cos(2\pi/3) + i\,\mathrm{sen}\,(2\pi/3))$.

(c) $|2 - 2i| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$; $\cos(\theta) = \frac{2}{\sqrt{2}}$; $\mathrm{sen}\,(\theta) = -\frac{2}{\sqrt{2}} \Longrightarrow \theta = 7\pi/4$.
Logo, $2 - 2i = 2\sqrt{2}(\cos(7\pi/4) + i\,\mathrm{sen}\,(7\pi/4)\ )$.

(d) $|-i| = \sqrt{0+1} = 1$; $\cos(\theta) = 0/1 = 0$; $\mathrm{sen}\,(\theta) = -1/1 = 1 \Longrightarrow \theta = 3\pi/2 \Longrightarrow -i = (\cos(3\pi/2) + i\,\mathrm{sen}\,(3\pi/2)\ )$.

8. (a) Sabe-se que $z^n = w$, $n \in \mathbb{N}$, $w = |w|(\cos\theta + i\,\text{sen}\,\theta) \Longrightarrow z = |w|^{1/n}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\,\text{sen}\left(\frac{\theta+2k\pi}{n}\right)\right)$, $k = 0, 1, 2, \ldots, n-1$.

Pelos dados do exercício, $w = 4\sqrt{3} + 4i$, pelo ex. 7.(a), $4\sqrt{3} + 4i = 8(\cos(\pi/6) + i\,\text{sen}\,(\pi/6)\ )$. Logo,

$z = 8^{1/4}\left(\cos\left(\frac{\frac{\pi}{6}+2k\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{6}+2k\pi}{4}\right)\right)$, $k \in \{0, 1, 2, 3\}$. Determinando as quatro raízes da equação,

$z_1 = 8^{1/4}\left(\cos\left(\frac{\frac{\pi}{6}+0\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{6}+0\pi}{4}\right)\right) = 8^{1/4}\left(\cos\left(\frac{\pi}{24}\right) + i\,\text{sen}\left(\frac{\pi}{24}\right)\right)$.

$z_2 = 8^{1/4}\left(\cos\left(\frac{\frac{\pi}{6}+2\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{6}+2\pi}{4}\right)\right) = 8^{1/4}\left(\cos\left(\frac{13\pi}{24}\right) + i\,\text{sen}\left(\frac{13\pi}{24}\right)\right)$.

$z_3 = 8^{1/4}\left(\cos\left(\frac{\frac{\pi}{6}+4\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{6}+4\pi}{4}\right)\right) = 8^{1/4}\left(\cos\left(\frac{25\pi}{24}\right) + i\,\text{sen}\left(\frac{25\pi}{24}\right)\right)$.

$z_4 = 8^{1/4}\left(\cos\left(\frac{\frac{\pi}{6}+6\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{6}+6\pi}{4}\right)\right) = 8^{1/4}\left(\cos\left(\frac{37\pi}{24}\right) + i\,\text{sen}\left(\frac{37\pi}{24}\right)\right)$.

(b) $w = 27 - 27i$, $|w| = \sqrt{27^2 + 27^2} = \sqrt{2 \cdot 27^2} = 27\sqrt{2}$; $\cos(\theta) = \frac{27}{27\sqrt{2}} = \frac{\sqrt{2}}{2}$; $\text{sen}\,(\theta) = \frac{-27}{27\sqrt{2}} = -\frac{\sqrt{2}}{2} \Longrightarrow \theta = 7\pi/4$.

$z = 27^{1/3}\left(\cos\left(\frac{\frac{7\pi}{4}+2k\pi}{3}\right) + i\,\text{sen}\left(\frac{\frac{7\pi}{4}+2k\pi}{3}\right)\right)$, $k \in \{0, 1, 2\}$. Determinando as três raízes da equação,

$z_1 = 3\left(\cos\left(\frac{\frac{7\pi}{4}+0\pi}{3}\right) + i\,\text{sen}\left(\frac{\frac{7\pi}{4}+0\pi}{3}\right)\right) = 3\left(\cos\left(\frac{7\pi}{12}\right) + i\,\text{sen}\left(\frac{7\pi}{12}\right)\right) = 3\left(\frac{\sqrt{2}-\sqrt{6}}{4} + i\frac{\sqrt{2}+\sqrt{6}}{4}\right)$ (ver exercício 4.(e)).

$z_2 = 3\left(\cos\left(\frac{\frac{7\pi}{4}+2\pi}{3}\right) + i\,\text{sen}\left(\frac{\frac{7\pi}{4}+2\pi}{3}\right)\right) = 3\left(\cos\left(\frac{5\pi}{4}\right) + i\,\text{sen}\left(\frac{5\pi}{4}\right)\right) = 3\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\ \right)$.

$z_3 = 3\left(\cos\left(\frac{\frac{7\pi}{4}+4\pi}{3}\right) + i\,\text{sen}\left(\frac{\frac{7\pi}{4}+4\pi}{3}\right)\right) = 3\left(\cos\left(\frac{23\pi}{12}\right) + i\,\text{sen}\left(\frac{23\pi}{12}\right)\right) = 3\left(\frac{\sqrt{2}+\sqrt{6}}{4} + i\frac{\sqrt{2}-\sqrt{6}}{4}\right)$.

Justificativa: sabemos que $\dfrac{23\pi}{12} = 2\pi - \pi/12$ corresponde a $345^\circ = 300^\circ + 45^\circ$, logo

$\cos(345^\circ) = \cos(300^\circ + 45^\circ) = \cos(300^\circ)\cos(45^\circ) - \text{sen}\,(300^\circ)\,\text{sen}\,(45^\circ) = \frac{1}{2}\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$.

$\text{sen}\,(345^\circ) = \text{sen}\,(300^\circ + 45^\circ) = \text{sen}\,(300^\circ)\cos(45^\circ) + \text{sen}\,(45^\circ)\cos(300^\circ) = -\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$.

(c) $w = -i$; $|w| = 1$; $\theta = \arg(w) = 3\pi/2$.

$z = 1^{1/5}\left(\cos\left(\frac{\frac{3\pi}{2}+2k\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+2k\pi}{5}\right)\right)$, $k \in \{0, 1, 2, 3, 4\}$. Determinando as cinco raízes da equação,

$z_1 = \left(\cos\left(\frac{\frac{3\pi}{2}+0\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+0\pi}{5}\right)\right) = \left(\cos\left(\frac{3\pi}{10}\right) + i\,\text{sen}\left(\frac{3\pi}{10}\right)\right)$. $\frac{3\pi}{10} \sim 54^\circ$, mesmo comentário do exercício 4.(b).

$z_2 = \left(\cos\left(\frac{\frac{3\pi}{2}+2\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+2\pi}{5}\right)\right) = \left(\cos\left(\frac{7\pi}{10}\right) + i\,\text{sen}\left(\frac{7\pi}{10}\right)\right)$. $\frac{7\pi}{10} \sim 126^\circ$, mesmo comentário do exercício 4.(b).

$z_3 = \left(\cos\left(\frac{\frac{3\pi}{2}+4\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+4\pi}{5}\right)\right) = \left(\cos\left(\frac{11\pi}{10}\right) + i\,\text{sen}\left(\frac{11\pi}{10}\right)\right)$. $\frac{11\pi}{10} \sim 198^\circ$, mesmo comentário do exercício 4.(b).

$z_4 = \left(\cos\left(\frac{\frac{3\pi}{2}+6\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+6\pi}{5}\right)\right) = \left(\cos\left(\frac{3\pi}{2}\right) + i\,\text{sen}\left(\frac{3\pi}{2}\right)\right) = -i$.

$z_5 = \left(\cos\left(\frac{\frac{3\pi}{2}+8\pi}{5}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+8\pi}{5}\right)\right) = \left(\cos\left(\frac{19\pi}{10}\right) + i\,\text{sen}\left(\frac{19\pi}{10}\right)\right)$. $\frac{19\pi}{10} \sim 242^\circ$, mesmo comentário do exercício 4.(b).