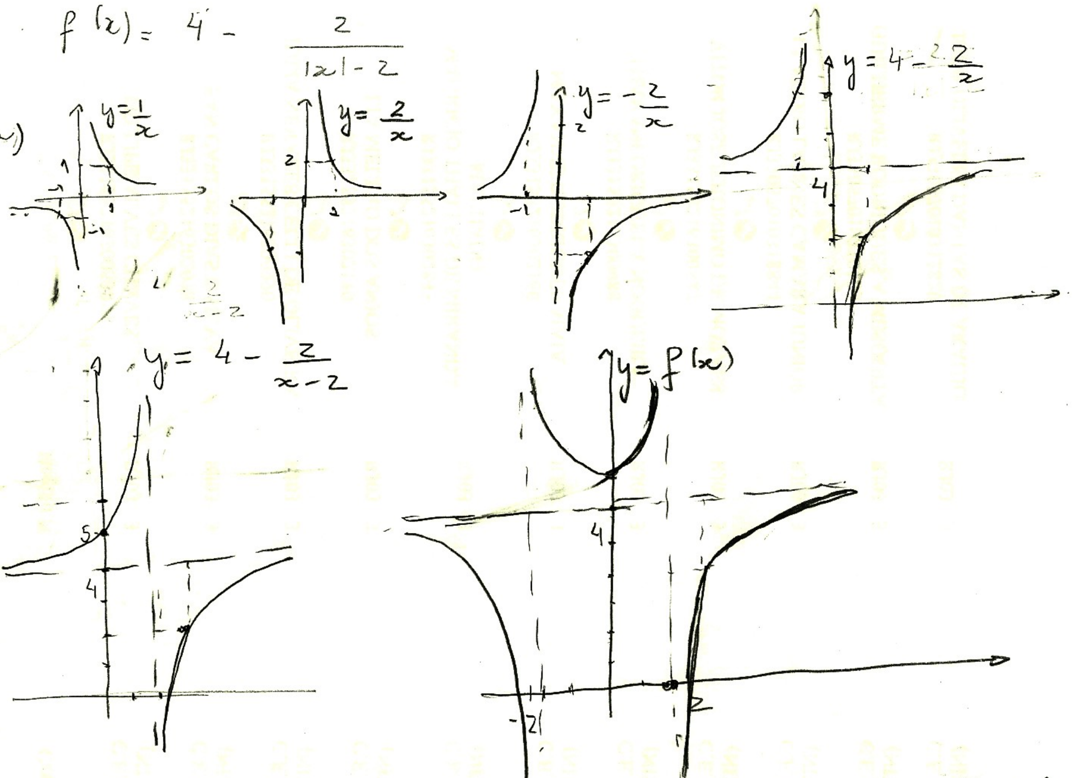
1) $f(x) = 4 - \dfrac{2}{|x|-2}$

(a)



(b) assíntotas verticais: $x = -2$ e $x = 2$, pois

$\lim_{x\to 2^-} 4 - \underbrace{\dfrac{2}{|x|-2}}_{\to 0^-} = +\infty$; $\lim_{x\to 2^+} 4 - \dfrac{2}{\underbrace{|x|-2}_{\to 0^+}} = -\infty$

$\lim_{x\to -2^-} 4 - \dfrac{2}{\underbrace{|x|-2}_{\to 0^+}} = -\infty$; $\lim_{x\to -2^+} 4 - \dfrac{2}{\underbrace{|x|-2}_{\to 0^-}} = \infty$

assíntota horizontal: $y = 4$, pois

$\lim_{x\to \infty} 4 - \underbrace{\dfrac{2}{|x|-2}}_{\to 0} = 4$ e $\lim_{x\to -\infty} 4 - \underbrace{\dfrac{2}{|x|-2}}_{\to 0} = 4$

(c) domínio: x; $|x|-2 \neq 0 \Rightarrow |x| \neq 2 \Leftrightarrow x \neq 2$ e $x \neq -2$

domínio $= (-\infty,-2) \cup (-2,2) \cup (2,\infty)$

imagem: $(-\infty,4) \cup (5,\infty)$

2) $f(x) = \dfrac{x\sqrt{4x^2-1}}{9-x^2}$

(a) $4x^2 - 1 \geq 0 \iff (2x-1)(2x+1) \geq 0$

$$\begin{array}{ccccccc} & + & 0 & - & 0 & + & \\ -3 & & -\frac{1}{2} & & \frac{1}{2} & & 3 \end{array}$$

e $9 - x^2 \neq 0 \iff x^2 \neq 9 \iff x \neq 3$
e $x \neq -3$

f está definida e é contínua nos intervalos:

$(-\infty,-3) \cup (-3,-\frac{1}{2}] \cup [+\frac{1}{2},3) \cup (3,\infty)$

(b) $\displaystyle\lim_{x\to\infty} \frac{x\sqrt{4x^2-1}}{4-x^2} = \lim_{x\to\infty} \frac{x\sqrt{x^2(4-\frac{1}{x^2})}}{x^2(\frac{4}{x^2}-1)} =$

$\displaystyle= \lim_{\substack{x\to+\infty \\ x>0 \\ |x|=x}} \frac{x|x|\sqrt{4-1/x^2}}{x^2(\frac{4}{x^2}-1)} = \lim_{x\to\infty} \frac{\cancel{x^2}\sqrt{4-\cancel{1/x^2}}}{\cancel{x^2}(\frac{4}{x^2}-1)} = \frac{2}{-1} = -2$

$\displaystyle\lim_{x\to-\infty} \frac{x\sqrt{4x^2-1}}{4-x^2} = \lim_{\substack{x\to-\infty \\ x<0 \\ |x|=-x}} \frac{x|x|\sqrt{4-\frac{1}{x^2}}}{x^2(\frac{4}{x^2}-1)} =$

$\displaystyle= \lim_{x\to-\infty} \frac{-x^2\sqrt{4-1/x^2}}{x^2(\frac{4}{x^2}-1)} = \frac{(-1)(2)}{-1} = 2$

Logo as assíntotas horizontais são $y=-2$ e $y=2$

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3) $f(x) = \begin{cases} ax+10 & x \leq 2 \\ \dfrac{2x^3+x^2-8x-4}{x^2-5x+6} & 2 < x < 3 \\ ax+b & x \geq 3 \end{cases}$

$\displaystyle\lim_{x\to2^-} f(x) = \lim_{x\to2^-} 2a+10 = 2a+10$; $f(2) = 2a+10$

$$\begin{array}{c|cccc} & 2 & 1 & -8 & -4 \\ 2 & 2 & 5 & 2 & 0 \end{array}$$

$\displaystyle\lim_{x\to2^+} f(x) = \lim_{x\to2^+} \frac{2x^3+x^2-8x-4}{(x-2)(x-3)} = \frac{0}{0}$

$\displaystyle= \lim_{x\to2^+} \frac{\cancel{(x-2)}(2x^2+5x+2)}{\cancel{(x-2)}(x-3)} = \frac{8+10+2}{2-3} = \frac{+20}{-1} = -20$

Logo para ser contínua em $x=2$; $2a+10=-20 \Rightarrow 2a=-30$ $\boxed{a=-15}$

$\displaystyle\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{\cancel{(x-2)}(2x^2+5x+2)^{\to 45}}{\cancel{(x-2)}(x-3)_{\to 0^-}} = -\infty$

Logo $\nexists \displaystyle\lim_{x\to3^-} f(x)$ e $\nexists$ b; f seja contínua em $x=3$.

4) (a) $$\lim_{x\to 1} \frac{\sqrt{x+3} - \sqrt{x^2-2x+5}}{\underbrace{x^3-1}_{\to 0}} = \frac{0}{0}, \text{ indeterminado}$$

$$\lim_{x\to 1} \frac{(\sqrt{x+3} - \sqrt{x^2-2x+5})(\sqrt{x+3} + \sqrt{x^2-2x+5})}{(x-1)(x^2+x+1)(\sqrt{x+3} + \sqrt{x^2-2x+5})} =$$

$$= \lim_{x\to 1} \frac{(x+3) - (x^2-2x+5)}{(x-1)(x^2+x+1)(\sqrt{x+3} + \sqrt{x^2-2x+5})}$$

$$= \lim_{x\to 1} \frac{x+3-x^2+2x-5}{(x-1)(x^2+x+1)(\sqrt{x+3} + \sqrt{x^2-2x+5})} =$$

$$= \lim_{x\to 1} \frac{3x-3}{(x-1)(x^2+x+1)(\sqrt{x+3} + \sqrt{x^2-2x+5})} =$$

$$= \lim_{x\to 1} \frac{3\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)(\sqrt{x+3} + \sqrt{x^2-2x+5})} = \frac{3}{3(2+2)} = \frac{1}{4}$$ //

(b) $$\lim_{x\to 0^+} \frac{x^2+\operatorname{sen}^2 x}{x \operatorname{sen} x} = \lim_{x\to 0^+} \frac{1 + \frac{\operatorname{sen}^2 x}{x^2}}{\frac{\operatorname{sen} x}{x}} =$$

$$\lim_{x\to 0} \frac{1 + \left(\frac{\operatorname{sen} x}{x}\right)^2}{\underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}} = \frac{1+1^2}{1} = 2$$ //

5) $f(x) = 2x - 3 - \operatorname{sen} 2x$

(a) $f(0) = 2 - 3 - 0 = -1$

$f(\pi/2) = 2\frac{\pi}{2} - 3 - \operatorname{sen}\pi = \pi - 3 > 0$

(i) f é contínua em $[0, \frac{\pi}{2}]$ pois é soma, composta de contínuas, multiplicação por constante

(ii) $k=0$, $\underbrace{-1}_{=f(0)} < k < \underbrace{\pi-3}_{=f(\pi-3)}$,

Logo as hipóteses do Teorema do Valor Intermediário são verdadeiras.

(b) $f(\pi/4) = 2\frac{\pi}{4} - 3 - \operatorname{sen}\frac{\pi}{2} = \frac{\pi}{2} - 3 - 1 = -4 + \frac{\pi}{2} < 0$

$k=0$; $\underbrace{-4+\pi/2}_{=f(\pi/4)} < k < \underbrace{\pi-3}_{=f(\pi/2)} \Rightarrow \exists\, c \in \boxed{(\pi/4, \frac{\pi}{2})}$

amplitude $= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ //