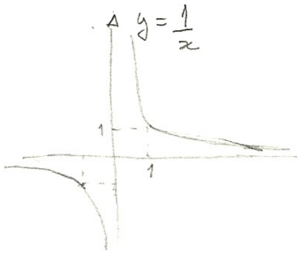
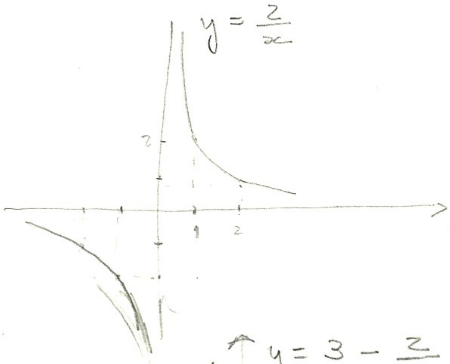
(1) (a) $g(x) = 3 - \dfrac{2}{x-1}$

$y = \dfrac{1}{x}$



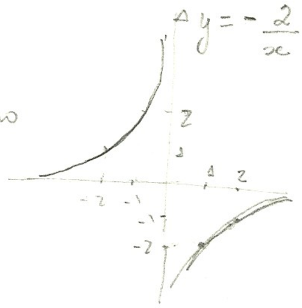
$\xrightarrow[\text{por fator } k=2]{\text{ampliação vertical}}$

$y = \dfrac{2}{x}$



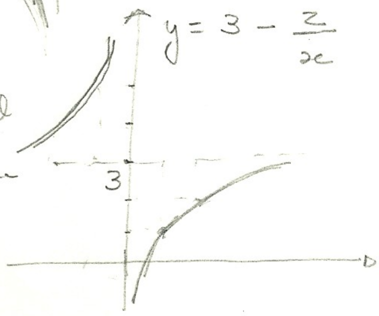
$\xrightarrow[\text{eixo } x]{\text{reflexão no}}$

$y = -\dfrac{2}{x}$



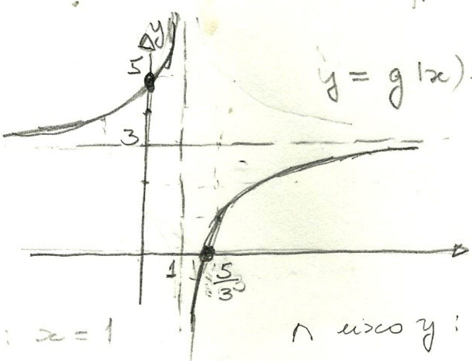
$\xrightarrow[\text{3 unidades p/ cima}]{\text{translação vertical}}$

$y = 3 - \dfrac{2}{x}$



$\xrightarrow[\text{1 unidade p/ direita}]{\text{translação horizontal}}$

$y = g(x) = 3 - \dfrac{2}{x-1}$



assíntota vertical: $x = 1$

assíntota horizontal: $y = 3$

$\cap$ eixo y: $g(0) = 3 - \dfrac{2}{-1} = 5$

$\cap$ eixo x: $3 - \dfrac{2}{x-1} = 0$

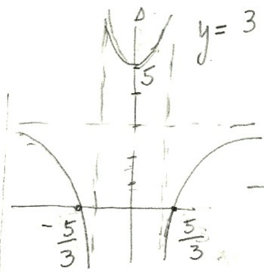
$\dfrac{2}{x-1} = 3$, $x - 1 = \dfrac{2}{3}$, $x = \dfrac{5}{3}$

(b) Partindo do gráfico da função do item (a):

p/ $x < 0$ reflexão no eixo y da parte $x > 0$

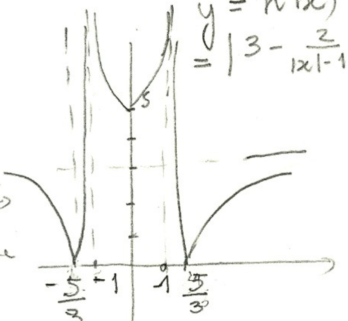
p/ $x > 0$ mantem o gráfico

$y = 3 - \dfrac{2}{|x|-1}$



$\xrightarrow[\text{eixo } x\text{, da parte negativa de } y]{\text{reflexão no}}$

$y = h(x) = \left| 3 - \dfrac{2}{|x|-1} \right|$



assíntotas verticais: $x = 1$ e $x = -1$

assíntota horizontal: $y = 3$

$\cap$ eixo y: $h(0) = 5$

$\cap$ eixo x: $x = \dfrac{5}{3}$ e $x = -\dfrac{5}{3}$

2) (a) $\underbrace{\lim_{x\to 2^-} \frac{\sqrt[3]{3x+2}-2}{x^3-3x^2+4}}_{=} = \frac{\sqrt[3]{8}-2}{8-12+4} = \frac{0}{0}$, indeterminado

	1	-3	0	4
2	1	-1	-2	0

$$= \lim_{x\to 2^-} \frac{(\sqrt[3]{3x+2}-2)}{(x-2)(x^2-x-2)} \times \frac{\left((\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+\sqrt[3]{3x+2}\right)^2}{(\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+(\sqrt[3]{3x+2})^2}$$

$$= \lim_{x\to 2^-} \frac{(3x+2-8)}{(x-2)(x^2-x-2)} \times \frac{1}{\underbrace{(\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+(\sqrt[3]{3x+2})^2}_{=A(x)}}$$

$$= \lim_{x\to 2^-} \frac{3(x-2)}{(x-2)(x^2-x-2)} \times \frac{1}{A(x)} = \lim_{x\to 2^-} \frac{1}{\underbrace{x^2-x-2}_{\to 0^-}} \times \frac{1}{\underbrace{A(x)}_{\to 4+4+4}}$$

$$= \lim_{x\to 2^-} \frac{3}{\underbrace{(x-2)(x+1)}_{\to 0^-}} \times \frac{1}{\underbrace{A(x)}_{\to 12}} = -\infty$$

$x^2-x-2=(x-2)(x+1)$

$+\quad -1 \quad -\quad 2 \quad +$

(b) $\underbrace{\lim_{x\to 0} \frac{x^2-2x\tan(3x)}{1-\cos(2x)}}_{=} = \frac{0-0}{1-1} = \frac{0}{0}$, indeterminado

$$= \lim_{x\to 0} \frac{1-\frac{2\tan(3x)}{x}}{\frac{1-\cos(2x)}{x^2}} = \lim_{x\to 1} \frac{1-2\times 3\,\frac{\sin(3x)}{3x}\cdot\frac{1}{\cos(3x)}}{4\left(\frac{1-\cos 2x}{4x^2}\right)}$$

$$= \frac{1-6\cdot 1\cdot \frac{1}{1}}{4\cdot\frac{1}{2}} = \frac{1-6}{2} = -\frac{5}{2}$$ //

3) (a) $x^2-9\ge 0$ e $x^2-5x+4\ne 0$

$x^2\ge 9 \iff |x|\ge 3 \iff x\ge 3$ ou $x\le -3$

$x^2-5x+4=0$, $x=\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm 3}{2}=\begin{cases}\frac{8}{2}=4\\ \frac{2}{2}=1\end{cases}$

$-3 \quad 1 \quad 3 \quad 4$

$\boxed{\operatorname{dom} f = (-\infty,-3]\cup[3,4)\cup(4,\infty)}$

(c) $\lim_{x\to 4} \frac{x^2+2x\sqrt{x^2-9}}{x^2-5x+4} = \frac{16+8\sqrt{7}}{16-20+4} = \frac{16+8\sqrt{7}}{0}$, abrir

$$\lim_{x\to 4^+} \frac{\overbrace{x^2+2x\sqrt{x^2-9}}^{\to 16+8\sqrt{7}}}{\underbrace{x^2-5x+4}_{\to 0^+}} = \infty$$

$+\quad 1 \quad -\quad 4 \quad +$

$$\lim_{x\to 4^-} \frac{\overbrace{x^2+2x\sqrt{x^2-9}}^{\to 16+8\sqrt{7}}}{\underbrace{x^2-5x+4}_{\to 0^-}} = -\infty$$

Assíntota vertical: $x=4$

ii, Perto de $x=1$, a função não está definida

Logo a única assíntota vertical é $x=4$.

b) $\lim_{x\to\infty} \frac{x^2+2x\sqrt{x^2-9}}{x^2-5x+4} = \lim_{x\to\infty} \frac{x^2+2x\sqrt{x^2\left(1-\frac{9}{x^2}\right)}}{x^2-5x+4} =$

$= \lim_{x\to\infty} \frac{x^2+2x|x|\sqrt{1-\frac{9}{x^2}}}{x^2-5x+4} = \lim_{\substack{x\to\infty \\ |x|=x}} \frac{x^2+2x^2\sqrt{1-\frac{9}{x^2}}}{x^2-5x+4} =$

$= \lim_{x\to\infty} \frac{x^2\left(1+2\sqrt{1-\frac{9}{x^2}}\right)}{x^2\left(1-\frac{5}{x}+\frac{4}{x^2}\right)} = \frac{1+2}{1} = 3$

$\lim_{x\to-\infty} \frac{x^2+2x\sqrt{x^2-9}}{x^2-5x+4} = \lim_{\substack{x\to-\infty \\ |x|=-x}} \frac{x^2-2x^2\sqrt{1-\frac{9}{x^2}}}{x^2-5x+4} =$

$= \lim_{x\to-\infty} \frac{x^2\left(1-2\sqrt{1-\frac{9}{x^2}}\right)}{x^2\left(1-\frac{5}{x}+\frac{4}{x^2}\right)} = \frac{1-2}{1} = -1$

Logo assíntotas horizontais: $y=-1$ e $y=3$

4) $\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} m = m$

$\lim_{x\to-\frac{\pi}{2}^+} f(x) = \lim_{x\to-\frac{\pi}{2}^+} \underbrace{(1+\operatorname{sen} x)}_{\to 0}\operatorname{sen}\left(\frac{1}{4x^2-\pi^2}\right)$

como $\lim_{x\to-\frac{\pi}{2}^+} (1+\operatorname{sen} x) = 0$ e $\operatorname{sen}\left(\frac{1}{4x^2-\pi^2}\right)$ é limitada,

pelo teorema do anulamento, $\lim_{x\to-\frac{\pi}{2}^+} f(x) = 0$

Para f ser contínua em $x=-\pi/2$ é preciso que $\underbrace{\lim_{x\to-\frac{\pi}{2}^-} f(x)}_{=m} = \underbrace{\lim_{x\to-\pi/2^-} f(x)}_{=0} = \underbrace{f\left(-\frac{\pi}{2}\right)}_{=m}$

Logo, $\boxed{m=0}$

$$\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to \pi/2^+} n = n$$

$$\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} \underbrace{(1+\overset{1}{\operatorname{sen} x})}_{\to 2}\operatorname{sen}\left(\overset{-\infty}{\frac{1}{4x^2-\pi^2}}\right),$$

como $\operatorname{sen}\dfrac{1}{4x^2-\pi^2}$ oscila entre -1 e 1,

$f(x)$ oscila entre 2 e -2, $\nexists \lim\limits_{x\to+\pi/2^-} f(x)$,

logo $\nexists$ n tal que f seja contínua em $x=\pi/2$.

5) $f(x) = \dfrac{2+3x-x^4}{x^3-1}$

, não está definida em $x=1$

$f(0) = \dfrac{2}{-1} = -2 < 0$

$f(-1) = \dfrac{2-3-1}{-1-1} = \dfrac{-2}{-2} = 1 > 0$

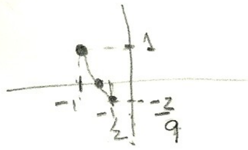
$= -\dfrac{1}{4}\times\dfrac{8}{9} = \dfrac{-2}{9}$

$f\left(-\frac{1}{2}\right) = \dfrac{2+\left(-\frac{3}{2}\right)-\frac{1}{4}}{-\frac{1}{8}-1} = \dfrac{2-\frac{7}{4}}{-\frac{1}{8}-1} = \dfrac{\frac{1}{4}}{-\frac{9}{8}} < 0$

Como f é contínua no intervalo $\left[-1,-\frac{1}{2}\right]$

$-\frac{2}{9} = f\left(-\frac{1}{2}\right) < 0 < f(-1) = 1$

As hipóteses do teorema do valor intermediário estão satisfeitas.



Logo, $\exists\ c \in \left(-1,-\frac{1}{2}\right) = I$

tal que $f(c) = 0$ e amplitude de $I = \dfrac{1}{2}$ //