(Q1) (a) $\lim_{x\to -1} \dfrac{\pi + 4\arctan(x)}{x^2+\cos(\pi x)} = \dfrac{\pi - 4\,\pi/4}{1+(-1)} = \dfrac{0}{0}$, indeterminado

$\overset{L'H}{=} \lim_{x\to -1} \dfrac{4\times \frac{1}{1+x^2}}{2x - \pi\,\mathrm{sen}(\pi x)} = \dfrac{\frac{4}{2}}{-2-0} = -1$ //

(b) $\lim_{x\to 0^+} (1-x)^{\frac{1}{x^2}} = 1^{\infty}$, ind.

$= \lim_{x\to 0^+} e^{\boxed{\frac{1}{x^2}\ln(1-x)}\; g(x)}$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \dfrac{\ln(1-x)}{x^2} = \dfrac{0}{0}$, ind.

$\overset{L'H}{=} \lim_{x\to 0^+} \dfrac{\frac{1}{1-x}(-1)}{2x} = \lim_{x\to 0^+} \dfrac{1}{(x-1).2x} = \dfrac{1}{(-1)\,0^+} = -\infty$

Logo, $\lim_{x\to 0^+} (1-x)^{1/x^2} = e^{-\infty} = 0$ //

(Q2) $f(x) = \dfrac{25(x-3)^{2/5}}{x-9}$

(a) $\mathrm{Dom}\, f = (-\infty, 9) \cup (9, \infty)$

- contínua no domínio pois é quociente de contínuas.

- AH :

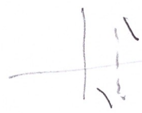
$\lim_{x\to\infty} \dfrac{25(x-3)^{2/5}}{x-9} = \dfrac{\infty}{\infty}$; $\lim_{x\to -\infty} 5(x-3)^{-3/5} = 0$

$\overset{L'H}{=} \lim_{x\to\infty}^{(-\infty)} \dfrac{5(x-3)^{-3/5}}{1} = 0$

Equação de AH : $y = 0$

- AV :

$\lim_{x\to 9^-} \dfrac{25(x-3)^{2/5}}{x-9} = \dfrac{25.\,6^{2/5}}{0^-} = -\infty$



$\lim_{x\to 9^+} \dfrac{25(x-3)^{2/5}}{x-9} = \dfrac{25.\,6^{2/5}}{0^+} = \infty$

Equação de AV : $x = 9$

- Crescimento :

$f'(x) = 25\dfrac{(x-9).\frac{2}{5}(x-3)^{-3/5}(1) - (x-3)^{2/5}(1)}{(x-9)^2} =$

$= 25.\dfrac{\frac{2(x-9)}{5(x-3)^{3/5}} - (x-3)^{2/5}}{(x-9)^2} =$

$= \cancel{25}^{5} \times \dfrac{2(x-9) - 5(x-3)}{\cancel{5}(x-3)^{3/5}(x-9)^2} = 5\,\dfrac{(2x-18-5x+15)}{(x-3)^{3/5}(x-9)^2} =$

continuação de (Q2)

$$f'(x) = 5\,\frac{-3x-3}{(x-3)^{3/5}(x-9)^2} = \frac{-15(x+1)}{(x-3)^{3/5}(x-9)^2}$$

	$(-\infty,-1)$	-1	$(-1,3)$	3	$(3,9)$	9	$(9,\infty)$
$-15(x+1)$	+	0	−	−	−	−	−
$(x-3)^{3/5}$	−	−	−	0	+	+	+
$(x-9)^2$	+	+	+	+	+	0	+
$f'(x)$	−	0	+	$\nexists$	−	$\nexists$	−
crescimento de f	↘ decresc.	↓	↗ cresc.	↓	↘ decresc.		↘ decresc.

$x=-1$ ponto de mín relativo c/ reta tangente horizontal

$x=3$ ponto de máx relativo c/ reta tangente vertical

decrescente: $(-\infty,-1) \cup (3,9) \cup (9,\infty)$

crescente: $(-1,3)$.

$\Delta = \sqrt{4+96} = 10$

$x = \frac{-2\pm 10}{2} = \begin{cases} \frac{8}{2} = 4 \\ \frac{-12}{2} = -6 \end{cases}$

– concavidade

$$f''(x) = \frac{24\,(x^2+2x-24)}{(x-3)^{8/5}(x-9)^3}$$

	$(-\infty,-6)$	-6	$(-6,3)$	3	$(3,4)$	4	$(4,9)$	9	$(9,\infty)$
$24(x^2+2x-24)$	+	0	−	−	−	−	+	0	+
$(x-3)^{8/5}$	+	+	+	0	+	+	+	+	+
$(x-9)^3$	−	−	−	−	−	−	−	0	+
$f''(x)$	−	0	+	$\nexists$	+	0	−	$\nexists$	+
	p/ baixo ∩	↓	p/ cima ∪		p/ cima ∪	↓	p/ baixo ∩		p/ cima ∪

$x=-6$ e $x=4$ são pontos de inflexão.

(b)

$f(-1) = \frac{25\,(-4)^{2/5}}{-10} = -\frac{5}{2}\cdot 2^{2/5}$

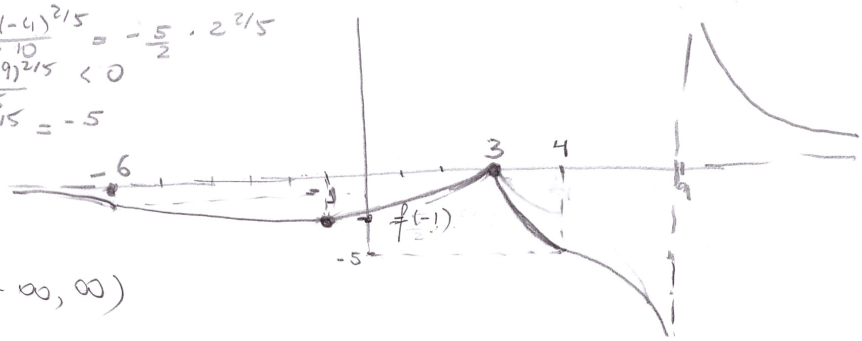
$f(-6) = \frac{25\,(-9)^{2/5}}{-15} < 0$

$f(4) = \frac{25(1)^{2/5}}{-5} = -5$

$f(0) =$

$f(3) = \frac{25(0)}{-6}$

imagem $= (-\infty, \infty)$



Q3) $f(x) = \operatorname{senh}(x)$ $\quad f(0) = 0$

$f'(x) = \cosh(x)$ $\quad f'(0) = 1$

$f''(x) = \operatorname{senh}(x)$ $\quad f''(0) = 0$

$f'''(x) = \cosh(x)$ $\quad f'''(0) = 1$

$\vdots$ $\quad f^{(2n)}(0) = 0 \quad , f^{(2n+1)}(0) = 1$

$$P(x) = 0 + \frac{1}{1!}(x-0) + \frac{1}{3!}(x-0)^3 + \frac{1}{5!}(x-0)^5 + \cdots$$

$$P(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!}$$

$\frac{27}{6} = \frac{9}{2}$

$$f(0.03) \simeq 0.03 + \frac{(0.03)^3}{6} = 0.03 + (10^{-2})^3 \times \frac{3^3}{6} = 0{,}03 + \frac{9}{2} \cdot 10^{-6}$$

$$= 0{,}03 + 4{,}5 \times 10^{-6} = 0.03 + 0{,}0000045$$

$$\boxed{f(0.03) \simeq 0.0300045}$$

Q4) Custo = 10× Área da base + 5 área lateral

$= 10 \times \pi r^2 + 5 \times 2\pi r h$

Custo $= 10\pi(r^2 + rh)$

Volume $= \pi r^2 h = 16000\pi$

$h = \frac{16.000}{r^2}$

Logo, custo $= 10\pi \left(r^2 + r\,\frac{16.000}{r^2}\right)$

$C(r) = 10\pi\left(r^2 + \frac{16.000}{r}\right)$ $\quad r > 0$

$c'(r) = 10\pi\left(2r - \frac{16000}{r^2}\right) = 10\pi\left(\frac{2r^3 - 16.000}{r^2}\right)$

$2r^3 - 16.000$

$r^3 = 8000$

$r = 20$

	$(0,20)$	20	$(20,\infty)$
$2r^3 - 16.000$	$-$	0	$+$
$c'(r)$	$\searrow$		$\nearrow$

$r = 20$ é ponto de mínimo relativo absoluto de f.

$h = \frac{16.000}{400} = 40$

Logo as dimensões são $r = 20$ cm e $h = 40$ cm.

(Q5) (a) $\int \left(\sqrt{x^3} + \frac{2}{x} - \frac{3}{x^2}\right) dx = \int \left(x^{3/2} + \frac{2}{x} - 3x^{-2}\right) dx =$

$$= \frac{x^{5/2}}{5/2} + \ln|x| - 3\,\frac{x^{-1}}{-1} + C = \frac{2}{5}x^{5/2} + \ln|x| + \frac{3}{x} + C$$

(b) $f'(x) = \frac{4 - 3\cos(x)}{\operatorname{sen}^2 x} = 4\csc^2 x - \frac{3\cos x}{\operatorname{sen} x} \cdot \frac{1}{\operatorname{sen} x}$

$f'(x) = 4\csc^2 x - 3\cot x \cdot \csc x$

$f(x) = \int (4\csc^2 x - 3\cot x \csc x)\, dx$

$f(x) = 4(-\cot x) + 3\csc x + C$