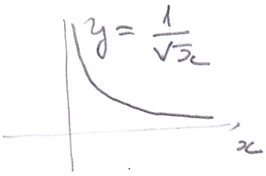
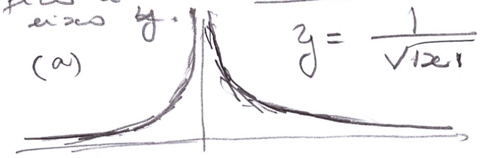
(Q1) (a) $f(x) = \dfrac{1}{\sqrt{x}}$ $\xrightarrow[\text{de parte do gráfico à direita do eixo } y.]{\text{reflexão no eixo } y}$ $\boxed{g(x) = \dfrac{1}{\sqrt{|x|}}}$





(b) $|x| \geq 0 \quad \forall x$ e $|x| = 0 \iff x = 0$.

logo dom $g = \{x;\ x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

im $g = (0, \infty)$.

(c) Como dom $g = (-\infty, 0) \cup (0, \infty) \Rightarrow 0 \notin$ dom $g \Rightarrow$

o gráfico não corta o eixo y.

Como im $g = (0, \infty) \Rightarrow y > 0 \Rightarrow$ não corta o eixo x

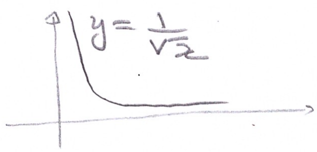
d) $\displaystyle\lim_{x \to \infty} \frac{1}{\sqrt{|x|}} = \lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$

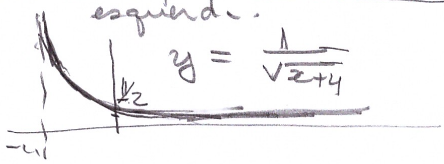
$\displaystyle\lim_{x \to -\infty} \frac{1}{\sqrt{|x|}} = \lim_{x \to -\infty} \frac{1}{\sqrt{-x}} = 0$ $\Rightarrow$ $\boxed{y = 0}$ é a única assíntota horizontal.

$\displaystyle\lim_{x \to 0^+} \frac{1}{\sqrt{|x|}} = \lim_{x \to 0^+} \frac{1}{\sqrt{x}} = \infty$ $\Rightarrow$ $\boxed{x = 0}$ é assíntota vertical.

ou $\displaystyle\lim_{x \to 0^-} \frac{1}{\sqrt{|x|}} = \lim_{x \to 0^-} \frac{1}{\sqrt{-x}} = \infty$.

(a) $f(x) = \dfrac{1}{\sqrt{x}}$ $\xrightarrow[\text{4 unidades p/ esquerda.}]{\text{translação horizontal}}$ $\boxed{h(x) = \dfrac{1}{\sqrt{x+4}}}$





(b) $\begin{cases} x + 4 \geq 0 \\ x + 4 \neq 0 \end{cases} \Rightarrow x + 4 > 0 \Rightarrow x > -4 \Rightarrow$ dom $h = (-4, \infty)$

$\sqrt{x+4} > 0 \Rightarrow \dfrac{1}{\sqrt{x+4}} > 0 \Rightarrow$ im $h = (0, \infty)$

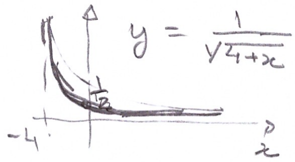
(c) $h(0) = \dfrac{1}{\sqrt{0+4}} = \dfrac{1}{2} \Rightarrow$ o gráfico corta o eixo y em $y = \frac{1}{2}$.

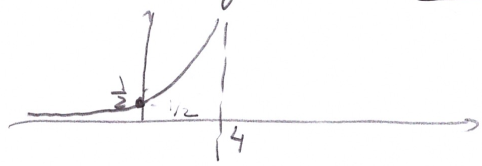
Como im $g = (0, \infty) \Rightarrow y > 0 \Rightarrow$ o gráfico não corta o eixo x.

(d) $\displaystyle\lim_{x \to \infty} \frac{1}{\sqrt{x+4}} = 0 \Rightarrow y = 0$ é assíntota horizontal.

lim ... é assíntota vertical

(1) (a) $h(x) = \dfrac{1}{\sqrt{x+4}} = \dfrac{1}{\sqrt{4+x}} \xrightarrow[\text{eixo } y]{\text{reflexão no}} \boxed{r(x) = \dfrac{1}{\sqrt{4-x}}}$





(b) $4 - x > 0 \Rightarrow 4 > x \Rightarrow x < 4 \Rightarrow \text{dom } r = (-\infty, 4)$
$\text{im } g = (0, \infty)$

(c) $r(0) = \dfrac{1}{\sqrt{4-0}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2} \Rightarrow$ corta o eixo y em $y = \dfrac{1}{2}$.

como $\text{im } g = (0,\infty) \Rightarrow$ não corta o eixo x.

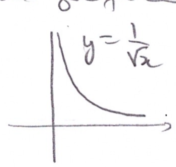
(d) $\lim\limits_{x \to -\infty} \dfrac{1}{\underbrace{\sqrt{4-x}}_{\to\infty}} = 0 \Rightarrow y = 0$ é assíntota horizontal

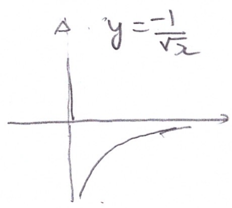
$\lim\limits_{x \to 4^-} \dfrac{1}{\underbrace{\sqrt{4-x}}_{0^+}} = \infty \Rightarrow x = 4$ é assíntota vertical.

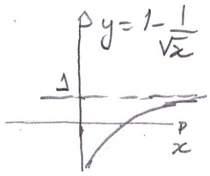
——— x ——— x ——— x ———

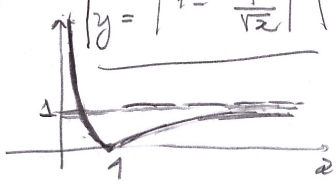
(2) (a) $y = \dfrac{1}{\sqrt{x}} \xrightarrow[\text{no eixo } x]{\text{reflexão}} y = \dfrac{-1}{\sqrt{x}} \xrightarrow[\text{1 un. p/ cima}]{\text{translação vertical}} y = 1 - \dfrac{1}{\sqrt{x}}$

$\xrightarrow[\text{da parte abaixo do gráfico}]{\text{reflexão no eixo } x} y = s(x) = \left|1 - \dfrac{1}{\sqrt{x}}\right|$









(b) $\text{dom } s = (0, \infty)$ $\quad$ $\text{im } s = [0, \infty)$

(c) como $\text{dom } s = (0,\infty) \Rightarrow \nexists\, s(0) \Rightarrow$ o gráfico não corta o eixo y

$\left|1 - \dfrac{1}{\sqrt{x}}\right| = 0 \Leftrightarrow 1 - \dfrac{1}{\sqrt{x}} = 0 \Leftrightarrow \dfrac{1}{\sqrt{x}} = 1 \Leftrightarrow \sqrt{x} = 1 \Leftrightarrow \boxed{x = 1}$

o gráfico corta o eixo x em $\boxed{x = 1}$.

d) $\lim\limits_{x \to \infty} \left|1 - \cancelto{0}{\dfrac{1}{\sqrt{x}}}\right| = 1 - 0 = 1 \Rightarrow y = 1$ é assíntota horizontal

$\lim\limits_{x \to 0^+} \left|1 - \cancelto{\infty}{\dfrac{1}{\underbrace{\sqrt{x}}_{0^+}}}\right| = \infty \Rightarrow x = 0$ é assíntota vertical.

(Q2) (a) $\lim_{x\to\infty} \left(\frac{4x^3+x}{2-x^3}\right) \operatorname{sen}\left(\frac{\pi}{6}+\frac{1}{x}\right)$

$\lim_{x\to\infty} \frac{4x^3+x}{2-x^3} = \lim_{x\to\infty} \frac{\cancel{x^3}\left(4+\frac{1}{x^2}\right)}{\cancel{x^3}\left(\frac{2}{x^3}-1\right)} = \frac{4}{-1} = -4$

$\lim_{x\to\infty} \operatorname{sen}\left(\frac{\pi}{6}+\frac{1}{x}\right) = \operatorname{sen}\left(\frac{\pi}{6}\right) = \frac{1}{2}$

logo $\lim_{x\to\infty} \left(\frac{4x^3+x}{2-x^3}\right) \operatorname{sen}\left(\frac{\pi}{6}+\frac{1}{x}\right) = (-4).\frac{1}{2} = -2$ //

(b) $\lim_{x\to 2} \frac{\sqrt{x+3}-\sqrt{2x+1}}{x^3-8} = \frac{\sqrt{5}-\sqrt{5}}{8-8} = \frac{0}{0}$, indeterminado

$= \lim_{x\to 2} \frac{\sqrt{x+3}-\sqrt{2x+1}}{(x-2)(x^2+2x+4)} \times \frac{\sqrt{x+3}+\sqrt{2x+1}}{\sqrt{x+3}+\sqrt{2x+1}} =$

$= \lim_{x\to 2} \frac{\overbrace{(x+3)-(2x+1)}^{=-x+2}}{(x-2)(x^2+2x+4)} \times \frac{1}{\sqrt{x+3}+\sqrt{2x+1}} =$

$= \lim_{x\to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)} \times \frac{1}{\sqrt{x+3}+\sqrt{2x+1}}$

$= \frac{-1}{\underbrace{4+4+4}_{12}} \times \frac{1}{\underbrace{\sqrt{5}+\sqrt{5}}_{2\sqrt{5}}} = -\frac{1}{24\sqrt{5}}$ //

(c) $\lim_{x\to -\infty} \frac{x-2\sqrt{x^2+1}}{x+1} = \lim_{x\to -\infty} \frac{x-2\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x+1} =$

$= \lim_{x\to -\infty} \frac{x-2\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}{x+1} =$ $\left(\begin{array}{l} x\to -\infty \\ x<0 \\ \sqrt{x^2}=|x|=-x \end{array}\right)$

$= \lim_{x\to -\infty} \frac{x+2x\sqrt{1+\frac{1}{x^2}}}{x+1} =$

$= \lim_{x\to -\infty} \frac{\cancel{x}\left(1+2\sqrt{1+\frac{1}{x^2}}\right)}{\cancel{x}\left(1+\frac{1}{x}\right)} = \frac{1+2}{1} = 3$ //

(Q3) Para que f seja contínua em $x=0$ é preciso que: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{a^2 \operatorname{sen}(4x)}{x} = \lim_{x\to 0^-} a^2 4. \frac{\operatorname{sen}(4x)}{4x} = a^2.4.1 = 4a^2$.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} b^2 + \left(\frac{1-\cos x}{x}\right) \underbrace{\operatorname{sen}\left(\frac{1}{x}\right)}_{\text{limitada}} = b^2+0 = b^2$

$f(0) = b$

Logo, $\boxed{b^2=b} \Leftrightarrow b^2-b=0 \Leftrightarrow b(b-1)=0 \Leftrightarrow b=1$ ou $b=0$

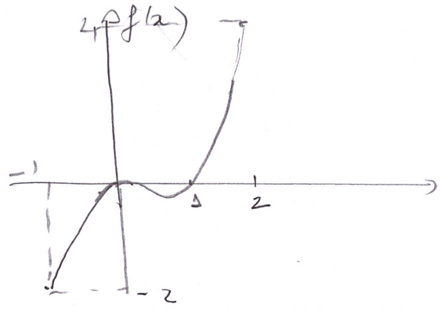
$4a^2=b \Rightarrow 4a^2=0$ ou $4a^2=1$

Logo quando $b=0$, $a=0$ e quando $b=1$, $a^2=\frac{1}{4}$, $a=\pm\frac{1}{2}$

(Q3)(continuação)

$(a,b)=(0,0)$ ou $(a,b)=(\frac{1}{2},1)$ ou $(a,b)=(-\frac{1}{2},1)$

(Q4) $f(-1)=(-1)^3-(-1)^2=-2$
$f(2)=(2)^3-2^2=8-4=4$



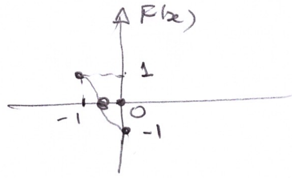
$y=4x+1$, chamando

$g(x)=4x+1$

$f(x)=g(x)$ se e só se

$f(x)-g(x)=0$

$\underbrace{x^3-x^2-4x-1}_{F(x)}=0$



$F(0)=0-0-0-1=-1$

$F(-1)=-1-1+4-1=1$

$F(0)=-1<0<1=F(-1)$

Como $F(x)$ é contínua em $[-1,0]$

e para $k=0$, $F(0)<0<F(-1)$,

pelo Teorema do valor intermediário,

$\exists\, c\in(-1,0)$ tal que $F(c)=0$ ou $f(c)=g(c)$.

Logo como $c\in(-1,0)$, $c<0$.

Determinando um intervalo de amplitude $\frac{1}{2}$.

$F(-\frac{1}{2})=(-\frac{1}{2})^3-(-\frac{1}{2})^2-4(-\frac{1}{2})-1$

$=-\frac{1}{8}-\frac{1}{4}+2-1=1-\frac{1}{8}-\frac{1}{4}=\frac{8-1-2}{8}=\frac{5}{8}>0$

Logo $F(0)<0<F(\frac{-1}{2})=\frac{5}{8}$, pelo Teorema do Valor Intermediário, como F é contínua,

$\exists\, c\in(-\frac{1}{2},0)$ tal que $F(c)=0$.

O intervalo de amplitude $\frac{1}{2}$ é $(-\frac{1}{2},0)$.