Q1) $f(x) = \begin{cases} x^2 + \operatorname{sen} x & x < 0 \\ kx\cos(x) & x \geq 0 \end{cases}$, $f(0) = k.0.\cos 0 = 0$

(a) $f'_-(0) = \lim\limits_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0^+} \dfrac{(x^2 + \operatorname{sen} x) - \overbrace{f(0)}^{=0}}{x}$

$= \lim\limits_{x \to 0^+} x + \dfrac{\operatorname{sen} x}{x} = 0 + 1 = 1$

$f'_+(0) = \lim\limits_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0^-} \dfrac{kx\cos x - \overbrace{f(0)}^{=0}}{x}$

$= \lim\limits_{x \to 0^-} k\cos x = k \cdot 1 = k$

(b) Para que f seja diferenciável em $x = 0$, é preciso que $f'_-(0) = f'_+(0)$

Logo $\boxed{k = 1}$

Q2)(a) $f(x) = (x+1)\ln(g(x))$, $g(0) = e$, $g'(0) = 3$

$f'(x) = (x+1) \cdot \dfrac{1}{g(x)} \cdot g'(x) + \ln(g(x))$

$f'(0) = (0+1) \cdot \dfrac{1}{g(0)} \cdot g'(0) + \ln(g(0))$

$= \dfrac{1}{e} \cdot 3 + \ln(e) = 1 + \dfrac{3}{e}$ //

(b) $F(x) = G\left(\operatorname{arctan}\left(\dfrac{x}{x-1}\right)\right)$.

$F'(x) = G'\left(\operatorname{arctan}\left(\dfrac{x}{x-1}\right)\right) \cdot \left(\dfrac{1}{1 + \left(\dfrac{x}{x-1}\right)^2}\right) \cdot \dfrac{\overbrace{(x-1)(1) - x(+1)}^{x-1-x=-1}}{(x-1)^2}$

$x = \dfrac{1}{2} \Rightarrow \dfrac{\frac{1}{2}}{\frac{1}{2} - 1} = \dfrac{\frac{1}{2}}{-\frac{1}{2}} = -1$.

$F'\left(\frac{1}{2}\right) = G'\left(\underbrace{\operatorname{arctan}(-1)}_{= -\pi/4}\right) \cdot \dfrac{1}{1 + (-1)^2} \cdot \dfrac{-1}{(-1/2)^2}$

$= \underbrace{G'\left(-\frac{\pi}{4}\right)}_{= 2} \cdot \dfrac{1}{2} \times \dfrac{-1}{\frac{1}{4}} = 2 \times \dfrac{1}{2} \times (-4) = -4$ //

(Q 3) $xy^2 - y^x = 9$

$xy^2 - e^{x \ln y} = 9$, derivando [illegible] y'

$x \cdot 2y \cdot y' + y^2 - e^{x \ln y}\left(x \cdot \frac{1}{y} \cdot y' + \ln y\right) = 0$

$x = 2,\ y = 3$;

$2 \times 2 \times 3y' + 9 - 3^2\left(\frac{2}{3}y' + \ln 3\right) = 0$

$12y' + 9 - 6y' - 9\ln 3 = 0$

$6y' = -9 + 9\ln 3$

$y' = \dfrac{-9 + 9\ln 3}{6}$

$y' = \dfrac{-3 + 3\ln 3}{2} \implies y'(2) = \dfrac{-3 + 3\ln 3}{2}$

Eq. da reta tangente

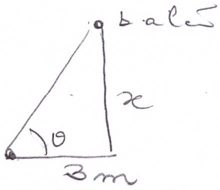
$y - y(2) = y'(2)(x - 2)$

$y - 3 = -\dfrac{3 + 3\ln 3}{2}(x - 2)$

$y = \left(\dfrac{-3 + 3\ln 3}{2}\right)x + 3 + 3 - 3\ln 3$

$y = \left(\dfrac{-3 + 3\ln 3}{2}\right)x + 6 - 3\ln 3$ //

(Q 4) $\dfrac{x}{3} = \tan\theta$



$x = 3\tan\theta$

$\dfrac{dx}{dt} = 3(\sec^2\theta)\dfrac{d\theta}{dt}$

$\theta = \frac{\pi}{3}$ rad $\quad \dfrac{d\theta}{dt} = 0.02$ rad/min

$\dfrac{dx}{dt} = 3\left(\sec^2\left(\frac{\pi}{3}\right)\right) \times 0.02$

$\sec\frac{\pi}{3} = \dfrac{1}{\cos\frac{\pi}{3}} = \dfrac{1}{\frac{1}{2}} = 2$

$\dfrac{dx}{dt} = 3 \times 4 \times 0.02 = 12 \times (0.02) = 0,24$

a velocidade do balão é 0,24 m/min

(Q5) $s(t) = \frac{t^3}{9} + t$

v_m = velocidade média = $\frac{s(6) - s(3)}{6-3}$.

$s(6) = \frac{6^3}{9} + 6 = \frac{2^3 \cdot 3^{3}}{9} + 6 = 24 + 6 = 30$

$s(3) = \frac{3^3}{9} + 3 = 3 + 3 = 6$,

$v_m = \frac{30-6}{6-3} = \frac{24}{3} = 8$ m/s.

Teorema do Valor Médio:

Hipóteses: $s(t) = \frac{t^3}{9} + t$ é contínua no intervalo $[3, 6]$ e diferenciável em $(3, 6)$.

Tese: existe $c \in (3, 6)$ tal que a velocidade no instante $c = s'(c) = \frac{s(6)-s(3)}{6-3} = 8$.

$s'(t) = \frac{3t^2}{9} + 1 = \frac{t^2}{3} + 1$

$s'(3) = 8 \Rightarrow \frac{t^2}{3} + 1 = 8$

$\frac{t^2}{3} = 7$

$t^2 = 21$

$t = \pm\sqrt{21}$, $t \geq 0$

$\boxed{t = \sqrt{21} \text{ seg}}$