(Q1)(a) $\lim\limits_{x \to 0} \dfrac{1-\cos(2x) + x\,\text{sen}(3x)}{x^2} = \dfrac{1-1+0}{0} = \dfrac{0}{0}$

$\overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{2\,\text{sen}\,2x + 3x\cos(3x) + \text{sen}(3x)}{2x} = \dfrac{0+0+0}{0} = \dfrac{0}{0}$

$\overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{4\cos 2x - 9\,\text{sen}(3x) + 3\cos 3x + 3\cos 3x}{2}$

$= \dfrac{4 - 0 + 3 + 3}{2} = \dfrac{10}{2} = 5$ //

Outra resolução:

$\lim\limits_{x \to 0} \dfrac{1-\cos 2x + x\,\text{sen}(2x)}{x^2} = \lim\limits_{x \to 0} \dfrac{1-\cos 2x}{x^2} + \dfrac{\text{sen}(2x)}{x}$

$= \lim\limits_{x \to 0} \dfrac{1-\cos(2x)}{(2x)^2} \cdot 4 + 2 \cdot \dfrac{\text{sen}\,2x}{2x} = \dfrac{1}{2} \cdot 2 + 2 = 3$ //

(b) $\lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^{3x} = 1^{\infty}$, indeterminado.

$\lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^{3x} = \lim\limits_{x \to \infty} e^{\underbrace{3x \cdot \ln\left(1+\frac{2}{x}\right)}_{f(x)}} =$

$\lim\limits_{x \to \infty} 3x \cdot \ln\left(1 + \dfrac{2}{x}\right) = \infty \cdot \ln(1) = \infty \cdot 0$, indeterminado

$= \lim\limits_{x \to \infty} \dfrac{3 \cdot \ln\left(\frac{x+2}{x}\right)}{\frac{1}{x}} = \lim\limits_{x \to \infty} \dfrac{3 \cdot \frac{x}{x+2} \cdot \left(\frac{-2}{x^2}\right)}{-\frac{1}{x^2}} =$

$= \lim\limits_{x \to \infty} 6 \cdot \dfrac{x}{x+2} = 6 \cdot 1 = 6$ // $\left\{ \lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^{3x} = e^{6} \right.$

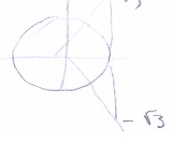
(Q2) $f(x) = x \cdot \text{arctan}(g(x))$

$f'(x) = x \cdot \dfrac{1}{1+(g(x))^2} \cdot g'(x) + \text{arctan}(g(x))$.

$f'(3) = 3 \, \dfrac{1}{1+(g(3))^2} \cdot g'(3) + \text{arctan}(g(3))$

$f'(3) = 3 \cdot \dfrac{1}{1+3} \cdot 5 + \text{arctan}(-\sqrt{3})$ $\left\{ \begin{array}{l} g(3) = -\sqrt{3} \\ g'(3) = 5 \end{array} \right.$

$= \dfrac{15}{4} - \dfrac{\pi}{3}$



(Q3) $y\sqrt{x} + x\sqrt{y} = 6$ $\qquad f(1) = 4$

Reta tangente: $y - f(1) = f'(1)(x-1)$

$f'(x) = ?$, derivando implicitamente a equação:

$$y \cdot \frac{1}{2\sqrt{x}} + \sqrt{x}\,y' + x \cdot \frac{1}{2\sqrt{y}} \cdot y' + \sqrt{y} = 0$$

$x = 1,\ y = 4$:

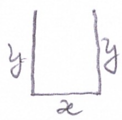
$$4 \cdot \frac{1}{2} + 1 \cdot y' + 1 \cdot \frac{1}{2 \cdot 2} y' + 2 = 0$$

$$\left(1 + \frac{1}{4}\right) y' = -4$$

$$\frac{5}{4}\, y' = -4 \implies y' = -\frac{16}{5}$$

Reta tangente: $\boxed{y - 4 = -\frac{16}{5}(x-1)}$

——————— × ——————— × ———————

(Q4) 

$xy = 8. \implies y = \frac{8}{x}$, $y > 0, x > 0$

Comprimento $= x + 2y = x + \frac{16}{x}$, $x > 0$ ou $x \in (0, \infty)$

$f(x) = x + \frac{16}{x}$

$f'(x) = 1 - \frac{16}{x^2} = \frac{x^2 - 16}{x^2}$

		-4	$(-4,4)$	4	$(4,\infty)$
x^2-16			$-$		$+$
x^2	$+$	$+$	$+$	$+$	$+$
$f'(x)$			$-$		$+$
			$\searrow$		$\nearrow$

, f tem um único ponto crítico no intervalo $(0, \infty)$: $x = 2$ é ponto de mínimo relativo e absoluto de f.

Solução: base $= 4$
altura $= \frac{4}{2} = 2.$

——————— × ——————— × ———————

(Q5) $\displaystyle\int \frac{2x^4 - 4x^3 - 3x^2 + 5\sqrt[3]{x}}{x^3}\, dx =$

$\frac{1}{3} - 3 = -\frac{8}{3}$

$-\frac{8}{3} + 1 = -\frac{5}{3}$

$$= \int \left(2x - 4 - \frac{3}{x} + 5x^{-8/3}\right) dx$$

$$= \frac{2x^2}{2} - 4x - 3\ln|x| + 5\frac{x^{-5/3}}{-5/3} + C$$

$$= x^2 - 4x - 3\ln|x| - 3\, x^{-5/3} + C \quad //$$

(Q6) $f(x) = 1 + \frac{3}{x} - \frac{1}{x^3} = \frac{x^3+3x^2-1}{x^3}$

dom $f = (-\infty, 0) \cup (0, \infty)$, contínua no domínio pois é quociente de polinômios (contínuos)

AV: $\displaystyle\lim_{x \to 0^+} \frac{x^3+3x^2-1}{x^3} = \frac{-1}{0^+} = -\infty$

$\displaystyle\lim_{x \to 0^-} \frac{x^3+3x^2-1}{x^3} = \frac{-1}{0^-} = +\infty$

$\Rightarrow$ $x = 0$ é A.V.

AH: $\displaystyle\lim_{x \to \infty} 1 + \frac{3}{x} - \frac{1}{x^3} = 1 + 0 + 0 = 1$

$\displaystyle\lim_{x \to -\infty} 1 + \frac{3}{x} - \frac{1}{x^3} = 1 + 0 - 0 = 1$

$\Rightarrow$ $y = 1$ é A.H.

Pontos críticos: x; $f'(x) = 0$.

$$f'(x) = 0 - \frac{3}{x^2} + \frac{3}{x^4} = 3\left(-\frac{1}{x^2} + \frac{1}{x^4}\right) = \frac{3(-x^2+1)}{x^4}$$

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$3(-x^2+1)$	$-$	0	$+$	$+$	$+$	0	$-$
x^4	$+$	$+$	$+$	0	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	$\nexists$	$+$	0	$-$
crescimento de f	$\searrow$		$\nearrow$		$\nearrow$		$\searrow$

$x = -1$ é ponto de mín relativo

$x = 1$ é " " máx relativo

$$f'(x) = 3\left(-\frac{1}{x^2} + \frac{1}{x^4}\right) = 3\left(-x^{-2} + x^{-4}\right)$$

$$f''(x) = 3\left(\frac{2}{x^3} - \frac{4}{x^5}\right) = 6\left(\frac{x^2-2}{x^5}\right)$$

	$(-\infty, -\sqrt{2})$	$-\sqrt{2}$	$(-\sqrt{2}, 0)$	0	$(0, \sqrt{2})$	$\sqrt{2}$	$(\sqrt{2}, \infty)$
$6(x^2-2)$	+	0	−	−	−	0	+
x^5	−	−	−	0	+	+	+
$f''(x)$	−	0	+	$\nexists$	−	0	+
concavidade	$\frown$		$\smile$		$\frown$		$\smile$

pontos de inflexão: $x = -\sqrt{2}$ e $x = \sqrt{2}$.

Cálculo de $f(x) = 1 + \frac{3}{x} - \frac{1}{x^3}$ em alguns pontos importantes:

$f(1) = 1 + 3 - 1 = 3$

$f(-1) = 1 - 3 + 1 = -1$

$f(\sqrt{2}) = 1 + \frac{3}{\sqrt{2}} - \frac{1}{2\sqrt{2}} = 1 + \frac{6-1}{2\sqrt{2}} = 1 + \frac{5}{\sqrt{2}}$

$f(-\sqrt{2}) = 1 - \frac{3}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = 1 \frac{-6+1}{2\sqrt{2}} = 1 - \frac{5}{\sqrt{2}}$

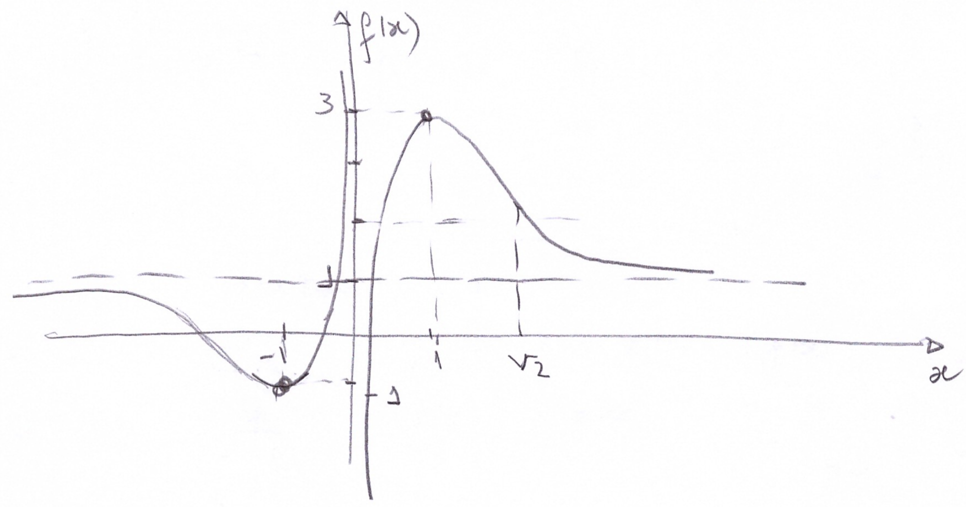


imagem $= (-\infty, \infty)$

não possui máx nem mín absoluto pois
$x \to 0^+,\ f(x) \to -\infty$
$x \to 0^-,\ f(x) \to +\infty$