(Q1)(a) $\lim\limits_{x\to-\infty} \dfrac{x\sqrt{x^2-1}}{2+x^2} = \lim\limits_{x\to-\infty} \dfrac{x\sqrt{x^2(1-\frac{1}{x^2})}}{2+x^2} =$

$= \lim\limits_{x\to-\infty} \dfrac{x\,|x|\sqrt{1-\frac{1}{x^2}}}{2+x^2} = \lim\limits_{x\to-\infty} \dfrac{-x^2\sqrt{1-1/x^2}}{2+x^2} =$

$x<0,\ |x|=-x$

$= \lim\limits_{x\to-\infty} \dfrac{-\sqrt{1-\frac{1}{x^2}}}{\frac{2}{x^2}+1} = \dfrac{-\sqrt{1-0}}{0+1} = \dfrac{-1}{1} = -1$ //

(b) $\lim\limits_{x\to0} \dfrac{\sin(3x^2)}{3x^2-\tan^2(2x)} = \lim\limits_{x\to0} \dfrac{3\cdot\frac{\sin(3x^2)}{3x^2}}{3-4\cdot\frac{\tan^2(2x)}{4x^2}}$ $\quad \to 1,\ \ 1^2=1$

$= \dfrac{3}{3-4} = -3$ //

(Q2)(a) $f(x) = \sqrt[3]{\sec^2(\frac{x}{3})} = \left(\sec\left(\frac{x}{3}\right)\right)^{2/3}$

$f'(x) = \dfrac{2}{3}\left(\sec\left(\frac{x}{3}\right)\right)^{-1/3}\left(\sec\left(\frac{x}{3}\right)\cdot\tan\left(\frac{x}{3}\right)\right)\cdot\dfrac{1}{3}$

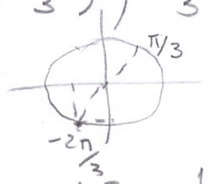
$f'(-2\pi) = \dfrac{2}{3}\left(\sec\left(-\frac{2\pi}{3}\right)\right)^{-1/3}\left(\sec\left(-\frac{2\pi}{3}\right)\cdot\tan\left(-\frac{2\pi}{3}\right)\right)\cdot\dfrac{1}{3}$

$f'(-2\pi) = \dfrac{2}{3}(-2)^{-1/3}\cdot\left((-2)\cdot\sqrt{3}\right)\dfrac{1}{3}$

$= -\dfrac{2}{3\cdot\sqrt[3]{2}}\cdot\left(\dfrac{-2\sqrt{3}}{3}\right) = \dfrac{+4\sqrt{3}}{9\sqrt[3]{2}}$ //

$= \dfrac{4\sqrt{3}}{9}\cdot\dfrac{\sqrt[3]{4}}{2}$

$= \dfrac{2}{9}\cdot\sqrt{3}\cdot\sqrt[3]{4}$ //



$\sec\left(\frac{-2\pi}{3}\right) = \dfrac{1}{\cos\left(-\frac{2\pi}{3}\right)} = \dfrac{1}{-1/2} = -2$

$\tan\left(\frac{-2\pi}{3}\right) = \dfrac{\sin\left(-\frac{2\pi}{3}\right)}{\cos\left(-\frac{2\pi}{3}\right)} = \dfrac{-\sqrt{3}/2}{-1/2} = +\sqrt{3}$

(Q2)(b) $F(x) = \dfrac{(x+g(x))^3}{4-g(4x)}$, $F'(1) = ?$

$F'(x) = \dfrac{(4-g(4x))\cdot\left(3(x+g(x))^2\cdot(1+g'(x))\right) - (x+g(x))^3\,(-g'(4x))\cdot4}{(4-g(4x))^2}$

$F'(1) = \dfrac{\underbrace{(4-g(4))}_{=1}\cdot\left(3(1+\overset{0}{g(1)})^2\right)\cdot(1+\overset{2}{g'(1)}) - (1+g(1))^3\,(-g'(4))\cdot4}{(4-\underbrace{g(4)}_{=3})^2}$

$g(1)=0$
$g(4)=3$
$g'(1)=2$
$g'(4)=4$

$= \dfrac{1\cdot3\cdot1^2\cdot3 - (1)(-4)\cdot4}{(4-3)^2}$

$= 9+16 = 25$

Q2)(c) $4xy^2 - 5yx^2 = 800$ (Eq.1) reta tangente // $y = \frac{5}{4}x$, $m = \frac{5}{4}$

derivando implicitamente, logo : $f'(x) = \frac{5}{4}$.

$4x.2y.y' + 4y^2 - 5y.2x - 5y'.x^2 = 0$

$8xy' + 4y^2 - 10xy - 5y'x^2 = 0$, substituindo $y' = \frac{5}{4}$,

$8xy.\frac{5}{4} + 4y^2 - 10xy - \frac{25}{4}x^2 = 0$

$\cancel{10xy} + 4y^2 - \cancel{10xy} - \frac{25x^2}{4} = 0 \iff 4y^2 = \frac{25x^2}{4} \iff y^2 = \frac{25x^2}{16}$

$y = \frac{\pm 5}{4}x$

$\boxed{y = \frac{5}{4}x}$, substituindo na Eq. 1.

$4x.\frac{25}{16}x^3 - \frac{25}{4}x^3 = 800 \iff 0x^2 = 800$, impossível

$\boxed{y = -\frac{5}{4}x}$,

$4x.\frac{25}{\cancel{16}\,4}x^2 + \frac{25}{4}x.x^2 = 800$

$\frac{5\cancel{0}x^3}{4} = 80\cancel{0} \iff x^3 = \frac{80\times 4}{5} = 64 \Rightarrow \boxed{x = 4}$

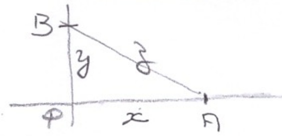
$y = -\frac{5}{4}\times 4 = -5$

Logo, o ponto é $(4, -5)$.

x ——— x ———

(Q 3) x – distância percorrida por A

y – " " " B



em $x = 20$ cm, $y = 15$ cm, $\frac{dx}{dt} = 5$ cm/min, $\frac{dy}{dt} = 8$ cm/min.

z – distância entre A e B, $\frac{dz}{dt} = ?$.

$z^2 = x^2 + y^2$

$\cancel{2}z.\frac{dz}{dt} = \cancel{2}x.\frac{dx}{dt} + \cancel{2}y.\frac{dy}{dt}$

$25.\frac{dz}{dt} = 20.5 + 15\times 8$

$25\frac{dz}{dt} = 100 + 120 = 220$

$\frac{dz}{dt} = \frac{220}{25} = \frac{44}{5} = \frac{88}{10} = 8,8$

, determinando z quando $x = 20$, $y = 15$

$z^2 = 20^2 + 15^2$

$z^2 = 400 + 225$

$z^2 = 625$

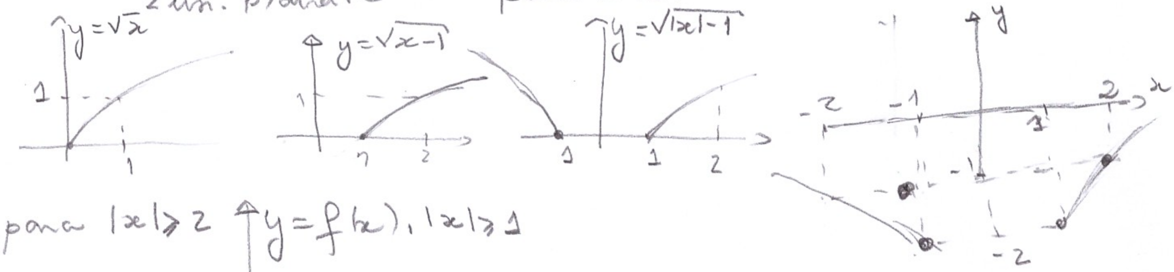
$\boxed{z = 25}$

A velocidade da distância entre A e B é 8,8 cm/min.

(Q4)(a) $|x| \geq 2$, $f(x) = \sqrt{|x|-1} - 2$.

$y=\sqrt{x} \xrightarrow{} y=\sqrt{x-1} \xrightarrow{} y=\sqrt{|x|-1} \xrightarrow{} y=\sqrt{|x|-1}-2$

Translação horizontal 2 un. p/direita — refletir parte $x>0$ para $x<0$. — Translação vertical 2 un. p/ baixo



para $|x| \geq 2$ $y = f(x), |x| \geq 1$

$\sqrt{|x|-1} - 2 = 0$

$\Leftrightarrow \sqrt{|x|-1} = 2 \Leftrightarrow |x|-1 = 4$

$\Leftrightarrow |x| = 5 \Leftrightarrow x = \pm 5$

(b) f é contínua em $x=2$ se $f(2) = \lim_{x \to 2} f(x)$ (1ª condição)

$f(2) = \sqrt{|2|-1} - 2 = 1 - 2 = -1$

$\lim_{x\to 2^+} f(x) = \lim_{x \to 2^+} \sqrt{|x|-1} - 2 = \sqrt{|2|-1} - 2 = -1$ $\Rightarrow \boxed{2a+b=-1}$

$\lim_{x \to 2^-} f(x) = \lim_{x\to 2^-} ax+b = 2a+b$

f é diferenciável em $x=2$ se $f'_+(2) = f'_-(2)$ (2ª condição)

$f'_+(2) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2^+} \frac{\sqrt{|x|-1} - 2 - (-1)}{x-2} =$

$x>2$, $|x| = x$

$= \lim_{x\to 2^+} \frac{\sqrt{x-1}-1}{x-2} \overset{\frac{0}{0}}{=} \lim_{x\to 2^+} \frac{\sqrt{x-1}-1}{x-2} \frac{(\sqrt{x-1}+1)}{(\sqrt{x-1}+1)} =$

$= \lim_{x \to 2^+} \frac{x-\cancel{1}-\cancel{1}}{(\cancel{x-2})} \cdot \frac{1}{\sqrt{x-1}+1} = \frac{1}{1+1} = \frac{1}{2}$

da 1ª condição $b = -1-2a$

$f'_-(2) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^-} \frac{ax+b-(-1)}{x-2} =$

$= \lim_{x\to 2^-} \frac{ax+1-2a+1}{x-2} = \lim_{x\to 2^-} \frac{ax-2a}{x-2} = \lim_{x\to 2^-} \frac{a(x-2)}{x-2} = a$

Logo, $\boxed{a = \frac{1}{2}}$, $b = -1-2 \cdot \frac{1}{2} \Rightarrow \boxed{b=-2}$

p/ $|x| \leq 2$, $f(x) = \frac{1}{2}x - 2$

$\lim_{x\to -2^-} f(x) = -1-2 = -3 \neq -1 = f(-2)$.

f não é diferenciável em $x=-2$ porque f não é contínua em $x=-2$.