(Q1)(a) $\lim\limits_{x\to-\infty} \dfrac{1+2x^2}{x\sqrt{x^2-1}} = \lim\limits_{x\to-\infty} \dfrac{1+2x^2}{x\sqrt{x^2(1-\frac{1}{x^2})}} =$

$= \lim\limits_{x\to-\infty} \dfrac{1+2x^2}{x\cdot|x|\sqrt{1-\frac{1}{x^2}}} = \lim\limits_{x\to-\infty} \dfrac{1+2x^2}{-x^2\sqrt{1-\frac{1}{x^2}}} =$

$x<0,\ |x|=-x$

$= \lim\limits_{x\to-\infty} \dfrac{\frac{1}{x^2}+2}{-\sqrt{1-\frac{1}{x^2}}} = \dfrac{0+2}{-\sqrt{1-0}} = \dfrac{2}{-1} = -2$ //

(b) $\lim\limits_{x\to 0} \dfrac{\tan(2x^2)}{8x^2-\operatorname{sen}^2(3x)} = \lim\limits_{x\to 0} \dfrac{2\cdot\frac{\tan(2x^2)}{2x^2}}{8-9\cdot\frac{\operatorname{sen}^2(3x)}{9x^2}}$

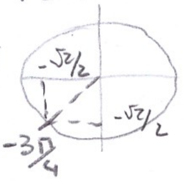
$= \dfrac{2\cdot 1}{8-9\cdot 1^2} = \dfrac{2}{-1} = -2$ //

(Q2)(a) $f'(x) = \sqrt[3]{\tan(\frac{x}{4})} = \left(\tan(\frac{x}{4})\right)^{1/3}$, $f'(-3\pi) = ?$

$f'(x) = \dfrac{1}{3}\left(\tan(\frac{x}{4})\right)\cdot\left(\sec^2(\frac{x}{4})\right)\cdot\left(\frac{1}{4}\right)$

$f'(-3\pi) = \dfrac{1}{3}\tan\left(-\frac{3\pi}{4}\right)\cdot\sec^2\left(-\frac{3\pi}{4}\right)\cdot\dfrac{1}{4} =$

$= \dfrac{1}{3}\cdot 1\cdot(-\sqrt{2})^2\cdot\dfrac{1}{4} = \dfrac{2}{3\cdot 4} = \dfrac{1}{6}$ //




$\tan(-\frac{3\pi}{4}) = \dfrac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$

$\sec(-\frac{3\pi}{4}) = \dfrac{1}{(-\frac{\sqrt{2}}{2})} = -\dfrac{2}{\sqrt{2}} = -\sqrt{2}$

(Q2)(b) $F(x) = \dfrac{(2+g(3x))^3}{x\cdot g(x)}$, $F'(1) = ?$

$g(1) = 2$
$g(3) = -1$
$g'(1) = 3$
$g'(3) = 0$

$F'(x) = \dfrac{x\cdot g(x)\cdot 3(2+g(3x))^2(3\cdot g'(3x)) - (2+g(3x))^3(x\cdot g'(x)+g(x))}{(x\cdot g(x))^2}$

$F'(1) = \dfrac{1\cdot g(1)\cdot 3(2+g(3))^2(3\cdot\overbrace{g'(3)}^{=0}) - (2+\overbrace{g(3)}^{-1})^3(\overbrace{g'(1)}^{3}+\overbrace{g(1)}^{2})}{(1\cdot g(1))^2}$

$= \dfrac{0-(1)^3(3+2)}{2^2} = -\dfrac{5}{4}$ //

(Q2)(c) $y^3 - xy + x^3 = 25$ (Eq.1) reta tangente horizontal: $y = f(x)$, $f'(x) = 0$, $x = ?$

derivando implicitamente,

$$3y^2 \cdot \underbrace{y'}_{=0} - x\underbrace{y'}_{=0} - y + 3x^2 = 0 \Rightarrow y = 3x^2.$$

Substituindo na eq. 1, $y = 3x^2$,

$$27x^6 - x \cdot 3x^2 + x^3 = 25$$

$$27x^6 - 2x^3 = 25 \overset{t = x^3}{\iff} 27t^2 - 2t - 25 = 0$$

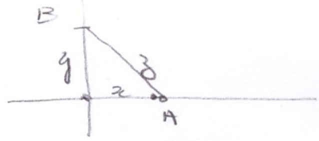
$$t = \frac{2 \pm \sqrt{4 + 2700}}{54} = \frac{2 \pm \sqrt{2704}}{54} = \frac{2 \pm 52}{54} \iff \begin{cases} x^3 = 1 \iff x = 1 \\ \text{ou} \\ x^3 = -\frac{50}{54} = -\frac{25}{27} \end{cases}$$

Um ponto é $(1, 3)$.

(Q3) $x$ - distância percorrida por A

$y$ - " " por B

$z$ - distância entre A e B.




$x = 0.15$ km

$y = 0.20$ km

$\frac{dx}{dt} = 80$ km/h

$\frac{dy}{dt} = 60$ km/h

$\frac{dz}{dt} = ?$

$$z^2 = x^2 + y^2 \quad \text{(Eq. 1)}$$

derivando a Eq. 1

$$2z\frac{dz}{dt} = 2\underbrace{x}_{0.15}\underbrace{\frac{dx}{dt}}_{=90} + 2\underbrace{y}_{0,20}\underbrace{\frac{dy}{dt}}_{60} \quad \text{(Eq. 2)}$$

determinando $z$ na Eq. 1,

$$z^2 = (0.15)^2 + (0.20)^2$$

$$z^2 = \frac{15^2}{100^2} + \frac{20^2}{100^2} = \frac{225 + 400}{100^2} = \frac{625}{100^2}$$

$$z = \frac{25}{100} = 0.25 \text{ km}$$

substituindo na Eq. 2:

$$0.25\frac{dz}{dt} = (0,15) \times 90 + (0,20) \times 60$$

$$0.25\frac{dz}{dt} = 13,5 + 12$$

$$\begin{array}{r} 1.5 \\ 9. \\ \hline 13.5 \end{array}$$

$$\frac{dz}{dt} = \frac{25,5}{0,25} = \frac{2550}{25} = \frac{510}{5}$$

$$\frac{dz}{dt} = \frac{1020}{10} = 102$$

A velocidade da distância nesse instante é de 102 km/h.

Q 4)(a) $|x| \geq 1$, $f(x) = \sqrt{2|x|-1} - 2$

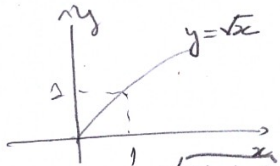
$$y = \sqrt{x} \xrightarrow{①} y = \sqrt{x-1} \xrightarrow{②} y = \sqrt{2x-1} \xrightarrow{③} y = \sqrt{2|x|-1} \xrightarrow{④} y = \sqrt{2|x|-1} - 2$$

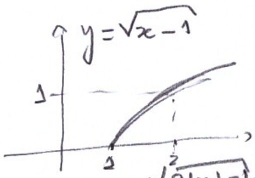
① translação 1 un. p/ direita

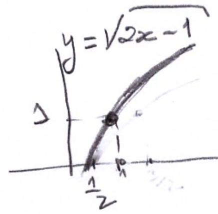
② redução horizontal fator $\frac{1}{2}$

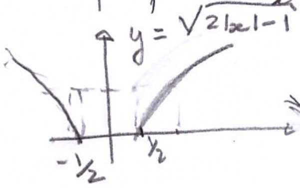
③ reflete parte $x>0$ p/ $x<0$

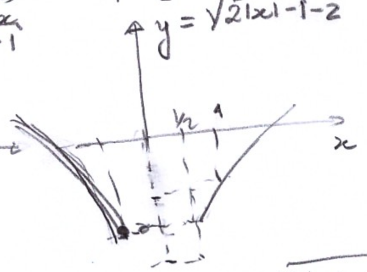
④ translação vertical 1 un. p/ direita

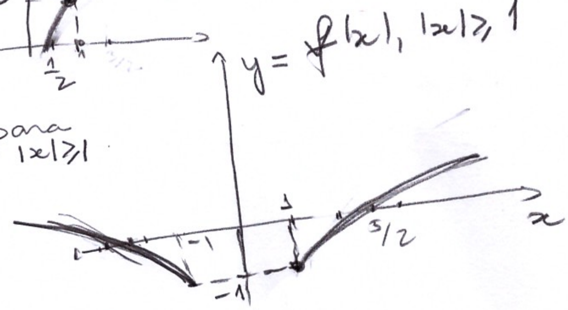



$\sqrt{2|x|-1} - 2 = 0 \iff \sqrt{2|x|-1} = 2$

$\iff 2|x| - 1 = 4 \iff 2|x| = 5$, $|x| = 5/2$, $x = \pm 5/2$

(b) $f$ é contínua em $x = 1$ se $\lim_{x\to 1} f(x) = f(1)$.

$f(1) = \sqrt{2-1} - 2 = -1$

$\lim_{x\to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{2|x|-1} - 2 = \sqrt{2.1-1} - 2 = -1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} ax + b = \lim_{x \to 1^-} a + b$.

Logo a 1ª condição é $\boxed{a + b = -1}$

$f$ é diferenciável em $x = 1$ se $f'_+(1) = f'_-(1)$

$$f'_+(1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x\to 1^+} \frac{\sqrt{2|x|-1} - 2 - (-1)}{x-1} = \lim_{x\to 1^+} \frac{\sqrt{2x-1} - 1}{x-1} = \frac{0}{0}$$

$\left( \begin{matrix} x > 1 > 0 \\ |x| = x \end{matrix} \right)$

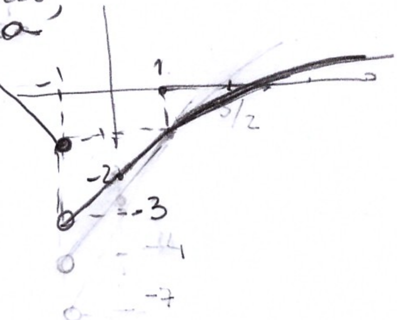
$$= \lim_{x\to 1^+} \frac{\sqrt{2x-1} - 1}{x-1} \cdot \frac{\sqrt{2x-1}+1}{\sqrt{2x-1}+1} = \lim_{x\to 1^+} \frac{2x-1-1}{(x-1)(\sqrt{2x-1}+1)} = \lim_{x\to 1^+} \frac{2(x-1)}{x-1} \times \frac{1}{2} = 1$$

$$f'_-(1) = \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{ax + b - (-1)}{x-1} = \lim_{x\to 1^-} \frac{ax - 1 - a + 1}{x-1} =$$

da 1ª condição, $b = -1 - a$

$$= \lim_{x\to 1^-} \frac{a(x-1)}{x-1} = a$$

Logo $\boxed{a = 1}$, $b = -1 - 1 \Rightarrow \boxed{b = -2}$




$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x - 2 = -1 - 2 = -3 \neq f(-1) = -1$

$f$ não é diferenciável em $x = -1$ porque $f$ não é contínua em $x = -1$.