1) a) $\lim\limits_{x\to-\infty} \dfrac{x\sqrt{x^2+x+1}}{2-2x+5x^2} = \lim\limits_{x\to-\infty} \dfrac{x\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}}{x^2\left(\frac{2}{x^2}-\frac{2}{x}+5\right)}$

$= \lim\limits_{x\to-\infty} \dfrac{x\,|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x^2\left(5-\frac{2}{x}+\frac{2}{x^2}\right)} \overset{x<0}{=} \lim\limits_{x\to-\infty} \dfrac{-x^2\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x^2\left(5-\frac{2}{x}+\frac{2}{x^2}\right)} =$

$= -1\,\dfrac{\sqrt{1+0+0}}{5-0+0} = -\dfrac{1}{5}$ //

(b) $\lim\limits_{x\to0^+} (2x+1)^{\cot x} = e^{\lim\limits_{x\to0^+} (\cot x)\ln(2x+1)} = e^L$

$L = \lim\limits_{x\to0^+} (\cot x)\ln(2x+1) = \lim\limits_{x\to0^+} \dfrac{\ln(2x+1)}{\tan x} \overset{L'H}{=}$

$= \lim\limits_{x\to0^+} \dfrac{\frac{1}{2x+1}\cdot 2}{\sec^2 x} = \dfrac{2}{1} = 2$

logo $\lim\limits_{x\to0^+} (2x+1)^{\cot x} = e^2$ //

2) $f(x) = \begin{cases} x^2-1 & \text{se } x \le 1 \\ a+bx & \text{se } x > 1 \end{cases}$

(a) $\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} x^2-1 = 0$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} a+bx = a+b$

$f(1) = 0$

$\Rightarrow a+b=0 \Rightarrow \boxed{b=-a}$

(b) $f'_-(1) = \lim\limits_{x\to1^-} \dfrac{f(x)-f(1)}{x-1} = \lim\limits_{x\to1^-} \dfrac{x^2-1-0}{x-1} = \lim\limits_{x\to1} (x+1) = 2$

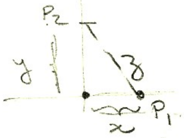
$f'_+(1) = \lim\limits_{x\to1^+} \dfrac{f(x)-f(1)}{x-1} = \lim\limits_{x\to1^+} \dfrac{a+bx-0}{x-1}$

mas para f ser diferenciável, f tem que ser contínua $\Rightarrow b=-a$.

$f'_+(1) = \lim\limits_{x\to1^+} \dfrac{a-bx}{x-1} = \lim\limits_{x\to1^+} \dfrac{a(1-x)}{x-1} = -a$

$f'_-(1) = f'_+(1) \Rightarrow \boxed{a=-2}$ e $\boxed{b=+2}$

3)



x - distância percorrida por P_1

y - " " por P_2

z - distância entre P_1 e P_2

$$x^2 + y^2 = z^2 \qquad\qquad x = 30 \;,\; y = 40$$

$$30^2 + 40^2 = z^2$$

$$\cancel{2}x \underbrace{\frac{dx}{dt}}_{=10} + \cancel{2}y \underbrace{\frac{dz}{dt}}_{20} = 2z\frac{dz}{dt} \qquad\qquad 10^2(9+16) = z^2$$

$$z = 50$$

$$30 \times 10 \times 40 \times 20 = 50 \times \frac{dz}{dt}$$

$$300 + 800 = 50\,\frac{dz}{dt}$$

$\dfrac{dz}{dt} = \dfrac{1100}{50} = 22 \Rightarrow$ a distância está variando à velocidade de 22 cm/seg.

4) (a) $f(x) = [g(\text{sen}(3x))]^2$

$$f'(x) = 2[g(\text{sen}(3x))] \cdot g'(\text{sen}(3x)) \cdot \cos(3x) \cdot 3$$

$x = -\frac{\pi}{9} \Rightarrow \text{sen}(3x) = \text{sen}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos(3x) = \cos\left(-\frac{\pi}{9}\right) = +\frac{1}{2}$

$$f'\left(-\frac{\pi}{9}\right) = 2\,g\left(-\frac{\sqrt{3}}{2}\right) \cdot g'\left(-\frac{\sqrt{3}}{2}\right) \cdot \left(+\frac{3}{2}\right)$$

g é ímpar $\quad g\left(-\frac{\sqrt{3}}{2}\right) = -g\left(\frac{\sqrt{3}}{2}\right) = -2$

Logo, $f'\left(-\frac{\pi}{9}\right) = 2 \cdot (-2) \cdot \left(\frac{-3}{2}\right) \cdot \left(+\frac{3}{2}\right) = +9$

$$f\left(-\frac{\pi}{9}\right) = g\left(-\frac{\sqrt{3}}{2}\right) = -2$$

Eq. da reta tangente:

$$y - f\left(-\frac{\pi}{9}\right) = f'\left(-\frac{\pi}{9}\right)\left(x + \frac{\pi}{9}\right)$$

$$y - (-2) = 9\left(x + \frac{\pi}{9}\right)$$

$$y + 2 = 9x + \pi$$

$$\boxed{-9x + y = \pi - 2} \iff \boxed{y = 9x + \pi - 2}$$

4) (b) $f(x) = e^{x^3+2x}$

$f'(x) = e^{x^3+2x} \cdot (3x^2+2)$, como $x^2 \geq 0 \Rightarrow$ $3x^2+2 \geq 2 > 0$; $e^{x^3+2x} > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow f'(x) > 0 \quad \forall x \neq 0$

$\Rightarrow f$ é crescente $\Rightarrow f$ admite inversa.

$e^{x^3+2x} = 1$

$x^3 + 2x = 0$

$x\underbrace{(x^2+2)}_{>0} = 0 \Rightarrow \boxed{x = 0}$

$(f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{e^0 \cdot (0+2)} = \dfrac{1}{2}$ //

(5) $f(x) = x \ln|x|$

~~domínio~~ : $|x| > 0 \Rightarrow \boxed{x \neq 0}$

domínio $\mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$

contínua no domínio.

Assíntotas verticais:

$\lim\limits_{x \to 0} \overset{0}{x} \overset{-\infty}{\ln|x|} = \lim\limits_{x \to 0^-} \dfrac{\ln|x|}{\frac{1}{x}} = \lim\limits_{x \to 0} \dfrac{\frac{1}{|x|} \cdot \frac{x}{|x|}}{-\frac{1}{x^2}} = \dfrac{\frac{x}{x^2}}{-\frac{1}{x^2}} = 0$

$\Rightarrow f$ não tem assíntota vertical.

Assíntotas horizontais:

$\lim\limits_{x \to \infty} \overset{\infty}{x} \overset{\infty}{\ln|x|} = \infty$; $\lim\limits_{x \to -\infty} \overset{-\infty}{x} \overset{\infty}{\ln|x|} = -\infty$.

Não tem assíntota horizontal

Crescimento

$f'(x) = x \dfrac{1}{|x|} \cdot \dfrac{x}{|x|} + \ln|x| = 1 + \ln|x|$

$1 + \ln|x| = 0$ | $x = \pm e^{-1}$

$\ln|x| = -1$ | $x = \dfrac{1}{e}$ ou $x = -\dfrac{1}{e}$

$|x| = e^{-1}$

5) continuação

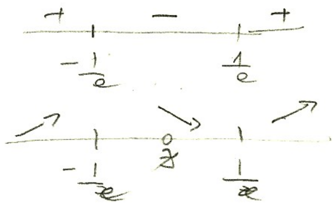
$1 + \ln|x| > 0$

$\ln|x| > -1$

$|x| > e^{-1}$

$x > e^{-1} = \frac{1}{e}$

ou $x < -e^{-1} = -\frac{1}{e}$



crescente : $(-\infty, -\frac{1}{e}) \cup (\frac{1}{e}, \infty)$

decrescente : $(-\frac{1}{e}, 0) \cup (0, \frac{1}{e})$

máx relativo $= f(-\frac{1}{e}) = -\frac{1}{e} \ln\left|-\frac{1}{e}\right| = -\frac{1}{e} \ln \frac{1}{e} =$

$= -\frac{1}{e}(\ln 1 - \ln e) = \frac{1}{e}$

mín relativo $= f(\frac{1}{e}) = \frac{1}{e} \ln\left|\frac{1}{e}\right| = -\frac{1}{e}$

Concavidade

$f(x) = 1 + \ln|x|$

$f'(x) = \frac{1}{|x|} \cdot \frac{x}{|x|} = \frac{1}{x}$

$f''(x)$: $-$ | 0 | $+$

não tem ponto de inflexão