(1) (a) $$\lim_{x\to-\infty} \frac{\sqrt{9x^{50}+3}}{5-4x^{25}} = \lim_{x\to-\infty} \frac{\sqrt{x^{50}\left(9+\frac{3}{x^{50}}\right)}}{x^{25}\left(\frac{5}{x^{25}}-4\right)} =$$

$$= \lim_{x\to-\infty} \frac{|x^{25}|\sqrt{9+\frac{3}{x^{50}}}}{x^{25}\left(\frac{5}{x^{25}}-4\right)} = \lim_{x\to-\infty} \frac{-\cancel{x^{25}}\sqrt{9+\frac{3}{x^{50}}}}{\cancel{x^{25}}\left(\frac{5}{x^{25}}-4\right)} =$$

$x<0$

$|x^{25}| = -x^{25}$

$$= \frac{-\sqrt{9}}{-4} = \frac{3}{4} \;//$$

(b) $$\lim_{x\to\infty}\left(1+\frac{1}{e^x}\right)^x = \lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{e^x}\right)} = e^L$$

$$L = \lim_{x\to\infty} \overset{\infty}{x}\,\overset{0}{\ln\left(1+\frac{1}{e^x}\right)} = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{e^x}\right)\to 0}{\frac{1}{x}\to 0} \overset{L'H}{=}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{e^x}}\right)\left(0-\frac{1}{e^x}\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\left(\frac{1}{\frac{e^x+1}{e^x}}\right)\cdot\left(-\frac{1}{e^x}\right)}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to\infty} \frac{x^2}{e^x+1} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$$

Logo $\lim_{x\to\infty}\left(1+\frac{1}{e^x}\right)^x = e^0 = 1$ //

(2) $x^2y - xy^3 = x^2+y^2$

$x^2y' + 2xy - 3xy^2y' - y^3 = 2x + 2yy'$

Substituindo por $x=-1$ e $y=1$

$y' - 2 + 3y' - 1 = -2 + 2y'$

$2y' = 1 \Rightarrow y' = \frac{1}{2}$

Eq. da reta tangente:

$y - 1 = \frac{1}{2}(x+1)$

$2y - 2 = x + 1$

$\boxed{2y - x = 3}$

$x = -1 \Rightarrow$

$y + y^3 = 1 + y^2$

$y^3 - y^2 + y - 1 = 0$

$y = 1 \Rightarrow 1 - 1 + 1 - 1 = 0$

| | 1 | -1 | 1 | -1 |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 |

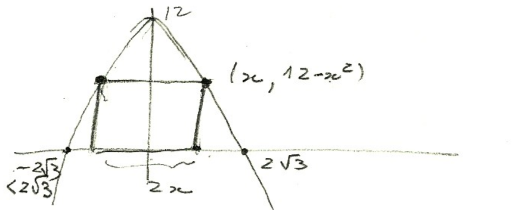
$(y-1)(y^2+1) = 0 \Rightarrow$

única solução: $y = 1$

3) $12-x^2=0$

$x^2=12$

$x=\pm\sqrt{12}=\pm 2\sqrt{3}$



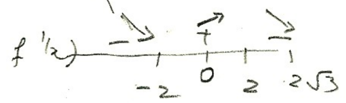

Área $= 2|x|(12-x^2)$, $0<x<2\sqrt{3}$

$A(x) = 2(12|x| - |x|^3)$

$A'(x) = 2(12-3x^2)$

$A'(x) = 6(4-x^2)$

$\Rightarrow$ $A(x)$ tem um máximo relativo e absoluto em $x=2$




crescente: $(0,2)$

decrescente $(2, 2\sqrt{3})$

$x=2$ é ponto de máximo relativo em $[0, 2\sqrt{3})$.

Dimensões de área máxima são: base $= 4$ altura $= 12-4 = 8$

4) $f(x) = g^2(\arctan(1-2x))$

$$f'(x) = 2g(\arctan(1-2x))\cdot g'(\arctan(1-2x))\cdot \frac{1}{1+(1-2x)^2}\cdot(-2)$$

$$f'(1) = 2g(\underbrace{\arctan(-1)}_{=-\pi/4})\cdot g'(\underbrace{\arctan(-1)}_{=-\pi/4})\cdot\frac{1}{1+1}\cdot(-2)$$

$$f'(1) = 2\times 2\times(-2)\cdot\frac{1}{2}(-2) = 8 \quad //$$

5) $f(x) = 1+\dfrac{x}{x^3-16}$

$x^3-16=0$

$x^3=16$

$x=\sqrt[3]{16} = 2\sqrt[3]{2}$

Dom $f$; $x$; $x^3-16\neq 0$

dom $f = (-\infty, 2\sqrt[3]{2})\cup(2\sqrt[3]{2}, \infty)$

contínua no domínio

Assíntotas horizontais

$$\lim_{x\to\infty} 1+\frac{x}{x^3-16} = \lim_{x\to\infty} 1+\frac{x}{x^3(1-\frac{16}{x^2})} =$$

$$= \lim_{x\to\infty} 1+\frac{1}{x^2(1-\frac{16}{x^2})} = 1+0 = 1$$

$$\lim_{x\to-\infty} 1+\frac{x}{x^3-16} = \lim_{x\to-\infty} 1+\frac{1}{x^2(1-\frac{16}{x^2})} = 1+0 = 1$$

$\Rightarrow$ assíntota horizontal: $\boxed{y=1}$

Assíntota vertical:

$$\lim_{x \to 2\sqrt[3]{2}^+} 1 + \frac{\overset{\to 2\sqrt[3]{2}}{x}}{\underset{\to 0^+}{x^3-16}} = \infty$$

| | | | |
|---|---|---|---|
| $x^3-16$ | $-$ | | $+$ |
| | | $2\sqrt[3]{2}$ | |

$$\lim_{x \to 2\sqrt[3]{2}^-} 1 + \frac{\overset{\to 2\sqrt[3]{2}}{x}}{\underset{\to 0^-}{x^3-16}} = -\infty$$

Logo $x = 2\sqrt[3]{2}$ é assíntota vertical.

Crescimento

$$f(x) = 1 + \frac{x}{x^3-16}$$

$$f'(x) = 0 + \frac{(x^3-16) - x(3x^2)}{(x^3-16)^2} = \frac{-2x^3-16}{(x^3-16)^2} = \frac{-2(x^3+8)}{(x^3-16)^2}$$

| | | $-2$ | | $2\sqrt[3]{2}$ | |
|---|---|---|---|---|---|
| $-2(x^3+8)$ | $+$ | $0$ | $-$ | $-$ | $-$ |
| $(x^3-16)^2$ | $+$ | $+$ | $+$ | $0$ | $+$ |
| $f'(x)$ | $+$ | $0$ | $-$ | $\nexists$ | $-$ |

crescente em $(-\infty, -2)$

decrescente em $(-2, 2\sqrt[3]{2}) \cup (2\sqrt[3]{2}, \infty)$

$f$ tem máximo relativo em $x = -2$.

Concavidade

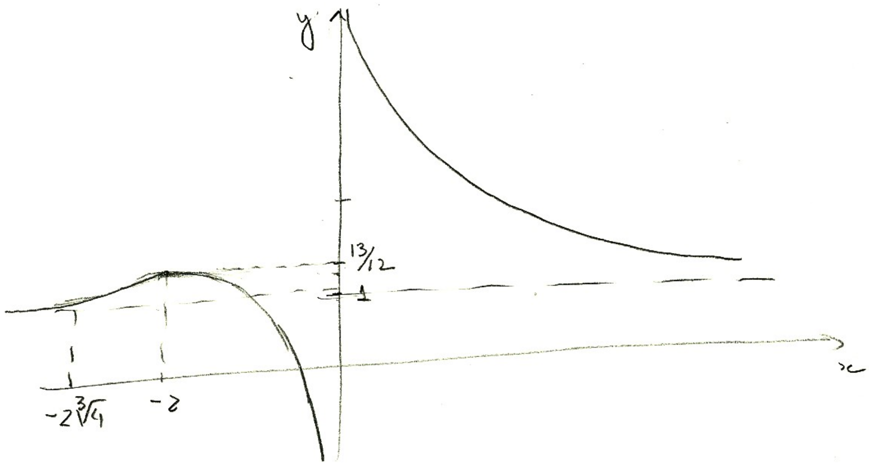
$$f''(x) = \frac{6x^2(x^3+32)}{(x^3-16)^3}$$

| | | $-2\sqrt[3]{4}$ | | $0$ | | $2\sqrt[3]{2}$ | |
|---|---|---|---|---|---|---|---|
| $6x^2$ | $+$ | $+$ | $+$ | $0$ | $+$ | $+$ | $+$ |
| $x^3+32$ | $-$ | $0$ | $+$ | | $+$ | $+$ | $+$ |
| $(x^3-16)^3$ | $-$ | $-$ | $-$ | | $-$ | $0$ | $+$ |
| $f''(x)$ | $+$ | $0$ | $-$ | $0$ | $-$ | $\nexists$ | $+$ |

côncava para cima: $(-\infty, -2\sqrt[3]{4}) \cup (2\sqrt[3]{2}, \infty)$

" para baixo $(-2\sqrt[3]{4}) \cup (2\sqrt[3]{2})$

ponto de inflexão: $x = -2\sqrt[3]{4}$.




$$f(-2) = 1 + \frac{-2}{-8-16} = 1 + \frac{2}{24} = 1 + \frac{1}{12} = \frac{13}{12}$$

6) $f'(x) = \dfrac{2x^3 + 3x^2 + 3x + 1}{x} = 2x^2 + 3x + 3 + \dfrac{1}{x}$

$$f(x) = \int \left(2x^2 + 3x + 3 + \frac{1}{x}\right) dx$$

$$f(x) = \frac{2x^3}{3} + \frac{3x^2}{2} + 3x + \ln|x| + C$$

$$f(1) = \frac{2}{3} + \frac{3}{2} + 3 + \ln 1 + C = 3$$

$$C = -\left(\frac{2}{3} + \frac{3}{2}\right) = -\frac{13}{6}$$

$$f(x) = \frac{2x^3}{3} + \frac{3x^2}{2} + 3x + \ln|x| - \frac{13}{6}$$