(1) $f(x) = \begin{cases} \dfrac{a}{\sqrt{x}} & \text{se } x > 1 \\ bx+2 & \text{se } x \leq 1 \end{cases}$

(a) $\left.\begin{array}{l} \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} bx + 2 = b+2 \\ \lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{a}{\sqrt{x}} = a \end{array}\right\} \Rightarrow \boxed{b+2=a}$

$f(1) = b+2$

(b) $f'_-(1) = \lim\limits_{x \to 1^-} \dfrac{f(x)-f(1)}{x-1} = \lim\limits_{x \to 1^-} \dfrac{(bx+2)-(b+2)}{x-1} =$

$= \lim\limits_{x \to 1^-} \dfrac{bx-b}{x-1} = \lim\limits_{x \to 1} b \dfrac{(x-1)}{x-1} = b$

$f'_+(1) = \lim\limits_{x \to 1^+} \dfrac{f(x)-f(1)}{x-1} = \lim\limits_{x \to 1^+} \dfrac{\frac{a}{\sqrt{x}} - (b+2)}{x-1} =$

$= \lim\limits_{x \to 1^+} \dfrac{\frac{a}{\sqrt{x}} - a}{x-1} = \lim\limits_{x \to 1^+} \dfrac{a\left(\frac{1}{\sqrt{x}} - 1\right)}{x-1} =$

$= \lim\limits_{x \to 1^+} a \dfrac{\frac{1-\sqrt{x}}{\sqrt{x}}}{x-1} = \lim\limits_{x \to 1} \dfrac{a}{\sqrt{x}} \dfrac{\overbrace{(1-x)}^{=-1}}{(x-1)} \cdot \dfrac{1}{1+\sqrt{x}} =$

$= -\dfrac{a}{1} \cdot \dfrac{1}{1+1} = -\dfrac{a}{2}$

Logo, $f'_-(1) = f'_+(1) \Rightarrow b = -\dfrac{a}{2} \Rightarrow a = -2b$.

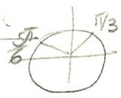
Como para f ser dif é preciso que f seja contínua, $b+2=a$

$\begin{cases} b+2=a \\ a=-2b \end{cases} \Rightarrow b+2=-2b \Rightarrow 3b=-2$ $\boxed{b = -\dfrac{2}{3}}$

$a = -2\left(-\dfrac{2}{3}\right)$ $\boxed{a = \dfrac{4}{3}}$

2) (a) $f(x) = \dfrac{\operatorname{sen}(g(x))}{\cos(2x)}$

$f'(x) = \dfrac{\cos(2x)(\cos(g(x))) \cdot g'(x) - \operatorname{sen}(g(x)) \cdot (-\operatorname{sen}(2x)) \times 2}{\cos^2 2x}$



$x = \dfrac{\pi}{6}$, $g\left(\dfrac{\pi}{6}\right) = \dfrac{5\pi}{6}$, $g'\left(\dfrac{\pi}{6}\right) = -3$

$f'\left(\dfrac{\pi}{6}\right) = \dfrac{\cos\left(\frac{2\pi}{6}\right)\left(\cos\left(\frac{5\pi}{6}\right)\right)(-3) + \operatorname{sen}\left(\frac{5\pi}{6}\right)\operatorname{sen}\left(2 \cdot \frac{\pi}{6}\right) \cdot (2)}{\cos^2\left(\frac{2\pi}{6}\right)}$

$f'\left(\dfrac{\pi}{6}\right) = \dfrac{\frac{1}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right)(-3) + \frac{1}{2} \cdot \frac{\sqrt{3}}{2}(2)}{\left(+\frac{1}{2}\right)^2} = \dfrac{+\frac{3\sqrt{3}}{4} + \frac{2 \times \sqrt{3}}{4}}{\frac{1}{4}} = 5\sqrt{3}$

2) b) $F(x) = \operatorname{arcsec}(2x-1)$, $F'(x) = \dfrac{1}{|1-2x|\sqrt{(2x-1)^2-1}} \times 2$

$G(x) = \arccos\left(\dfrac{1}{1-2x}\right)$

$$G'(x) = \frac{-1}{\sqrt{1-\frac{1}{(1-2x)^2}}} \cdot \left(\frac{-1}{(1-2x)^2}\right) \cdot (-2)$$

$$G'(x) = \frac{-2}{\sqrt{\frac{(1-2x)^2-1}{(1-2x)^2}}} \cdot \frac{1}{(1-2x)^2}$$

$$G'(x) = \frac{-2}{\frac{\sqrt{(1-2x)^2-1}}{|1-2x|}} \cdot \frac{1}{|1-2x|^2}$$

$\left\{ \left((1-2x)^2 = |1-2x|^2 \right.\right.$ pois $a^2 = |a|^2$

$$G'(x) = \frac{-2}{\sqrt{(1-2x)^2-1}} \cdot \frac{1}{|1-2x|}$$

$$F'(x) + G'(x) = \frac{2}{|1-2x|\sqrt{(2x-1)^2-1}} - \frac{2}{|1-2x|\sqrt{(1-2x)^2-1}} = 0 \;//$$

c) $f(t) = \dfrac{2^t \sec(t^2)}{\sqrt[4]{2t-1}} = \dfrac{2^t \sec(t^2)}{(2t-1)^{1/4}}$

$$\ln(f(t)) = \ln\left(\frac{2^t \sec(t^2)}{(2t-1)^{1/4}}\right)$$

$$\ln(f(t)) = \underbrace{\ln(2^t)}_{= t\ln 2} + \ln(\sec(t^2)) - \frac{1}{4}\ln(2t-1)$$

$$\frac{1}{f(t)} \cdot f'(t) = \ln 2 + \frac{1}{\sec(t^2)} \cdot \sec(t^2)\tan(t^2) \cdot 2t - \frac{1}{4(2t-1)} \times 2$$

$$f'(t) = \left(\ln 2 + 2t\tan t^2 - \frac{1}{2(2t-1)}\right) \frac{2^t \sec(t^2)}{\sqrt[4]{2t-1}} \;//$$

3) $f(x) = \ln(x-1)$

i) a função é contínua em $[2, e+1]$ pois é composta de contínuas, $\ln x$ é contínua.

ii) $f'(x) = \dfrac{1}{x-1}$ é diferenciável para todo $x > 1$ logo é diferenciável em $(2, e+1)$

Logo as hipóteses do TVM (teorema do valor médio) estão satisfeitas.

coef. angular da reta tangente $= f'(c) = \dfrac{f(e+1) - f(2)}{(e+1) - 2}$,

e $c \in (2, e+1)$. $c = ?$

$$f(e+1) = \ln(e+1-1) = \ln(e) = 1$$
$$f(2) = \ln(2-1) = \ln 1 = 0$$
$$\Rightarrow f'(c) = \frac{1}{e-1}$$

$$f'(c) = \frac{1}{c-1} = \frac{1}{e-1} \Rightarrow c - 1 = e - 1 \Rightarrow \boxed{c = e}$$

Eq. da reta tangente , $f(e) = \ln(e-1)$

$$y - f(e) = f'(e)(x - e)$$
$$\boxed{y - \ln(e-1) = \frac{1}{e-1}(x - e)}$$

4) $y^2\, 3^{2x} - 36(x-1)\, 2^y = 9$.

$y(1) = ?$ $x = 1 \Rightarrow y^2 . 3^2 - 36 \times 0 \times 2^y = 9$

$9y^2 = 9 \Rightarrow y^2 = 1,\ y < 0 \Rightarrow \boxed{y = -1}$

$y'(1) = ?$ derivando implicitamente a equação:

$$y^2 . (6 \ln 3)\, 3^{2x} + 2y . y' . 3^{2x} -$$
$$- 36(x-1) . (\ln 2)\, 2^y - 36 . 2^y = 0$$

$x = 1,\ y = -1 \Rightarrow$

$$1 . (6 \ln 3) . 9 + (-2)\, y' . 9 - 0 - 36 . 2^{-1} = 0$$
$$54 \ln 3 - 18 y' - 18 = 0$$
$$18 y' = -18 + 54 \ln 3$$
$$\boxed{y' = -1 + 3 \ln 3}$$

$$(3^{2x})' = (e^{(2x)\ln 3})' = e^{(2x)\ln 3} . 2 = (6 \ln 3) . 3^{2x}$$

$$(2^y)' = (e^{y \ln 2})' = e^{y . \ln 2} . \ln 2 = (\ln 2)\, 2^y$$