1) a) $\lim_{x \to 1} \frac{1-x+\ln x}{x^2-1} \cdot \underbrace{\text{sen}\left(\frac{1}{x-1}\right)}_{\text{limitada pois } |\text{sen}\,\theta| \leq 1,\ \forall \theta}$

$$\lim_{x \to 1} \frac{1-x+\ln x}{x^2-1} \overset{L'H}{=} \lim_{x \to 1} \frac{-1+\frac{1}{x}}{2x-1} = \frac{-1+1}{2-1} = \frac{0}{1} = 0$$

Logo, pelo teorema do anulamento,

$$\lim_{x \to 1} \underbrace{\frac{1-x+\ln x}{x^2-1}}_{\to 0} \underbrace{\boxed{\text{sen}\left(\frac{1}{1-x}\right)}}_{\text{limitada}} = 0$$

(b) $\lim_{x \to 0^+} (\tan x)^{x^2} = \lim_{x \to 0^+} (e)^{x^2 \ln(\tan x)} = e^L$

$$L = \lim_{x \to 0^+} x^2 \ln(\tan x) = \lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{x^2}} =$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{\tan x} \cdot \sec^2 x}{-\frac{2}{x^3}} = \lim_{x \to 0^+} -\frac{x^3}{\tan x} \cdot \sec^2 x =$$

$$= \lim_{x \to 0^+} -\frac{x^3}{\frac{\text{sen}\,x}{\cos x}} \cdot \sec^2 x = \lim_{x \to 0^+} -\frac{1}{\frac{\text{sen}\,x}{x}} \cdot x^2 \cdot \cos x \sec^2 x = 0$$

$$\lim_{x \to 0^+} (\tan x)^{x^2} = e^0 = 1$$

(c) $\lim_{x \to -\infty} \frac{\sqrt{1+4x^2}}{x - 2\sqrt{x^2+1}} = \lim_{x \to -\infty} \frac{\sqrt{x^2\left(\frac{1}{x^2}+4\right)}}{x - 2\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}$

$$= \lim_{\substack{x \to -\infty \\ x < 0 \\ |x| = -x}} \frac{|x|\sqrt{\frac{1}{x^2}+4}}{x - 2|x|\sqrt{1+1/x^2}} = \lim_{x \to -\infty} \frac{-x\sqrt{\frac{1}{x^2}+4}}{x + 2x\sqrt{1+\frac{1}{x^2}}}$$

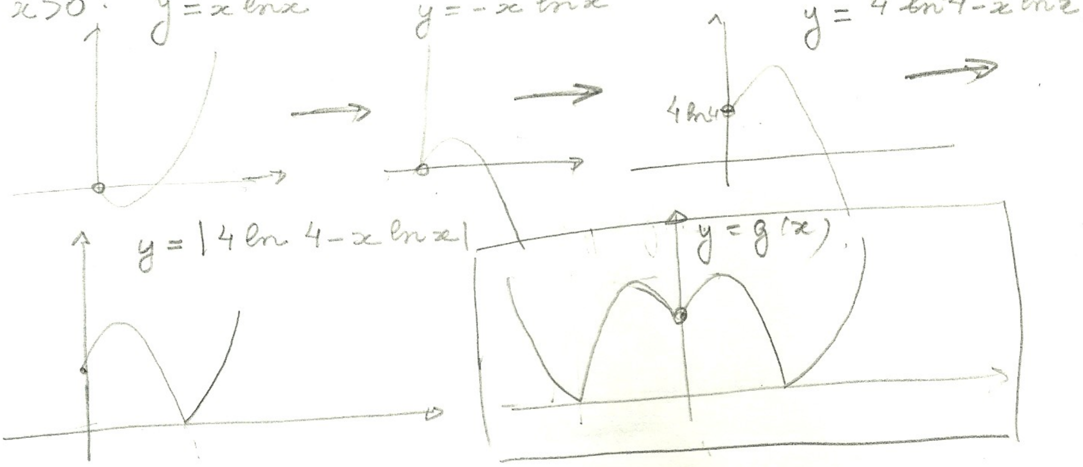
$$= \lim_{x \to -\infty} \frac{-\sqrt{\frac{1}{x^2}+4}}{1 + 2\sqrt{1+\frac{1}{x^2}}} = \frac{-\sqrt{4}}{1+2\sqrt{1}} = \frac{-2}{3} \quad //$$

2) (a) $g(x) = \left| 4\ln(4) - |x| \ln|x| \right|$

g é par pois domínio de $g = (-\infty,0) \cup (0,\infty)$, simétrico em relação à origem o.

$g(-x) = \left|4\ln(4) - |-x|\ln|-x|\right| = \left|4\ln(4) - 4|x|\ln|x|\right| = g(x)$

$x>0$: $y = x\ln x$ $\quad y = -x\ln x$ $\quad y = 4\ln 4 - x\ln x$



(b) $f(x) = x\ln x$

$f'(x) = x \cdot \frac{1}{x} + \ln x = 1 + \ln x > 0 \iff$

$\iff \ln x > -1 \iff x > e^{-1} \iff \boxed{x > \frac{1}{e}}$

Para $x > \frac{1}{e}$, f é crescente e como f é contínua, então f admite inversa em $[\frac{1}{e}, \infty)$

$f(1) = 1 \ln 1 = 0$

$$(f^{-1})'(0) = \frac{1}{f'(1)} = \frac{1}{1+\ln 1} = \frac{1}{1+0} = 1$$

3) $\arctan(xy) = \frac{\pi}{4} + y - x$

$x = 1$, $\arctan y = \frac{\pi}{4}$, $y = \tan\frac{\pi}{4} = 1 \Rightarrow y(1) = 1$

$\left(\frac{1}{1+x^2y^2}\right)(x \cdot y' + y) = y' - 1$, $x = 1, y = 1 \Rightarrow$

$\frac{1}{2}(y'+1) = y' - 1$

$\frac{1}{2}y' + \frac{1}{2} = y' - 1$

$\frac{1}{2} + 1 = y'\left(1 - \frac{1}{2}\right)$

$\frac{3}{2} = y'\left(\frac{1}{2}\right)$

$\Rightarrow \boxed{y'(1) = 3}$

Reta tangente

$\boxed{y - 1 = 3(x-1)}$

4) (a) $f(x) = 144 \cdot \dfrac{x+6}{x^2+22x+105} = 144 \cdot \dfrac{x+6}{(x+15)(x+7)}$

dom $f = (-\infty,-15) \cup (-15,-7) \cup (-7,\infty)$ $\quad x \neq -15$, $x \neq -7$

contínua no domínio.

$x^2+22x+105$: $+$ $\;-15\;$ $-$ $\;-7\;$ $+$

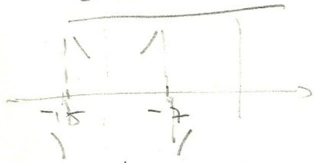
AV: $\lim\limits_{x \to -15^-} 144 \dfrac{(x+6)^{\to -9}}{\underbrace{(x+15)(x+7)}_{\to 0^+}} = \boxed{-\infty}$

$\lim\limits_{x \to -15^+} \dfrac{144 \cdot (x+6)^{\to -9}}{\underbrace{(x+15)(x+7)}_{\to 0^-}} = \boxed{+\infty}$

$\lim\limits_{x \to -7^-} \dfrac{144 (x+6)^{\to -1}}{\underbrace{(x+15)(x+7)}_{0^-}} = \boxed{\infty}$

$\lim\limits_{x \to -7^+} \dfrac{144 (x+6)^{\to -1}}{\underbrace{(x+15)(x+7)}_{0^+}} = \boxed{-\infty}$

A.V: equações: $\boxed{x = -15 \text{ e } x = -7}$



AH: $\lim\limits_{x \to \infty} 144 \dfrac{x+6}{x^2+22x+105} = \lim\limits_{x \to \infty} 144 \dfrac{1}{2x+22} = 0$

$\lim\limits_{x \to -\infty} 144 \dfrac{x+6}{x^2+22x+105} = 0$

AH: equação: $\boxed{y = 0}$

$f'(x) = 144 \dfrac{(x^2+22x+105)(1) - (x+6)(2x+22)}{(x^2+22x+105)^2}$

$f'(x) = 144 \cdot \dfrac{-(x^2+12x+27)}{(x^2+22x+105)^2}$

	$(-\infty,-15)$	-15	$(-15,-9)$	-9	$(-9,-7)$	-7	$(-7,-3)$	-3	$(-3,\infty)$
$-144(x^2+12x+27)$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$
$(x^2+22x+105)^2$	$+$	0	$+$	$+$	$+$	0	$+$	$+$	$+$
$f'(x)$	$-$	$\nexists$	$-$	0	$+$	$\nexists$	$+$	0	$-$
	$\searrow$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$

decrescente: $(-\infty,-15) \cup (-15,-9) \cup (-3,\infty)$

crescente: $(-9,-7) \cup (-7,-3)$

mín rel em $x = -9$

máx rel em $x = -3$

$$
\begin{array}{r}
2x + 22 \\
x + 6 \\
\hline
2x^2 + 22x \\
12x + 132 \\
\hline
2x^2 + 34x + 132
\end{array}
$$

$$
\begin{array}{r}
-2x^2 - 34x - 132 \\
x^2 + 22x + 105 \\
\hline
-x^2 - 12x - 27
\end{array}
$$

$= -(x^2+12x+27)$

$x^2+12x+27 = 0$

$x = \dfrac{-12 \pm \sqrt{144-108}}{2}$ $\quad 27 \cdot 4 = 108$

$x = \dfrac{-12 \pm 6}{2} = \begin{cases} \dfrac{-6}{2} = -3 \\ \dfrac{-18}{2} = -9 \end{cases}$

4) (continuação)

(b) $f''(x) = \dfrac{288\,(x^3+18x^2+91x-36)}{(x^2+22x+105)^3}$

$f''(x)$ é contínua em $[0,1]$

$f''(0) = \dfrac{288(-36)}{(105)^3} < 0$

$f''(1) = \dfrac{288\,(1+18+91-36)}{(1+22+105)^3} > 0$

Pelo teorema do valor intermediário, existe um número c entre 0 e 1 tal que $f''(c) = 0$, como f'' é contínua, existe um intervalo contendo c, tal que $f''(x)$ muda de sinal.

Logo c é um ponto de inflexão.

(c) $f(-9) = \dfrac{144\,(-9+6)}{(-9+15)(-9+7)} = \dfrac{144\,(-3)}{6\times(-2)} = 36$

$f(-3) = \dfrac{144\,(-3+6)}{(-3+15)(-3+7)} = \dfrac{144\times 3}{12\times 4} = 9$

$f(0) = \dfrac{144\times 6}{15\times 7} = \dfrac{288}{35}$

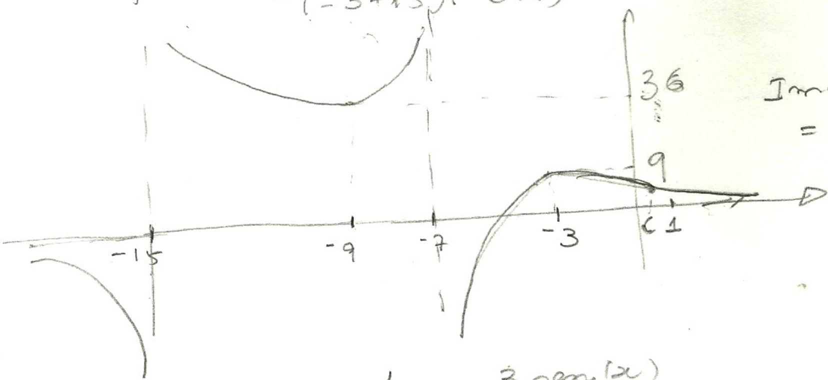


Imagem $= (-\infty, 9] \cup [36, \infty)$

5) $f'(x) = \dfrac{1}{x} + 3\,\text{sen}(x)$

$f(x) = \int\left(\dfrac{1}{x} + 3\,\text{sen}\,x\right)dx = \ln|x| - 3\cos x + C$

$f\left(\frac{\pi}{3}\right) = \ln\left|\frac{\pi}{3}\right| - 3\underbrace{\cos\frac{\pi}{3}}_{=1/2} + C = 3$

$C = 3 + \dfrac{3}{2} - \ln\dfrac{\pi}{3} = \dfrac{9}{2} - \ln\dfrac{\pi}{3}$

$$\boxed{f(x) = \ln|x| - 3\cos x + \frac{9}{2} - \ln\frac{\pi}{3}}$$