1) $(x-a+2)^{10} = (x+(2-a))^{10}$

coeficiente de x^6 é $\binom{10}{4}(2-a)^4 = 7560$

$$\binom{10}{4} = \frac{10!}{4!\,6!} = \frac{\cancel{6!}\,7\times\cancel{8}\times\cancel{9}^{3}\times 10}{\cancel{2}\times 3\times 4\times\cancel{6!}} = 210$$

Logo $210(2-a)^4 = 7560$

$$(2-a)^4 = \frac{756\cancel{0}}{21\cancel{0}} = \frac{252}{7} = 36$$

$$(2-a)^2 = 6$$

$$2-a = \pm\sqrt{6}$$

$$a = 2\pm\sqrt{6}$$

Logo $\boxed{a = 2+\sqrt{6}}$ ou $\boxed{a = 2-\sqrt{6}}$

2) $p(x) = (16x^4-1)^2\left(x^3-x^2+\frac{1}{8}\right)^3$

(a) grau de $(16x^4-1)^2$ é 8

" " $\left(x^3-x^2+\frac{1}{8}\right)^3$ é 9

Logo o grau de $p(x)$ é 17.

coef. do termo de maior grau é $16^2\times 1^3 = 256$

coef. do termo independente é $(-1)^2\times\left(\frac{1}{8}\right)^3 = \frac{1}{8^3}$

(b) $16x^4-1 = (4x^2-1)(4x^2+1)$

$4x^2+1=0 \iff x^2=-\frac{1}{4} \Rightarrow x=\pm\sqrt{-\frac{1}{4}} = \pm\frac{1}{2}i$

Logo duas soluções complexas: $\boxed{x=\frac{1}{2}i}$ e $\boxed{x=-\frac{1}{2}i}$

$4x^2-1=0 \Rightarrow x^2=\frac{1}{4} \Rightarrow \boxed{x=\frac{1}{2}}$ ou $\boxed{x=-\frac{1}{2}}$

$x^3-x^2+\frac{1}{8} = \frac{1}{8}(8x^3-8x^2+1) = 0$

possíveis raízes: $1;\ -1;\ \frac{1}{2},\ -\frac{1}{2},\ \frac{1}{4},\ -\frac{1}{4},\ \frac{1}{8},\ -\frac{1}{8}$

$x=1 \Rightarrow 8-8+1\neq 0$ | $x=\frac{1}{2}$: $8\times\frac{1}{8} - 8\times\frac{1}{4} + 1 = 1-2+1 = 0$

$x=-1 \Rightarrow -8-8+1\neq 0$

	8	−8	0	1
$\frac{1}{2}$	8	−4	−2	0

$$x^3 - x^2 + \frac{1}{8} = \frac{1}{8}\left(x - \frac{1}{2}\right)(8x^2 - 4x - 2)$$

$$8x^2 - 4x - 2 = 0$$

$$x = \frac{4 \pm \sqrt{16+64}}{16} = \frac{4 + \sqrt{80}}{16} = \frac{4 \pm 4\sqrt{5}}{16} = \frac{1}{4} \pm \frac{\sqrt{5}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{4} \Rightarrow x = \frac{1}{4} \pm \frac{\sqrt{5}}{4}$$

soluções: $x = \frac{1}{4} + \frac{\sqrt{3}}{4}$ ou $\boxed{x = \frac{1}{4} - \frac{\sqrt{5}}{4}}$

Raízes reais: $\frac{1}{2}$; $-\frac{1}{2}$; $\frac{1}{4} + \frac{\sqrt{3}}{4}$; $\frac{1}{4} - \frac{\sqrt{5}}{4}$

Raízes complexas: $\frac{1}{2}i$ e $-\frac{1}{2}i$

(d)

Raiz	Multiplicidade
$\frac{1}{2}$	$2 + 3 = 5$
$-\frac{1}{2}$	2
$\frac{1}{4} - \frac{\sqrt{5}}{4}$	3
$\frac{1}{4} + \frac{\sqrt{5}}{4}$	3
$\frac{1}{2}i$	2
$-\frac{1}{2}i$	2

(e) $(16x^4 - 1)^2 \left(x^3 - x^2 + \frac{1}{8}\right)^3 =$

$$= \left[(4x^2 - 1)(4x^2+1)\right]^2 \left[\left(x - \frac{1}{2}\right)\left(x - \left(\frac{1}{4} + \frac{\sqrt{5}}{4}\right)\right)\left(x - \left(\frac{1}{4} - \frac{\sqrt{5}}{4}\right)\right)\right]^3$$

$$= (2x-1)^2(2x+1)^2(4x^2+1)^2\left(x - \frac{1}{2}\right)^3\left(x - \left(\frac{1}{4} + \frac{\sqrt{5}}{4}\right)\right)^3\left(x - \left(\frac{1}{4} - \frac{\sqrt{5}}{4}\right)\right)^3$$

3) $x + x(x^2-3) + x(x^2-3)^2 + x(x^2-3)^3 + \cdots$

(a) razão $r = x^2 - 3$

A série é convergente se $|x^2-3| < 1$

$-1 < x^2 - 3 < 1$

$2 < x^2 < 4$

$x^2 > 2 \iff |x| > \sqrt{2} \iff x > \sqrt{2}$ ou $x < -\sqrt{2}$

$x^2 < 4 \iff |x| < 2 \iff -2 < x < 2$

(b) primeiro termo $= a = x$

$S = \dfrac{a}{1-r} = \dfrac{x}{1-(x^2-3)} = \dfrac{x}{4-x^2}$;

Logo $\dfrac{x}{4-x^2} \leqslant \dfrac{x}{3}$ $\quad\left\{\begin{array}{l} -2 < x < -\sqrt{2} \\ \text{ou} \\ \sqrt{2} < x < 2 \end{array}\right.$

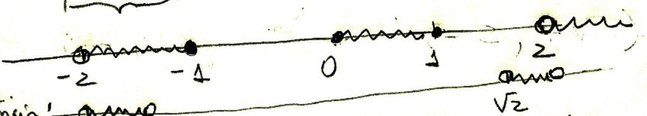
$\dfrac{x}{4-x^2} - \dfrac{x}{3} \leqslant 0$

$\dfrac{3x - x(4-x^2)}{3(4-x^2)} \leqslant 0$

$\dfrac{x(3-4+x^2)}{3(4-x^2)} \leqslant 0$

$\dfrac{x(x^2-1)}{3(4-x^2)} \leqslant 0$

		-2		-1		0		1		2	
x	−	−	−	−	−	0	+	+	+	+	+
x^2-1	+	+	+	0	−	−	−	0	+	+	+
$3(4-x^2)$	−	0	+	+	+	+	+	+	+	0	−
$\dfrac{x(x^2-1)}{3(4-x^2)}$	+	∄	−	0	+	0	−	0	+	∄	−



: I_1

: I_2

intervalo de convergência: −2 −√2

solução final é $I_1 \cap I_2 = (-2, -\sqrt{2})$

4)(a) $z = \sqrt{2} - \sqrt{2}\, i \implies |z| = \sqrt{2+2} = 2$

$\cos\theta_1 = \frac{\sqrt{2}}{2}$

$\text{sen}\,\theta_1 = \frac{-\sqrt{2}}{2}$ $\implies \theta_1 = -\frac{\pi}{4} \equiv 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

logo $\boxed{z = 2\left(\cos\frac{7\pi}{4} + i\,\text{sen}\,\frac{7\pi}{4}\right)}$

$w = \frac{\sqrt{3}}{3} + \frac{1}{3} i \implies |w| = \sqrt{\frac{3}{9} + \frac{1}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$\cos\theta_2 = \dfrac{\frac{\sqrt{3}}{3}}{\frac{2}{3}} = \frac{\sqrt{3}}{2} \implies \theta_2 = \frac{\pi}{6}$

$\text{sen}\,\theta_2 = \dfrac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

logo $\boxed{w = \frac{2}{3}\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right)}$

$\frac{81}{64} z^3 \cdot w^6 = \frac{81}{64} \cdot |z|^3 \cdot |w|^6 \left(\cos(3\theta_1 + 6\theta_2) + i\,\text{sen}\,(3\theta_1 + 6\theta_2)\right)$

mas $3\theta_1 + 6\theta_2 = 3 \times \frac{7\pi}{4} + 6 \times \frac{\pi}{6} = \frac{21\pi}{4} + \pi = \frac{25\pi}{4} = \frac{24\pi + \pi}{4} = 6\pi + \frac{\pi}{4} \equiv \frac{\pi}{4}$

logo $\frac{81}{64} \cdot z^3 \cdot w^6 = \frac{\cancel{81}}{\cancel{64}} \cdot 2^3 \cdot \frac{\cancel{2^6}}{\cancel{3^6}_{\,9}} \left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right)$

$= \frac{8}{9}\left(\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right) = \boxed{\frac{4\sqrt{2}}{9} + \frac{4\sqrt{2}}{9} i}$

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(b) $z^3 = -1 + i = w \implies |w| = \sqrt{1+1} = \sqrt{2}$

$\alpha = \arg(w)$

$\cos\alpha = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\text{sen}\,\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\implies \boxed{\alpha = \frac{3\pi}{4}}$

$|z| = |w|^{1/3} = (\sqrt{2})^{1/3} = \sqrt[6]{2}$

$\arg z = \frac{\alpha + 2k\pi}{3} = \frac{\frac{3\pi}{4} + 2k\pi}{3} = \frac{\pi}{4} + \frac{2k\pi}{3}$

$k = 0: \quad \theta_1 = \frac{\pi}{4} \implies z_1 = \sqrt[6]{2}\left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right)$

$k = 1 \quad \theta_2 = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12} \implies z_2 = \sqrt[6]{2}\left(\cos\frac{11\pi}{12} + i\,\text{sen}\,\frac{11\pi}{12}\right)$

$k = 2 \quad \theta_3 = \frac{\pi}{4} + \frac{4\pi}{3} = \frac{19\pi}{12} \implies z_3 = \sqrt[6]{2}\left(\cos\frac{19\pi}{12} + i\,\text{sen}\,\frac{19\pi}{12}\right)$