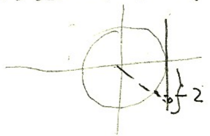
1) (a) $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$. Falso pois

$\frac{2\pi}{3} \notin \left(-\frac{\pi}{2}, 0\right)$

(b) $\arctan(-2) = a$. Verdadeiro pois



$-2 < 0$, e se $x < 0$, sabemos que $\arctan x \in \left(-\frac{\pi}{2}, 0\right)$.

(c) $\frac{\sqrt{x^6(x-1)^4}}{x} = \frac{|x^3|\,|(x-1)^2|}{x} = \frac{|x \cdot x^2|\,(x-1)^2}{x} =$

$= \frac{|x|\,x^2(x-1)^2}{x}$, mas $\frac{|x|}{x} = 1$ apenas se $x > 0$,

logo para $x < 0$ a afirmação é falsa.

2) $1 + \frac{x}{|x-6|} + \left(\frac{x}{|x-6|}\right)^2 + \dots$

$r = \frac{x}{|x-6|} \Rightarrow \left|\frac{x}{|x-6|}\right| < 1$.

$\left|\frac{x}{|x-6|}\right| = \frac{|x|}{|x-6|}$, logo $\frac{|x|}{|x-6|} < 1 \iff$

$|x| < |x-6|$

$\underbrace{-|x-6| < x}_{I} \underbrace{< |x-6|}_{II}$

I) $-|x-6| < x \iff |x-6| > -x \iff$

$x - 6 > -x$ | ou | $x - 6 < -(-x)$

$2x > 6$ | ou | $x - 6 < x$

$x > 3$ | | $-6 < 0$, verdadeiro $\forall x \in \mathbb{R}$

II) $x < |x-6| \iff |x-6| > x$

$x - 6 > x$ ou $x - 6 < -x$

$-6 > 0$ ou $2x < 6$

Falso $\forall x \in \mathbb{R}$; $x < 3$

Solução $= I \cap II$

Intervalo de convergência é $\boxed{(-\infty, 3)}$

3) a) $\cos(2x) - \operatorname{sen}(4x) = 0$

$\cos(2x) - 2\operatorname{sen}(2x)\cos 2x = 0$

$\cos(2x)\,(1 - 2\operatorname{sen} 2x) = 0$

$\cos(2x) = 0$ ou $1 - 2\operatorname{sen} 2x = 0$

$2x = \frac{\pi}{2} + k\pi$

$\boxed{x = \frac{\pi}{4} + \frac{k\pi}{2}}$

$2\operatorname{sen} 2x = 1$

$\operatorname{sen} 2x = \frac{1}{2}$

$\left\{ \begin{array}{l} 2x = \frac{\pi}{6} + 2k\pi \\ \text{ou } 2x = \frac{5\pi}{6} + 2k\pi \end{array} \right.$

$\boxed{x = \frac{\pi}{3} + k\pi}$

ou $\boxed{x = \frac{5\pi}{3} + k\pi}$

(b) $\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \frac{1}{3}$

$3e^{2x} - 3e^{-2x} = e^{2x} + e^{-2x}$

$2e^{2x} - 4e^{-2x} = 0$

$e^{2x} - 2e^{-2x} = 0$

$e^{2x} - \frac{2}{e^{2x}} = 0$

$\frac{e^{4x} - 2}{e^{2x}} = 0$

$e^{4x} = 2$

$\ln(e^{4x}) = \ln 2$

$4x = \ln 2$

$\boxed{x = \frac{1}{4}\ln 2}$

4) a) $\dfrac{8x^3+8x^2+2x-9}{x^2+x-2} \leq 4$

$$\frac{8x^3+8x^2+2x-9}{x^2+x-2} - 4 \leq 0$$

$$\frac{8x^3+8x^2+2x-9-4x^2-4x+8}{x^2+x-2} \leq 0$$

$$\frac{8x^3+4x^2-2x-1}{x^2+x-2} \leq 0$$

$8x^3+4x^2-2x-1=0$

possíveis raízes racionais : $1, -1, \frac{1}{2}; -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{8}$

$x=1 \Rightarrow 8+4-2-1 \neq 0$

$x=-1 \Rightarrow -8+4+2-1 \neq 0$

$x=\frac{1}{2} \Rightarrow 8 \times \frac{1}{8} + 4 \times \frac{1}{4} - 2 \times \frac{1}{2} - 1 =$

$= 1+1-1-1 = 0$

Aplicando Briot-Ruffini :

	8	4	−2	−1
$\frac{1}{2}$	8	8	2	0

$(x-\frac{1}{2})(8x^2+8x+2) =$

$(2x-1)(4x^2+4x+1)$

$(2x-1)(2x+1)^2$

$4x^2+4x+1=0$

$x = \dfrac{-4 \pm \sqrt{16-16}}{8} = -\dfrac{4}{8} = -\dfrac{1}{2}$

$x^2+x-2=$

$= \dfrac{-1 \pm \sqrt{1+8}}{2} = \dfrac{-1 \pm 3}{2} = \begin{cases} \frac{2}{2}=1 \\ \frac{-4}{2}=-2 \end{cases}$

$\frac{1}{2}, -\frac{1}{2}, 1, -2$

$$E(x) = \frac{(2x-1)(2x+1)^2}{(x-1)(x+2)} \leq 0$$

		−2		−½		½		1	
$2x-1$	−	−	−	−	−	0	+	+	+
$(2x+1)^2$	+	+	+	0	+	+	+	+	+
$(x-1)(x+2)$	+	0	−	−	−	−	−	0	+
$E(x)$	−	∄	+	0	+	0	−	∄	+

Solução : $(-\infty, -2) \cup \{-\frac{1}{2}\} \cup (\frac{1}{2}, 1)$

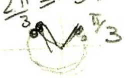
4) (b) $4\,\text{sen}\,x \cos x < \sqrt{3}$

$2 \cdot 2\,\text{sen}\,x \cos x < \sqrt{3}$

$2\,\text{sen}\,2x < \sqrt{3}$

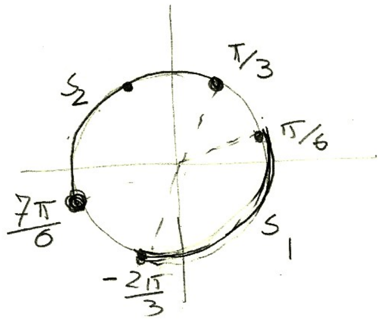
$\text{sen}\,2x < \frac{\sqrt{3}}{2}$

$\frac{2\pi}{3} \equiv \frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$



$-\frac{4\pi}{3} + 2k\pi < 2x < \frac{\pi}{3} + 2k\pi$

$$\boxed{-\frac{2\pi}{3} + k\pi < x < \frac{\pi}{6} + k\pi}$$



$k = 0 \Rightarrow -\frac{2\pi}{3} < x < \frac{\pi}{6}$

$k = 1 \Rightarrow -\frac{2\pi}{3} + \pi < x < \frac{\pi}{6} + \pi$

$\frac{\pi}{3} < x < \frac{7\pi}{6}$

5) $w = 2 - 2\sqrt{3}\,i \Rightarrow |w| = \sqrt{4+12} = 4$

$\cos\theta = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = \frac{5\pi}{3}$

$\text{sen}\,\theta = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

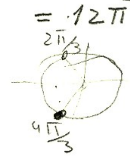
$-\frac{\pi}{3} \equiv 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Logo $w = 4\left(\cos\frac{5\pi}{3} + i\,\text{sen}\,\frac{5\pi}{3}\right)$

$w^8 = 4^8\left(\cos\frac{8 \times 5\pi}{3} + i\,\text{sen}\,\frac{8 \times \pi}{3}\right)$

$\frac{40\pi}{3} = \frac{36\pi + 4\pi}{3} = 12\pi + \frac{4\pi}{3} \equiv \frac{4\pi}{3}$

$w^8 = 4^8\left(\cos\frac{4\pi}{3} + i\,\text{sen}\,\frac{4\pi}{3}\right)$



$w^8 = 4^8\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\,i\right)$

$i^3 = i^2 \cdot i = -i$

$w^8 i^3 = \frac{4^8}{2}(-1 - \sqrt{3}\,i)(-i) = \frac{4^8}{2}(+i + \sqrt{3}\,i^2)$

$w^8 i = \frac{4^8}{2}(-\sqrt{3} + i)$

$\frac{4^8}{2} = \frac{2^{16}}{2} = 2^{15}$

$w^8 i = -2^{15}\sqrt{3} + 2^{15}\,i$ //