(1)(a) $\sqrt{(x^6+x^3+3)} = \sqrt{x^6\left(1+\frac{1}{x^3}+\frac{3}{x0}\right)} =$

$= \sqrt{x^6}\sqrt{1+\frac{1}{x^3}+\frac{3}{x^6}} = |x^3|\sqrt{1+\frac{1}{x^3}+\frac{3}{x0}}$

Mas $|x^3| = x^3$ apenas quando $x>0$

Logo para $x<0$: $\sqrt{x^6+x^3+3} = -x^3\sqrt{1+\frac{1}{x^3}+\frac{3}{x^6}}$

quando $x<0$

E a afirmação é FALSA

(b) $x^4 < x^6 \iff x^6 > x^4 \iff x^6 - x^4 > 0 \iff$

$x^4(x^2-1) > 0$, $x^4 \geqslant 0 \iff x^2 - 1 \geqslant 0 \iff$

$\iff x^2 \geqslant 1 \iff \sqrt{x^2} \geqslant \sqrt{1} \iff |x| \geqslant 1.$

A equivalência é VERDADEIRA.

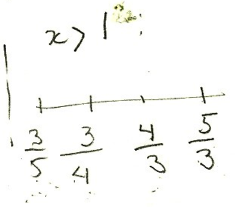
2) x; $x^{3/4}$; $x^{4/3}$; $x^{3/5}$; $x^{5/3}$; $x^{-3/4}$; $x^{-4/3}$; $x^{-5/3}$

(a) domínio $= \mathbb{R}$ para x; $x^{4/3}$; $x^{3/5}$; $x^{5/3}$

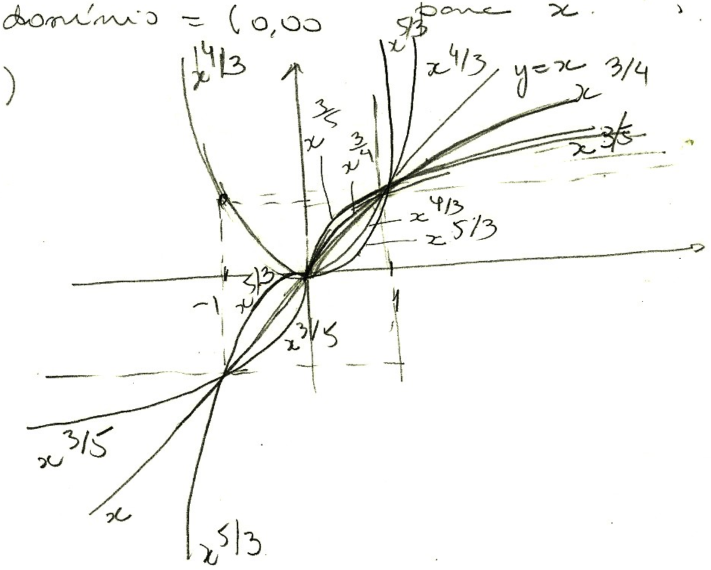
domínio $= \mathbb{R} - \{0\}$ para $x^{-4/3}$; $x^{-5/3}$

domínio $= [0,\infty)$ para $x^{3/4}$ e

domínio $= (0,\infty$ para $x^{-3/4}$.



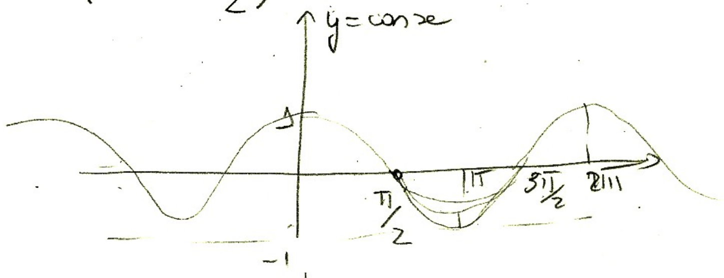
(b)

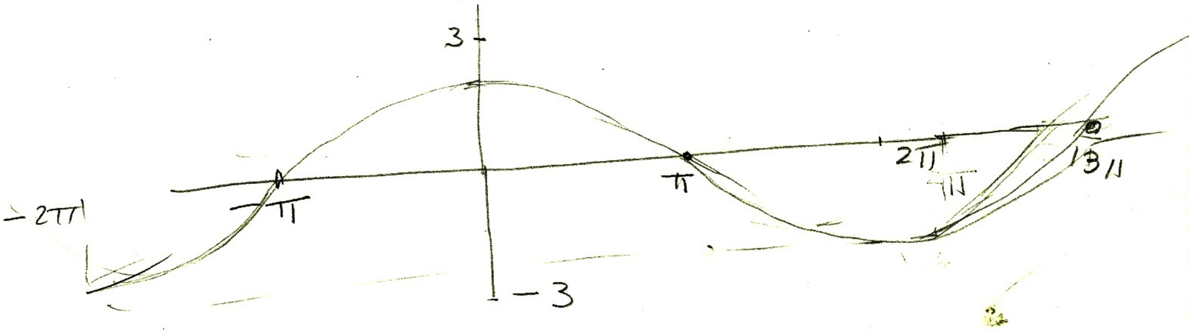


3) $f(x) = 3\cos^2\left(\frac{x}{4}\right) - 3sen^2\left(\frac{x}{4}\right)$

(a) $f(x) = 3\cos^2\frac{x}{2} - 3sen^2\frac{x}{2} = 3\left(\cos^2\frac{x}{4} - sen^2\frac{x}{4}\right)$

$= 3\left(\cos\frac{x}{2}\right) = 3\cos\frac{x}{2}$





(b) $f(x) = 0 \iff 3\cos\frac{x}{2} = 0$

$\iff \frac{x}{2} = \frac{\pi}{2} + k\pi$

$x = \pi + 2k\pi$

Solução : $\{\pi, 3\pi\}$

4)(a) $\frac{4}{x^3} + \frac{8}{x^2} - \frac{1}{x} \leqslant 2$

$$\frac{4}{x3} + \frac{8}{x\cancel{3}} - \frac{1}{x} - 2 \leqslant 0$$

$$\frac{4 + 8x - x^2 - 2x^3}{x^3} \leqslant 0$$

$$\frac{2x^3 + x^2 - 8x - 4}{x3} \geqslant 0$$

$2x^3 + x^2 - 8x - 4 = 0$

possíveis raízes: $\{\pm 1 ; \pm 2 ; \pm 4 ; \pm \frac{1}{2}\}$

$x = 1 \Rightarrow 1 + 1 - 8 - 4 \neq 0$

$x = -1 \Rightarrow -2 + 1 + 8 - 4 \neq 0$

$x = 2 \Rightarrow 16 + 4 - 16 - 4 = 0$

$$\begin{array}{c|cccc} & 2 & 1 & -8 & -4 \\ \hline 2 & 2 & 5 & +2 & 0 \end{array}$$

$2x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{25 - 16}}{4} > 0$

$x = \frac{-5 \pm 3}{4} = \begin{cases} \frac{-8}{4} = -2 \\ -\frac{2}{4} = -\frac{1}{2} \end{cases}$

$$f(x) = \frac{(x-2)(x+2)(2x+1)}{x3} \geqslant 0$$

$$\frac{(x^2-4)(2x+1)}{x3} \geqslant 0$$

		-2		-1/2		0		2	
x^2-4	+	0	−	−	−	−	−	0	+
$2x+1$	−	−	−	0	+	+	+	+	+
$x3$	−	−	−	−	−	0	+	+	+
$f(x)$	+	0	−	0	+	∄	−	0	+

solução:

$(-\infty, 0] \cup [-\frac{1}{2}, 0) \cup [2, \infty)$

(b) $\frac{x}{x - |x-1|} < 0$

$x \geqslant 1$

$\frac{x}{x-(x+1)} < 0$

$\frac{x}{x-x+1} < 0$

$\frac{x}{1} < 0$, $\nexists x$ $\overset{x<0}{e\, x \geqslant 1}$

ou

p/ $x < 1$

$\frac{x}{x+x-1} < 0$

$\frac{x}{2x-1} < 0$

$+ \quad | \quad - \quad | \quad +$

$\quad\;\; 0 \qquad \frac{1}{2}$

solução $\boxed{0 < x < 1/2}$

4)c) $\frac{1}{\operatorname{sen} x} \leq 2$

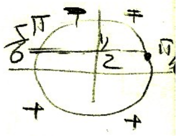
$\frac{1}{\operatorname{sen} x} - 2 \leq 0$

$\frac{1 - 2\operatorname{sen} x}{\operatorname{sen} x} \leq 0$

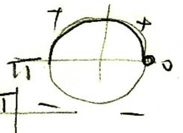
$1 - 2\operatorname{sen} x < 0$

$2 \operatorname{sen} x > 1$

$\operatorname{sen} x > \frac{1}{2}$



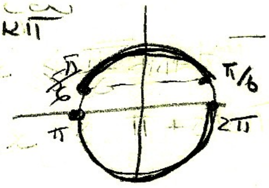
$\operatorname{sen} x > 0$



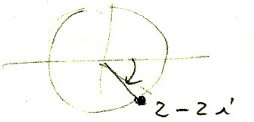
		π/6		5π/6		π		2π	−
$1 - 2\operatorname{sen} x$	+	0	−	0	+	+	+	+	+
$\operatorname{sen} x$	+		+		+	0	−	+	−
$\frac{1 - 2\operatorname{sen} x}{\operatorname{sen} x}$	+		−		+		−		−

Solução $\frac{\pi}{6} + 2k\pi < x \leq \frac{5\pi}{0} + 2k\pi$

ou $\pi + 2k\pi < x < 2\pi + 2k\pi$



5) $z = 2 - 2i$



$|z| = \sqrt{4+4} = 2\sqrt{2}$

$\theta = -\frac{\pi}{4} \equiv 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \Rightarrow z = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i \operatorname{sen}\frac{7\pi}{4}\right)$

$z^5 = (2\sqrt{2})^5\left(\cos\frac{35\pi}{4} + i \operatorname{sen}\frac{35\pi}{4}\right)$

$w^4 = \frac{1}{2^4}\left(\cos\frac{3\pi}{8} + i \operatorname{sen}\frac{3\pi}{8}\right) = \frac{1}{2^4}\left(\cos\frac{3\pi}{2} + i \operatorname{sen}\frac{3\pi}{2}\right)$

$z^5 . w^4 = \frac{(2\sqrt{2})^5}{2^4}\left(\cos\left(\frac{35\pi}{4} + \frac{3\pi}{2}\right) + i \operatorname{sen}\left(\frac{35\pi}{4} + \frac{3\pi}{2}\right)\right)$

$= 2(\sqrt{2})^5 \left(\cos\frac{\pi}{4} + i \operatorname{sen}\frac{\pi}{4}\right)$

$= 2 \times \sqrt{2}^4 \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= \overset{4}{\cancel{8}}\sqrt{2} \times \frac{\sqrt{2}}{\cancel{2}}\,(1 + i)$

$= 8 + 8i$ //

$\frac{35\pi}{4} + \frac{3\pi}{2} =$

$= \frac{41\pi}{4} =$

$= \frac{40\pi + \pi}{4}$

$= 10\pi + \frac{\pi}{4}$

$\equiv \frac{\pi}{4}$