(Q1) (a) $\lim\limits_{x\to-\infty} \dfrac{4x^4 - 2x^3\sqrt{x^2} - 3x + 1}{9 - 12x^4} =$

$= \lim\limits_{\substack{x\to-\infty \\ x<0,\ |x|=-x}} \dfrac{4x^4 - 2x^3|x| - 3x + 1}{9 - 12x^4} =$

$= \lim\limits_{x\to-\infty} \dfrac{4x^4 + 2x^3.x - 3x + 1}{9 - 12x^4} = \lim\limits_{x\to-\infty} \dfrac{6x^4 - 3x + 1}{9 - 12x^4} =$

$= \lim\limits_{x\to-\infty} \dfrac{\cancel{x^4}.\left(6 - \frac{3}{x^3}\xrightarrow{}0 + \frac{1}{x^4}\xrightarrow{}0\right)}{\cancel{x^4}\left(\frac{9}{x^4}\xrightarrow{}0 - 12\right)} = \dfrac{6}{-12} = -\dfrac{1}{2}$ //

(b) $\lim\limits_{x\to\pi/2} \dfrac{\tan(x-\pi/2)}{\text{sen}(2x-\pi)} = \dfrac{\tan(0)}{\text{sen}\,0} = \dfrac{0}{0}$, ind.

$y = x - \frac{\pi}{2}$, $x \to \frac{\pi}{2}$, $y \to 0$, $x = y + \frac{\pi}{2}$

$= \lim\limits_{y\to 0} \dfrac{\tan y}{\text{sen}\,2y} = \lim\limits_{y\to 0} \dfrac{\frac{\text{sen}\,y}{y}.\frac{1}{\cos y}}{2.\frac{\text{sen}\,2y}{2y}}$

$2x - \pi =$
$= 2(y + \frac{\pi}{2}) - \pi$
$= 2y + \pi - \pi$
$= 2y$

$= \dfrac{1.\frac{1}{1}}{2.1} = \dfrac{1}{2}$ //

(Q2) $\lim\limits_{x\to 2^-} \dfrac{\sqrt[3]{4x} - 2}{|x-2|} = \dfrac{\sqrt[3]{8} - 2}{|2-2|} = \dfrac{0}{0}$, ind.

($x < 2$, $x - 2 < 0$ e $|x-2| = -(x-2)$.)

$\lim\limits_{x\to 2^-} \dfrac{\sqrt[3]{4x} - 2}{-(x-2)} = \lim\limits_{x\to 2^-} \dfrac{(\sqrt[3]{4x} - 2)}{-(x-2)} \times \dfrac{((\sqrt[3]{4x})^2 + \sqrt[3]{4x}.2 + 2^2)}{((\sqrt[3]{4x})^2 + \sqrt[3]{4x}.2 + 2^2)}$

$= \lim\limits_{x\to 2^-} \dfrac{4x - 8}{-(x-2)} \times \dfrac{1}{((\sqrt[3]{4x})^2 + \sqrt[3]{4x}.2 + 2^2)}$

$= \lim\limits_{x\to 2^-} \dfrac{4\cancel{(x-2)}}{-\cancel{(x-2)}} \times \dfrac{1}{(\sqrt[3]{4x})^2 + \sqrt[3]{4x}.2 + 4} =$

$= -4 \times \dfrac{1}{4+4+4} = \dfrac{-4}{12} = -\dfrac{1}{3}$

$\lim\limits_{x\to 2^+} \dfrac{\sqrt[3]{4x} - 2}{|x-2|} = \lim\limits_{x\to 2^+} \dfrac{\sqrt[3]{4x} - 2}{x-2} =$

análogo anterior,

$= \lim\limits_{x\to 2^+} \dfrac{4\cancel{(x-2)}}{\cancel{(x-2)}} \times \dfrac{1}{(\sqrt[3]{4x})^2 + \sqrt[3]{4x}.2 + 4} =$

$= 4 \times \dfrac{1}{4+4+4} = \dfrac{1}{3}$

Não é possível definir $f(2)$ porque $\lim\limits_{x\to 2^-} g(x) \neq \lim\limits_{x\to 2^+} g(x)$.

Q3) (a) $f(x) = \dfrac{2(1+3x)}{\sqrt{1-3x}}$

$$f'(x) = 2\cdot\frac{\sqrt{1-3x}\times 3 - (1+3x)\times\frac{-3}{2\sqrt{1-3x}}}{1-3x} =$$

$$= 2\,\frac{6(1-3x)+3(1+3x)}{2\sqrt{1-3x}\,(1-3x)^1} =$$

$$= 2\,\frac{6-18x+3+9x}{2(1-3x)^{3/2}} = \frac{9-9x}{\sqrt{(1-3x)^3}}$$

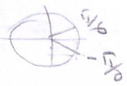
$$= \frac{9(1-x)}{\sqrt{(1-3x)^3}} \quad //$$

(b) $g(x) = x\,\text{sen}(\pi\cdot p(x))$.

$g'(x) = x(\cos(\pi\cdot p(x)))\cdot\pi\cdot p'(x) + \text{sen}(\pi\cdot p(x))$

$g'(3) = 3(\cos(\pi\cdot p(3)))\cdot\pi\cdot p'(3) + \text{sen}(\pi\cdot p(3))$

$g'(3) = 3(\cos(-\frac{\pi}{6}))\cdot\pi\cdot\frac{\sqrt{3}}{\pi} + \text{sen}(-\frac{\pi}{6})$



$g'(3) = 3\cdot\frac{\sqrt{3}}{2}\times\sqrt{3} + (-\frac{1}{2}) = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4 \quad //$

(c) $y\tan x = 2y^5 - \frac{4}{\pi}x$, $x = \frac{\pi}{4}$, $y=1$

derivando implicitamente,

$$y\cdot\sec^2 x + y'\cdot\tan x = 10y^4y' - \frac{4}{\pi}$$

p/ $x = \frac{\pi}{4}$, $y = 1$:

$$1\cdot\underbrace{\sec^2\frac{\pi}{4}}_{=2} + y'\cdot\underbrace{\tan\frac{\pi}{4}}_{=1} = 10y' - \frac{4}{\pi}$$

$\sec\frac{\pi}{4} = \dfrac{1}{\frac{\sqrt{2}}{2}} = \dfrac{2}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

$$2 + y' = 10y' - \frac{4}{\pi}$$

$$9y' = 2 + \frac{4}{\pi}$$

$$\boxed{y' = \frac{2\pi+4}{9\pi}}$$

(Q4) coef angular de retas paralelas são iguais, logo

$f'(x) = -\frac{1}{3}$, $x = ?$

reta dada:
$y = 2 - \frac{x}{3}$
$y = 2 - \frac{1}{3}x$
coef. angular $= -\frac{1}{3}$

$f(x) = x - x^{2/3}$

$f'(x) = 1 - \frac{2}{3}x^{-1/3}$

Logo, $1 - \frac{2}{3}x^{-1/3} = -\frac{1}{3}$

$\frac{2}{3}x^{-1/3} = 1 + \frac{1}{3}$

$\frac{2}{3}x^{-1/3} = \frac{4}{3}$

$2x^{-1/3} = 4$

$x^{-1/3} = 2 \Rightarrow \frac{1}{x^{1/3}} = 2 \Rightarrow x^{1/3} = \frac{1}{2}$

$x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$y = f\left(\frac{1}{8}\right) = \frac{1}{8} - \left(\frac{1}{8}\right)^{2/3} = \frac{1}{8} - \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$

O ponto é $\left(\frac{1}{8}, -\frac{1}{8}\right)$.

Equação da reta tangente: $\boxed{y + \frac{1}{8} = -\frac{1}{3}\left(x - \frac{1}{8}\right)}$

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(Q5) $\frac{dV}{dt} = 10\ dm^3/min$, $h = 8\ dm$

$V = \frac{1}{3}\pi r^2 . h$, $h = 2r$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi \times \frac{h^2}{4} . h$

$V = \frac{1}{12}\pi . h^3$

$\underbrace{\frac{dV}{dt}}_{=10} = \frac{1}{12}\pi . 3\underbrace{h^2}_{=8} . \frac{dh}{dt}$

$10 = \frac{1}{\cancel{12}_{\,4}} \times \cancel{3} \times \pi \times \cancel{64}^{\,16} . \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{16\pi} = \frac{5}{8\pi}$ dm/min //