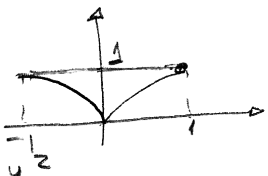
1) $I=\int_{-1}^{0}\int_{\sqrt{-x}}^{1} 3\cos(y^3)\,dy\,dx + \int_{0}^{1}\int_{\sqrt{x}}^{1} 3\cos(y^3)\,dy\,dx$



$y=\sqrt{-x},\ x<0$ $\quad$ $y=\sqrt{x},\ x>0$

$y^2=-x$ $\quad$ $x=y^2$

$x=-y^2$

$I=\int_0^1\int_{-y^2}^{y^2} 3\cos y^3\,dx\,dy = \int_0^1 x\Big]_{-y^2}^{y^2} 3\cos y^3\,dy =$

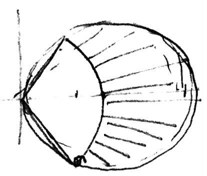
$= \int_0^1 (y^2-(-y^2))\cdot 3\cos y^3\,dy = \int_0^1 2y^2\cdot\cos(y^3)\,dy =$

$= 2\,\text{sen}\,y^3\Big]_0^1 = 2(\text{sen}(1)-\text{sen}(0)) = 2\,\text{sen}(1)$

2) $x^2+y^2=6x$

$x^2-6x+9+y^2=9$

$(x-3)^2+y^2=9$



Em coordenadas polares:

$x^2+y^2=6x$ $\quad$ $x^2+y^2=9$

$r^2=6r\cos\theta$ $\quad$ $r=3$

$r=6\cos\theta$

$\cap\begin{cases} r=6\cos\theta \\ r=3\end{cases}$ $\quad 6\cos\theta=3$, $\cos\theta=\frac{1}{2}$ $\quad \theta=-\pi/3$ ou $\theta=\pi/3$

Massa $\rho(x,y)=\frac{6}{\sqrt{x^2+y^2}}$, $\rho(r,\theta)=\frac{6}{r}$

$M=\iint_D \rho(x,y)\,dx\,dy = \iint_{D_{r\theta}} \rho(r,\theta)\,r\,dr\,d\theta =$

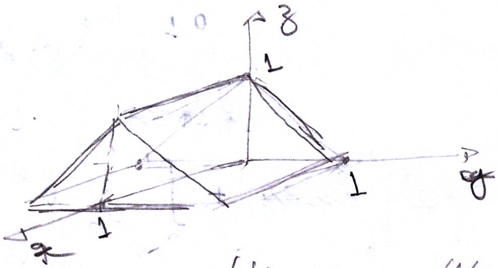
$= \int_{-\pi/3}^{\pi/3}\int_3^{6\cos\theta}\frac{1}{r}\cdot r\,dr\,d\theta = \int_{-\pi/3}^{\pi/3} r\Big]_3^{6\cos\theta} d\theta =$

$= \int_{-\pi/3}^{\pi/3}(6\cos\theta-3)\,d\theta = 6\,\text{sen}\,\theta - 3\theta\Big]_{-\pi/3}^{\pi/3} =$

$= 6\left(\overset{\sqrt3/2}{\text{sen}\frac{\pi}{3}} - \overset{=\sqrt3/2}{\text{sen}\left(-\frac{\pi}{3}\right)}\right) - 3\left(\frac{\pi}{3}-\left(-\frac{\pi}{3}\right)\right)$

$= 6\times 2\times\sqrt3 - 3\times\underline{2\pi} - 6\sqrt3 - 2\pi$ //

3) $\rho(x,y) = 1$



distância de (x,y,z) ao eixo y:

$d(x,y,z) = \sqrt{x^2+z^2}$

Momento de Inércia $I_y = \iiint_W \rho(x,y,z)\,(d(x,y,z))^2\,dx\,dy\,dz$

$= \iiint_W (1)(x^2+z^2)\,dx\,dy\,dz =$

$$D_{xz}: \begin{cases} 0 \leq x \leq 1 \\ 0 \leq z \leq 1 \end{cases}$$

$y + z = 1 \Rightarrow y = 1 - z$

$-y + z = 1 \Rightarrow y = z - 1$

$z - 1 \leq y \leq 1 - z$

$= \underbrace{\int_0^1\int_0^1}_{D_{xz}} \int_{z-1}^{1-z} (x^2+z^2)\,dy\,dx\,dz$

$= \int_0^1\int_0^1 (x^2+z^2)\left(y\Big|_{z-1}^{1-z}\right)dx\,dz =$

$= \int_0^1\int_0^1 (x^2+z^2)(1-z-z+1)\,dx\,dz =$

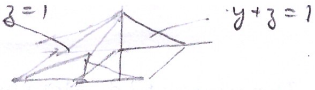
$= \int_0^1\int_0^1 (x^2+z^2)(2-2z)\,dx = 2\int_0^1 \left(\frac{x^3}{3} + xz^2\right)(1-z)\,dz$

$= 2\int_0^1 \left(\frac{1}{3} + z^2\right)(1-z)\,dz = 2\int_0^1 \left(\frac{1}{3} - \frac{z}{3} + z^2 - z^3\right)dz$

$= 2\left[\frac{z}{3} - \frac{z^2}{6} + \frac{z^3}{3} - \frac{z^4}{4}\Big|_0^1\right] =$

$= 2\left[\frac{1}{3} - \frac{1}{6} + \frac{1}{3} - \frac{1}{4}\right] = 2\,\frac{4-2+4-3}{12} = 2 \times \frac{3}{12} = \frac{1}{2}$

As 6 possíveis ordens: (Questão 3)



D_{xz}: $I = \int_0^1 \int_0^1 \int_{z-1}^{1-z} (x^2+z^2)\, dy\, dx\, dz$

D_{xz}: $I = \int_0^1 \int_0^1 \int_{z-1}^{1-z} (x^2+z^2)\, dy\, dz\, dx$

D_{xy}: $I = \int_{-1}^0 \int_0^1 \int_0^{1+y} (x^2+z^2)\, dz\, dx\, dy + \int_0^1 \int_0^1 \int_0^{1-y} (x^2+z^2)\, dz\, dx\, dy$

D_{xy}: $I = \int_0^1 \int_{-1}^0 \int_0^{1+y} (x^2+z^2)\, dz\, dy\, dx + \int_0^1 \int_0^1 \int_0^{1-y} (x^2+z^2)\, dz\, dy\, dx$

D_{yz}: $I = \int_0^1 \int_{z-1}^{1-z} \int_0^1 (x^2+z^2)\, dx\, dy\, dz$

D_{yz}: $I = \int_{-1}^0 \int_0^{y+1} \int_0^1 (x^2+z^2)\, dx\, dz\, dy + \int_0^1 \int_0^{1-y} \int_0^1 (x^2+z^2)\, dx\, dz\, dy$

4) $x^2+y^2+z^2=4z$

$x^2+y^2+z^2-4z+4=4$

$x^2+y^2+(z-2)^2=4$

Em coordenadas esféricas:

$x^2+y^2+z^2=4z$ $\qquad$ $z=2$

$\rho^2=4\rho\cos\varphi$ $\qquad$ $\rho\cos\varphi=2$

$\rho=4\cos\varphi$. $\qquad$ $\rho=2\sec\varphi$

1º octante : $x\geq 0, y\geq 0$: $0\leq\theta\leq\pi/2$.

$\cap\begin{cases}\rho=4\cos\varphi\\ \rho=2\sec\varphi\end{cases} \Rightarrow 4\cos\varphi=2\sec\varphi$

$2\cos\varphi=\frac{1}{\cos\varphi}$

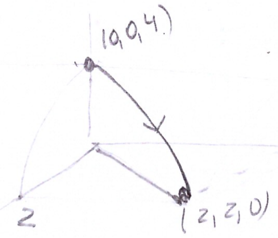
$\cos^2\varphi=\frac{1}{2} \Rightarrow \cos\varphi=\frac{\sqrt{2}}{2} \Rightarrow \varphi=\pi/4$

$$\iiint_V \frac{x^2}{z^2}\,dx\,dy\,dz=\int_0^{\pi/2}\int_0^{\pi/4}\int_{2\sec\varphi}^{4\cos\varphi}\frac{\rho^2\cos^2\theta\,\mathrm{sen}^2\varphi}{\rho^2\cos^2\varphi}\cdot\rho^2\,\mathrm{sen}\,\varphi\,d\rho\,d\varphi\,d\theta$$

$$=\int_0^{\pi/2}\int_0^{\pi/4}\int_{2\sec\varphi}^{4\cos\varphi}\rho^2\cos^2\theta\,\tan^2\varphi\,\mathrm{sen}\,\varphi\,d\rho\,d\varphi\,d\theta \quad //$$

(5) $\vec{r}(t)=(t,t,4-t^2)$

$0\leq t\leq 2$



$z=4-x^2$

$x=t,\ z=4-t^2$

$y=x\ ,\ y=t$

$\vec{r}\,'(t)=(1,1,-2t)$

$\|\vec{r}\,'(t)\|=\sqrt{1+1+4t^2}$

$=\sqrt{2+4t^2}$

$$\int_C x\,ds=\int_0^2 t\,\|\vec{r}\,'(t)\|\,dt$$

$$=\int_0^2 t\sqrt{2+4t^2}\,dt=$$

$$=\frac{1}{8}\int_0^2 8t\cdot(2+4t^2)^{1/2}\,dt$$

$$=\frac{1}{8}\left.\frac{(2+4t^2)^{3/2}}{\frac{3}{2}}\right]_0^2=$$

$$=\frac{1}{12}\left[(2+16)^{3/2}-(2)^{3/2}\right]=\frac{1}{12}\left(18^{3/2}-2^{3/2}\right)$$

$$=\frac{1}{12}\left(18\sqrt{18}-2\sqrt{2}\right)=\frac{1}{6}\left(\underbrace{9\sqrt{18}}_{27\sqrt{2}}-\sqrt{2}\right)=\frac{1}{6}\left(26\sqrt{2}\right)=\frac{13\sqrt{2}}{3}$$