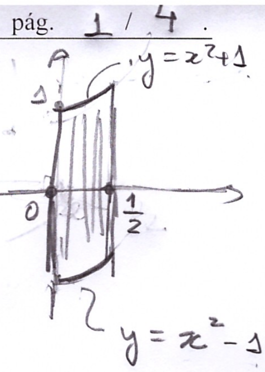
(Q1) $D_{xy} = \{(x,y) \in \mathbb{R}^2;\ 0 \leq 2x \leq 1 \text{ e } -1 \leq y - x^2 \leq 1$



$y - x^2 = 1 \Rightarrow y = x^2 + 1$

$y - x^2 = -1 \Rightarrow y = x^2 - 1$

$u = 2x$

$v = y - x^2$

$T^{-1}(x,y) = (2x,\ y - x^2) = (u,v)$

$\begin{cases} 0 \leq u \leq 1 \\ -1 \leq v \leq 1 \end{cases}$

$(x,y) = T(u,v)$

$$J(T) = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} 2 & 0 \\ -2x & 1 \end{vmatrix}} = \frac{1}{2}$$

$$\iint_{D_{xy}} e^{y-x^2}\,dx\,dy = \iint_{D_{uv}} e^{v}\frac{1}{2}\,du\,dv = \frac{1}{2}\int_{-1}^{1}\int_{0}^{1} e^{v}\,du\,dv =$$

$$= \frac{1}{2}\int_{-1}^{1} e^{v}\, u\Big]_0^1 dv = \frac{1}{2}\int_{-1}^{1} e^{v}\,dv = \frac{1}{2}e^{v}\Big]_{-1}^{1} = \frac{1}{2}(e - e^{-1})$$ //

(Q2) $x^2 + y^2 = 4y$

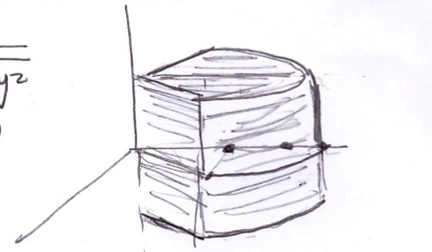
$x^2 + y^2 - 4y + 4 = 4$

$x^2 + (y-2)^2 = 4$

$z = 1,\quad z = -1,\quad y = x,\quad y = -x$.

$\rho(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2}}$

contém $(0,1,0)$



(a) $M = \iiint_W \dfrac{1}{\sqrt{x^2+y^2}}\,dx\,dy\,dz$

Coordenadas cilíndricas: $x^2 + y^2 = r^2$, $r^2 = 4r\,\text{sen}\,\theta$

$\boxed{r = 4\,\text{sen}\,\theta}$

projeção no plano xy:

$\begin{cases} \frac{\pi}{4} \leq \theta \leq 3\pi/4 \\ 0 \leq r \leq 4\,\text{sen}\,\theta \\ -1 \leq z \leq 1 \end{cases}$

$$M = \int_{\pi/4}^{3\pi/4}\int_0^{4\,\text{sen}\,\theta}\int_{-1}^{1} \frac{1}{r}\cdot r\,dz\,dr\,d\theta =$$

$$= \int_{\pi/4}^{3\pi/4}\int_0^{4\,\text{sen}\,\theta} \underbrace{z\Big]_{-1}^{1}}_{=2} dr\,d\theta = 2\int_{\pi/4}^{3\pi/4} r\Big]_0^{4\,\text{sen}\,\theta} d\theta = 2\int_{\pi/4}^{3\pi/4} 4\,\text{sen}\,\theta\,d\theta = 8(-\cos\theta)\Big]_{\pi/4}^{3\pi/4}$$

$$= 8\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = 8\sqrt{2}$$

O sólido W é simétrico em relação ao plano xy e em relação ao plano yz, logo é simétrico em relação ao eixo y.

$\rho(-x,y,z) = \rho(x,y,z)$ — ρ é simétrica em relação ao plano yz

$\rho(x,y,-z) = \rho(x,y,z)$ — ρ " " " " " " xy

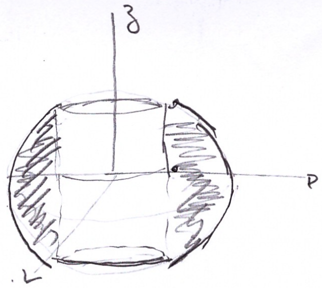
$\Longrightarrow$ ρ é simétrica em relação ao eixo y.

Portanto o centro de massa está no eixo y: $(\bar{x},\bar{y},\bar{z}) = (0,\bar{y},0)$

$$\bar{y} = \frac{\iiint_W y\,\rho(x,y,z)\,dx\,dy\,dz}{M} = \frac{1}{M}\iiint \frac{y}{\sqrt{x^2+y^2}}\,dx\,dy\,dz$$ //

(Q3) $V = \iiint_{W} dx\,dy\,dz$

$$= \iiint_{W_{\rho\varphi\theta}} \rho^2 \,\text{sen}\,\varphi \; d\rho\, d\varphi\, d\theta$$

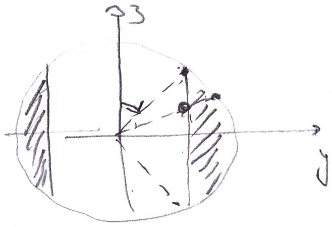


$x^2+y^2+z^2 = 9$

$\rho^2 = 9$

$\boxed{\rho = 3}$

$x^2+y^2 = \frac{9}{4}$



$\rho^2\cos^2\theta\,\text{sen}^2\varphi + \rho^2\text{sen}^2\theta\,\text{sen}^2\varphi = \frac{9}{4}$

$\rho^2\text{sen}^2\varphi = \frac{9}{4}$

$|\rho\,\text{sen}\,\varphi| = \frac{3}{2}$, variação máxima $0 \le \varphi \le \pi$, $\text{sen}\,\varphi \ge 0$

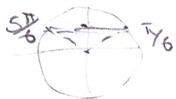
$|\text{sen}\,\varphi| = \text{sen}\,\varphi$

$\rho\,\text{sen}\,\varphi = \frac{3}{2}$

$\boxed{\rho = \frac{3}{2}\csc\varphi}$

$\frac{3}{2}\csc\varphi = 3$

$\text{sen}\,\varphi = \frac{1}{2}$



$\varphi = \frac{\pi}{6}$ ou $\varphi = \frac{5\pi}{6}$

$W_{\rho\varphi\theta}$: $0 \le \theta \le 2\pi$

$\frac{\pi}{6} \le \varphi \le \frac{5\pi}{6}$

$\frac{3}{2}\csc\varphi \le \rho \le 3$

$$V = \int_0^{2\pi}\int_{\pi/6}^{5\pi/6}\int_{\frac{3}{2}\csc\varphi}^{3} \rho^2\,\text{sen}\,\varphi \; d\rho\, d\varphi\, d\theta = \int_0^{2\pi}\int_{\pi/6}^{5\pi/6} \frac{\rho^3}{3}\Big]_{\frac{3}{2}\csc\varphi}^{3} \text{sen}\,\varphi \; d\varphi\, d\theta =$$

$$= \int_0^{2\pi}\int_{\pi/6}^{5\pi/6} \left(\frac{1}{3}\cdot 27 - \frac{1}{3}\cdot\frac{27}{8}\csc^3\varphi\right)\text{sen}\,\varphi\; d\varphi\, d\theta$$

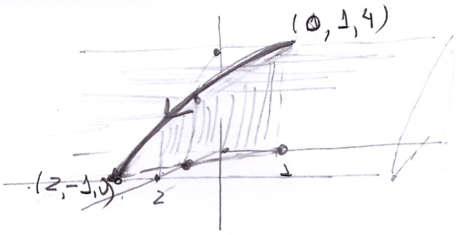
$$= \int_0^{2\pi}\int_{\pi/6}^{5\pi/6} \left(9\,\text{sen}\,\varphi - \frac{9}{8}\csc^2\varphi\right) d\varphi\, d\theta$$

$$= \int_0^{2\pi} \left(-9\cos\varphi + \frac{9}{8}\cot\varphi\right)\Big]_{\pi/6}^{5\pi/6} d\theta =$$

$$= \int_0^{2\pi}\left\{\left[-9\times\left(-\frac{\sqrt{3}}{2}\right) + \frac{9}{8}\left(\frac{-\sqrt{3}/2}{1/2}\right)\right] - \left[-9\times\frac{\sqrt{3}}{2} - \frac{9}{8}\times\frac{\frac{\sqrt{3}}{2}}{1/2}\right]\right\} d\theta$$

$$= \int_0^{2\pi}\left(9\sqrt{3} - \frac{9}{4}\sqrt{3}\right) d\theta = 9\sqrt{3}\times\frac{3}{4}\times\int_0^{2\pi} d\theta = \frac{27\sqrt{3}}{4}\times 2\pi$$

$$= \frac{27\sqrt{3}}{2}\pi$$



(Q4) $\cap \begin{cases} z = 4 - x^2 & , z \geq 0 \\ x + y = 1 & , x \geq 0 \end{cases}$

p/ $z = 0 \Rightarrow 4 - x^2 = 0 \overset{x \geq 0}{\Longrightarrow} x = 2$

$x = 2 \Rightarrow y = 1 - 2 \Rightarrow y = -1$

p/ $x = 0$, $y = 1$, $z = 4$.

Parametrização de C:

$\vec{r}(t) = (t, 1-t, 4-t^2)$ $\quad 0 \leq t \leq 2$

$I_z = \int_C \underbrace{(\sqrt{x^2+y^2})^2}_{=(d(x,y,z))^2} \cdot \rho(x,y,z)\, ds$, $d(x,y,z) = \sqrt{x^2+y^2}$ = distância ao eixo z

$\rho(x,y,z) = \sqrt{1+2x^2}$.

$\rho(\vec{r}(t)) = \sqrt{1+2t^2}$

$(\sqrt{x^2+y^2})^2 \longrightarrow (t^2 + (1-t)^2) = t^2 + 1 - 2t + t^2 = 2t^2 - 2t + 1$

$\vec{r}\,'(t) = (1, -1, -2t)$

$\|\vec{r}\,'(t)\| = \sqrt{1+1+4t^2} = \sqrt{2+4t^2} = \sqrt{2(1+2t^2)}$

$I_z = \int_0^2 (d(r(t)))^2 \cdot \rho(\vec{r}(t)) \cdot \|r'(t)\|\, dt$

$= \int_0^2 (2t^2 - 2t + 1)\sqrt{1+2t^2}\,\sqrt{2(1+2t^2)}\, dt =$

$= \sqrt{2}\int_0^2 (2t^2 - 2t + 1)(1+2t^2)\,dt =$

$= \sqrt{2}\int_0^2 (4t^4 - 4t^3 + 4t^2 - 2t + 1)\,dt$

$\begin{array}{r} 2t^2 - 2t + 1 \\ 2t^2 + 1 \\ \hline 4t^4 - 4t^3 + 2t^2 \\ 2t^2 - 2t + 1 \\ \hline \end{array}$

$= \sqrt{2}\left[4\frac{t^5}{5} - \frac{4t^4}{4} + \frac{4t^3}{3} - \frac{2t^2}{2} + t \Big]_0^2 \right]$

$= \sqrt{2}\left[\frac{4}{5}\cdot 2^5 - 2^4 + \frac{4}{3}2^3 - 4 + 2 \right]$

$= \sqrt{2}\left[2^3\left[\frac{16}{5} - 2 + \frac{4}{3}\right] - 2 \right]$

$= \sqrt{2}\left[2^4\left(\frac{8}{5} - 1 + \frac{2}{3}\right) - 2 \right] =$

$= \sqrt{2}\left[16 \times \frac{24-15+10}{15} - 2 \right] = \sqrt{2}\left[\frac{16 \times 19}{15} - 2\right]$

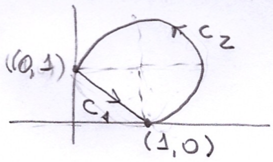
$= \frac{274\sqrt{2}}{15}$

$16 \times 19 - 30$
$= 2(8 \times 19 - 15)$

$\begin{array}{r} 19 \\ \times 8 \\ \hline 152 \\ -15 \\ \hline 137 \\ \times 2 \\ \hline 274 \end{array}$

(Q5) $\vec{F}(x,y) = (y, 2x)$

C_1: de $(0,1)$ p/ $(1,0)$: $\quad x + y = 1$



$\vec{r}_1(t) = (t, 1-t) \qquad 0 \leq t \leq 1.$

$\vec{F}(\vec{r}_1(t)) = (1-t, 2t)$

$\vec{r}_1\,'(t) = (1, -1)$ $\quad \Rightarrow \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1\,'(t) = 1 - t - 2t = 1 - 3t$

C_2: de $(1,0)$ p/ $(0,1)$: $\qquad (x-1)^2 + (y-1)^2 = 1$

$\vec{r}_2(t) = (1 + \cos t, 1 + \operatorname{sen} t) \quad -\frac{\pi}{2} \leq t \leq \pi$

$\vec{F}(\vec{r}_2(t)) = (1 + \operatorname{sen} t, 2 + 2\cos t)$

$\vec{r}_2\,'(t) = (-\operatorname{sen} t, \cos t)$

$$\vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2\,'(t) = -\operatorname{sen} t - \operatorname{sen}^2 t + 2\cos t + 2\cos^2 t$$
$$= -\operatorname{sen} t - 1 + \cos^2 t + 2\cos t + 2\cos^2 t$$
$$= -1 - \operatorname{sen} t + 2\cos t + 3\cos^2 t$$

$$\text{Trabalho} = \int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} =$$

$$= \int_0^1 \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1\,'(t)\,dt + \int_{-\pi/2}^{\pi} \vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2\,'(t)\,dt$$

$$= \int_0^1 (1 - 3t)\,dt + \int_{-\pi/2}^{\pi} (-1 - \operatorname{sen} t + 2\cos t + 3\underbrace{\cos^2 t}_{3 \cdot \frac{(1 + \cos 2t)}{2}})\,dt$$

$$= \int_0^1 (1 - 3t)\,dt + \int_{-\pi/2}^{\pi} (+\tfrac{1}{2} - \operatorname{sen} t + 2\cos t + \tfrac{3}{2}\cos 2t)\,dt$$

$$= t - \frac{3t^2}{2} \Big|_0^1 + \left(+\frac{t}{2} + \cos t + 2\operatorname{sen} t + \frac{3}{4}\operatorname{sen} 2t\right)\Big]_{-\pi/2}^{\pi}$$

$$= 1 - \frac{3}{2} + \left[\left(+\frac{\pi}{2} - 1 + 0 + 0\right) - \left(-\frac{\pi}{4} + 0 - 2 + 0\right)\right]$$

$$= -\frac{1}{2} + \frac{\pi}{2} + \frac{\pi}{4} + 2 - 1 =$$

$$= \frac{1}{2} + \frac{3\pi}{4} \;//$$