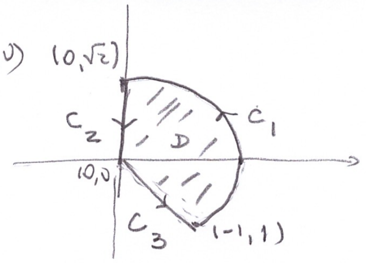
1) C_1: x^2+y^2 de $(1,-1)$ p/ $(0,\sqrt{2})$; C_2: de $(0,\sqrt{2})$ p/ $(0,0)$

$\vec{F} = (y+x^{100}, 2x+y^{100}) = (P,Q)$.

é de classe C^1 em $\mathbb{R}^2 \supset D$.

$\partial D = C_1 \cup C_2 \cup C_3 = C$



Pelo Teorema de Green,

$$\int_{C=\partial D} \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\int_{C_1\cup C_2} \vec{F}\cdot d\vec{r} = \underbrace{\iint_D \Big(\underbrace{\frac{\partial Q}{\partial x}}_{=2} - \underbrace{\frac{\partial P}{\partial y}}_{=1}\Big) dx\,dy}_{=\text{área de } D = \frac{1}{4}\pi\times(\sqrt{2})^2 + \frac{1}{8}\pi(\sqrt{2})^2} - \int_{C_3} \vec{F}\cdot d\vec{r}$$

$$\iint_D dx\,dy = \frac{1}{4}\pi\times 2 + \frac{1}{8}\pi\times 2 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}.$$

C_3: $\vec{r}(t) = (t, -t)$ $0 \le t \le 1$.

$\vec{r}\,'(t) = (1, -1)$

$\vec{F}(\vec{r}(t)) = (-t + t^{100}, 2t + (t)^{100})$

$$\int_{C_3} \vec{F}\cdot d\vec{r} = \int_0^1 (-t+t^{100}, 2t+t^{100})\cdot(1,-1)\,dt =$$

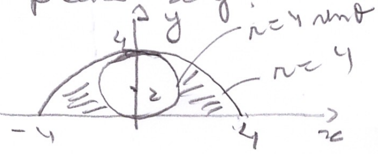
$$= \int_0^1 (-t+t^{100}-2t-t^{100})\,dt = \int_0^1 -3t\,dt = -\frac{3t^2}{2}\Big]_0^1 = -\frac{3}{2}$$

Portanto, $\displaystyle\int_{C_1\cup C_2} \vec{F}\cdot d\vec{r} = \frac{3\pi}{4} + \frac{3}{2}$ //

---

(2) S: $z = x^2+y^2$; $z \le 16$, $y \ge 2$, exterior a $x^2+y^2 = 2x$

projeção no plano xy:



$$\text{Massa} = M = \iint_S \frac{1}{\sqrt{z(1+4z)}}\,dS$$

$$= \iint_S \frac{1}{\sqrt{(x^2+y^2)(1+4(x^2+y^2))}}\,dS =$$

S: $\varphi(x,y) = (x, y, x^2+y^2)$

$\dfrac{\partial \varphi}{\partial x}\times\dfrac{\partial \varphi}{\partial y} = (-2x, -2y, 1)$

D é a projeção de S no plano xy

$$= \iint_D \frac{1}{\sqrt{x^2+y^2}\,\sqrt{1+4(x^2+y^2)}} \left\|\frac{\partial \varphi}{\partial x}\times\frac{\partial \varphi}{\partial y}\right\| dx\,dy$$

$$= \iint_D \frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{\cancel{\sqrt{1+4x^2+4y^2}}} \times \cancel{\sqrt{1+4x^2+4y^2}}\,dx\,dy$$

Coordenadas polares:

$x^2+y^2 = 4y$

$r^2 = 4r\,\text{sen}\,\theta$

$r = 4\,\text{sen}\,\theta$

$$= \iint_D \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy = \int_0^{\pi}\int_{4\,\text{sen}\,\theta}^{4} \frac{1}{r}\,r\,dr\,d\theta$$

$$= \int_0^{\pi} r\Big]_{4\,\text{sen}\,\theta}^{4} d\theta = \int_0^{\pi} (4 - 4\,\text{sen}\,\theta)\,d\theta$$

$$= 4\theta + 4\cos\theta\Big]_0^{\pi} = (4\pi - 4) - (0+4) = 4\pi - 8 \text{ //}$$

(3) $\vec{F}(x,y,z) = (1+z e^{y-x}, 1 - z e^{y-x}, -e^{y-x})$

$\vec{F}$ é de classe C^1 em $\mathbb{R}^3$. $\mathbb{R}^3$ é simplesmente conexo.

$$\text{rot}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1+ze^{y-x} & 1-ze^{y-x} & -e^{y-x} \end{vmatrix} =$$

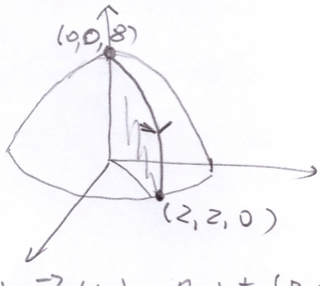
$$= \left(-e^{y-x} + e^{y-x},\ e^{y-x} - e^{y-x},\ +ze^{y-x} - z e^{y-x}\right) = (0,0,0) = \vec{0}$$

Logo, pelo Teorema das Quatro Equivalências, $\int_C \vec{F}\cdot d\vec{r}$ independe do caminho

vértice: $z = 8 - x^2 - y^2$, $x=0$, $y=0 \Rightarrow z = 8$

plano xy: $z = 0$, $x = y \Rightarrow 8 - 2x^2 = 0$

$x^2 = 4 \overset{x \geq 0}{\Longrightarrow} x = 2$



$$\int_C \vec{F}\cdot d\vec{r} = \int_{A=(0,0,8)}^{B=(2,2,0)} \vec{F}\cdot d\vec{r} =$$

C: $\vec{r}(t) = A + t(B-A)$
$= (0,0,8) + t(2,2,-8)$
$= (2t, 2t, 8-8t)$, $0 \leq t \leq 1$

$$= \int_0^1 \vec{F}(2t,2t,8-8t)\cdot(2,2,-8)\,dt =$$

$$= \int_0^1 (1+(8-8t)e^0, 1-(8-8t)e^0 - e^0)\cdot(2,2,-8)\,dt$$

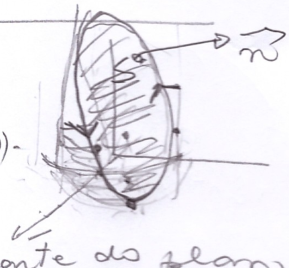
$$= \int_0^1 (9-8t, -7+8t, -1)\cdot(2,2,-8)\,dt = \int_0^1 (18 - 16t - 14 + 16t + 8)\,dt =$$

$$= \int_0^1 12\,dt = 12$$

---

(4) $F(x,y,z) = (z, y, f(z))$

$$\text{rot}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & y & f(z) \end{vmatrix} = (0-0, 1-0, 0-0) = (0,1,0)$$



$\vec{F}$ é de classe C^1 em $\mathbb{R}^3$, S = parte do plano delimitado por $C = \partial S$

$\vec{n}$ aponta no sentido oposto à origem

Pelo Teorema de Stokes,

$$\int_{C=\partial S} \vec{F}\cdot d\vec{r} = \iint_S \text{rot}\,\vec{F}\cdot\vec{n}\,dS$$

S: $\varphi(x,y) = (x, y, 2-x-y)$
$(x,y) \in D$; $x^2+y^2 \leq 1$

$\frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial y} = (1,1,1)$ mesmo sentido de $\vec{n}$.

$$= \iint_D (0,1,0)\cdot(1,1,1)\,dx\,dy =$$

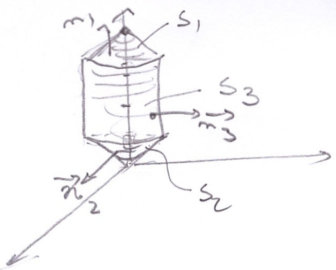
$$= \iint_D dx\,dy = \text{área de } D$$

$$= \pi \times (1)^2 = \pi$$

5) $S_1: \; z = 4 - \sqrt{x^2+y^2}$

$S_2: \; z = \sqrt{x^2+y^2}$

$S_3: \; x^2+y^2 = 1$



$\vec{F} = (x+z, \, y+z, \, 2z)$.

$\vec{F}$ é de classe C^1 em $\mathbb{R}^3$, $W \subset \mathbb{R}^3$.

$\partial W = S_1 \cup S_2 \cup S_3$.

Pelo Teorema de Gauss,

$$\iint_{S = S_1 \cup S_2 \cup S_3} \vec{F} \cdot \vec{n} \, dS = \iiint_W \underbrace{\operatorname{div} \vec{F}}_{=(1+1+2)=4} \, dV$$

Logo

$$\iint_{S_1 \cup S_2} \vec{F} \cdot \vec{n} \, dS = 4 \iiint_W dV - \iint_{S_3} \vec{F} \cdot \vec{n} \, dS.$$

$$\iiint_W dV = \text{volume de } W = \frac{1}{3} \times \pi \times (1)^2 \times 1 + \frac{1}{3} \pi \times (1)^2 \times 1 + 2\pi = \frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$$

$\begin{cases} z = 4 - \sqrt{x^2+1} \\ x^2+y^2 = 1 \end{cases}$ 

$\Rightarrow z = 3$, altura $= 4-3 = 1$

base $= 1$ (cone superior)

$\begin{cases} z = \sqrt{x^2+y^2} \\ x^2+y^2 = 1 \end{cases}$

$z = 1$ = altura, $r = 1$ = raio $\Big\}$ cone inferior

cilindro: altura = 2, raio = 1

$\pi \times 1^2 \times 2 = 2\pi$.

ou por integral tripla em cilíndricas:

$$\int_0^{2\pi} \int_0^1 \int_r^{4-r} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 \underbrace{r(4-r-r)}_{= 4r - 2r^2} \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{4r^2}{2} - \frac{2r^3}{3} \Big|_0^1 d\theta = \left(2 - \frac{2}{3}\right) \times 2\pi = \frac{8\pi}{3}$$

Cálculo de $\iint_{S_3} \vec{F} \cdot \vec{n} \, dS =$

$S_3: \varphi(\theta, z) = (\cos\theta, \operatorname{sen}\theta, z)$

$0 \le \theta \le 2\pi$, $1 \le z \le 3$.

$$\frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial z} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos\theta, \operatorname{sen}\theta, 0)$$

$\frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial z}(\pi/2, z) = (0, 1, 0)$ mesmo sentido de $\vec{n}_3$.

$$= \iint_D \vec{F}(\cos\theta, \operatorname{sen}\theta, z) \cdot (\cos\theta, \operatorname{sen}\theta, 0) \, dz \, d\theta$$

$$= \iint_D (\cos\theta + z, \operatorname{sen}\theta + z, 2z) \cdot (\cos\theta, \operatorname{sen}\theta, 0) \, dz \, d\theta$$

$$= \int_0^{2\pi} \int_1^3 (\cos^2\theta + z\cos\theta + \operatorname{sen}^2\theta + z \operatorname{sen}\theta) \, dz \, d\theta$$

$$= \int_1^3 \int_0^{2\pi} (1 + z(\cos\theta + \operatorname{sen}\theta)) \, dz \, d\theta =$$

$$= \int_1^3 \theta + z(\operatorname{sen}\theta - \cos\theta) \Big|_0^{2\pi} dz = \int_1^3 (2\pi + z \cdot 0) \, dz = 2\pi z \Big|_1^3 = 2\pi \times 2 = 4\pi$$

Portanto $\iint_{S_1 \cup S_2} \vec{F} \cdot \vec{n} \, dS = 4 \times \frac{8\pi}{3} - 4\pi = \frac{20\pi}{3}$ //