(Q1) (a) $\lim\limits_{x\to 1} \underbrace{\dfrac{(x-1)^2}{\operatorname{sen}(x-1)}} \operatorname{sen}\left(\dfrac{1}{x-1}\right)$

$\lim\limits_{x\to 1} {}'' \dfrac{(x-1)}{\operatorname{sen}(x-1)} \cdot (x-1)'' =$ $\quad y = x-1$

$= \lim\limits_{y\to 0} \dfrac{y}{\operatorname{sen} y} \cdot y = \lim\limits_{y\to 0} \dfrac{1}{\frac{\operatorname{sen} y}{y}} \cdot y = \dfrac{1}{1} \cdot 0 = 0$

como $\left|\operatorname{sen}\left(\frac{1}{x-1}\right)\right| \le 1$, $\operatorname{sen}\left(\frac{1}{x-1}\right)$ é limitado,

logo pelo teorema do anulamento,

$$\lim_{x\to 1} \underbrace{\frac{(x-1)^2}{\operatorname{sen}(x-1)}}_{\to 0} \underbrace{\operatorname{sen}\left(\frac{1}{x-1}\right)}_{\text{limitado}} = 0 \quad //$$

(b) $\lim\limits_{x\to e^+} (\underbrace{\ln x}_{\to 1})^{\overbrace{\frac{1}{x-e}}^{\to\infty}} = \lim\limits_{x\to e^+} e^{\underbrace{\frac{1}{x-e}\ln(\ln x)}_{=L}}$

$L = \lim\limits_{x\to e^+} \dfrac{\ln(\ln x)}{x-e} \overset{L'H}{=} \lim\limits_{x\to e^+} \dfrac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \dfrac{1}{1}\cdot\dfrac{1}{e} = \dfrac{1}{e}$

Logo, $\lim\limits_{x\to e^+} (\ln x)^{\frac{1}{x-e}} = e^L = e^{\frac{1}{e}}$ //

(c) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{4x^2+3}}{\sqrt[3]{1-x^3}+2x} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2\left(4+\frac{3}{x^2}\right)}}{\sqrt[3]{x^3\left(\frac{1}{x^3}-1\right)}+2x} =$

$= \lim\limits_{\substack{x\to-\infty \\ x<0,\ |x|=-x}} \dfrac{|x|\sqrt{4+\frac{3}{x^2}}}{x\sqrt[3]{\frac{1}{x^3}-1}+2x} =$

$= \lim\limits_{x\to-\infty} \dfrac{-\cancel{x}\sqrt{4+\frac{3}{x^2}}}{\cancel{x}\left(\sqrt[3]{\frac{1}{x^3}-1}+2\right)} = \dfrac{-\sqrt{4}}{\sqrt[3]{-1}+2} = \dfrac{-2}{1} = -2$ //

(Q2) (a) $f(x) = \dfrac{\cos(\pi(g(x))^2)}{x}$.

$f'(x) = \dfrac{x\left(-\operatorname{sen}(\pi(g(x))^2)\right)\cdot\pi\cdot 2g(x)\cdot g'(x) - \cos(\pi(g(x))^2)}{x^2}$

$f'\left(\frac{1}{2\pi}\right) = \dfrac{\frac{1}{2\pi}\left(-\operatorname{sen}\left(\pi\cdot\frac{3}{9}\right)\right)\cdot\pi\cdot 2\frac{\sqrt{3}}{3}\cdot 1 - \cos\left(\pi\cdot\frac{3}{9}\right)}{\frac{1}{4\pi^2}}$ $\quad g\left(\frac{1}{2\pi}\right) = \frac{\sqrt{3}}{3}$

$= \dfrac{-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{3} - \frac{1}{2}}{\frac{1}{4\pi^2}} = \dfrac{-1}{\frac{1}{4\pi^2}} = -4\pi^2$ // $\quad g'\left(\frac{1}{2\pi}\right) = 1$

$\operatorname{sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(Q2)(b) Eq. da reta tangente.

$$y - f\left(\frac{1}{2\pi}\right) = f'\left(\frac{1}{2\pi}\right)\left(x - \frac{1}{2\pi}\right)$$

$$f\left(\frac{1}{2\pi}\right) = \frac{\cos\left(\pi \cdot \frac{3}{9}\right)}{\frac{1}{2\pi}} = \frac{+\frac{1}{2}}{\frac{1}{2\pi}} = \pi$$

Logo, $\boxed{y - \pi = -4\pi^2\left(x - \frac{1}{2\pi}\right)}$

(Q3) $8 \cdot x \arctan y = \pi x y + 2\pi$ $\qquad f(2) = 1 \Rightarrow \begin{cases} x = 2 \\ y = 1 \end{cases}$

derivando implicitamente,

$$8x \cdot \frac{1}{1+y^2} \cdot y' + 8\arctan y = \pi x y' + \pi y$$

para $x = 2$ e $y = 1$,

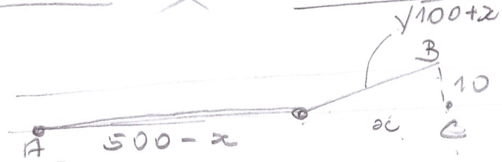
$$16 \cdot \frac{1}{1+1} \cdot y' + 8\underbrace{\arctan 1}_{=\pi/4} = 2\pi y' + \pi$$

$$8y' - 2\pi y' = \pi - 2\pi$$

$$(8 - 2\pi) y' = -\pi \Rightarrow \boxed{y' = \frac{\pi}{2\pi - 8}}$$

$$y' = -\frac{\pi}{8 - 2\pi}$$

(Q4)



v_B = velocidade da bicicleta = 5 m/seg

v_T = velocidade do triciclo = 3 m/seg

$$v_B = \frac{500 - x}{t_B} \Rightarrow t_B = \frac{500 - x}{5} = \text{tempo de bicicleta}$$

$$v_T = \frac{\sqrt{100 + x^2}}{t_T} \Rightarrow t_T = \frac{\sqrt{100+x^2}}{3} = \text{tempo de triciclo}$$

$$\text{Tempo Total} = T(x) = \frac{500 - x}{5} + \frac{\sqrt{100 + x^2}}{3}$$

$0 \leq x \leq 500$.

$$T'(x) = -\frac{1}{5} + \frac{1}{3} \times \frac{2x}{2\sqrt{100+x^2}} = \frac{-3\sqrt{100+x^2} + 5x}{15}$$

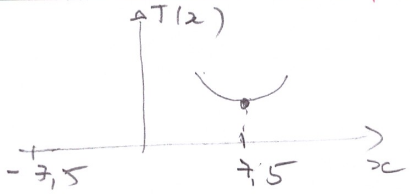
$$-3\sqrt{100+x^2} + 5x = 0$$

$$5x = 3\sqrt{100 + x^2}$$

$$25x^2 = 9(100 + x^2)$$

$$16x^2 = 900$$

$$x^2 = \frac{900}{16} \Rightarrow x = \pm\frac{30}{4} = \pm 7{,}5 \rightarrow \text{únicos pontos críticos}$$



$-3\sqrt{100+x^2}+5x \geq 0$

$5x \geq 3\sqrt{100+x^2}$, $x \geq 0$

$x \geq 7,5$

$T'(x) > 0$, crescente $x > 7,5$

$T'(x) < 0$, decrescente $x < 7,5$

Logo $x = 7,5$ é ponto de mínimo relativo e absoluto para $x \in [0, 500]$.

Logo, a troca deve ser feita 7,5 m antes de C ou

$500 - 7,5 = 482,5$ m depois de A.

(Q5) $f(x) = x\, e^{-\frac{x^2}{2}} = \dfrac{x}{e^{\frac{x^2}{2}}}$

(a) Como $e^{\frac{x^2}{2}} > 0 \ \forall x$, o denominador não se anula e dom $f = \mathbb{R} = [-\infty, \infty]$.

É contínua para todo $x \in \mathbb{R}$ pois é quociente e composta de contínuas.

Paridade: (i) $\mathbb{R}$ é simétrico em relação à origem $0 \in \mathbb{R}$.

(ii) $f(-x) = -x \cdot e^{-\left(\frac{(-x)^2}{2}\right)} = -x\, e^{-\frac{x^2}{2}} = -f(x)$.

Logo a função f é ímpar.

(b) A.V. → não há assíntota vertical pois a função é contínua em $\mathbb{R}$.

A.H. → $\displaystyle\lim_{x\to\infty} \frac{x}{e^{x^2/2}} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{e^{x^2/2}\cdot\frac{2x}{2}} = 0$

$\displaystyle\lim_{x\to -\infty} f(x) = 0$ (f é ímpar)

(Q 5) (b) (continuação)

$f(x) = x \cdot e^{-x^2/2}$

$f'(x) = x \cdot e^{\frac{-x^2}{2}} \cdot \frac{-2x}{2} + e^{\frac{-x^2}{2}} = (-x^2+1)\, e^{-x^2/2}$

		-1		1	
$-x^2+1$	$-$	0	$+$	0	$-$
$e^{-x^2/2}$	$+$	$+$	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	0	$-$
f é	$\searrow$	$\downarrow$	$\nearrow$	$\downarrow$	$\searrow$

$x = 1$ é ponto de mín relativo

$x = 1$ é ponto de max relativo

crescente : $[-1, 1]$

decrescente : $(-\infty, -1] \cup [1, \infty)$

$f''(x) = (-x^2+1)\, e^{-x^2/2} \cdot \frac{-2x}{2} + (-2x)\, e^{-x^2/2}$

$f''(x) = (+x^3 - x - 2x)\, e^{-x^2/2} = (+x^3 - 3x)\, e^{-x^2/2} = x(x^2-3)\, e^{\frac{-x^2}{2}}$

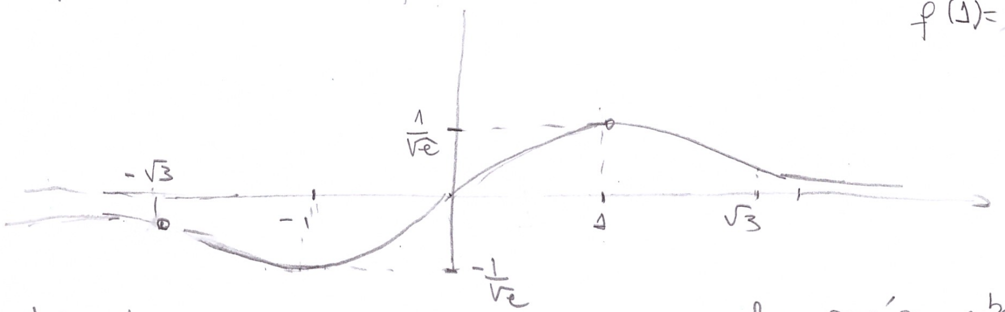
		$-\sqrt{3}$		0		$+\sqrt{3}$	
x	$-$	$-$	$-$	0	$+$	$+$	$+$
x^2-3	$+$	0	$-$	$-$	$-$	0	$+$
$e^{-x^2/2}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$f''(x)$	$-$	0	$+$	0	$-$	0	$+$
concavidade	$\cap$		$\cup$		$\cap$		$\cup$

côncava p/ cima: $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

côncava p/ baixo: $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

pontos de inflexão em: $x = -\sqrt{3}$, $x = 0$, $x = \sqrt{3}$

$f(1) = e^{-1/2} = \frac{1}{\sqrt{e}}$



ponto máximo absoluto = 1 , valor máx absoluto = $\frac{1}{\sqrt{e}}$

ponto mínimo absoluto = -1 , " mín " = $-\frac{1}{\sqrt{e}}$

imagem = $\left[-\frac{1}{\sqrt{e}}, \frac{1}{\sqrt{e}}\right]$ //