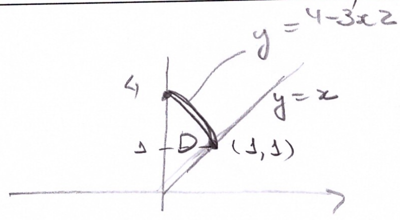
(Q1) $\begin{cases} y = 4 - 3x^2 \\ y = x \end{cases}$ $\cap$

$4 - 3x^2 = x$



(a) $3x^2 + x - 4 = 0$

$x = \dfrac{-1 \pm \sqrt{1+48}}{6} = \dfrac{-1 \pm 7}{6} = \begin{cases} \frac{6}{6} = 1 \\ \frac{-8}{6} = -\frac{4}{3} \text{ (não convém)} \end{cases}$

$y = 4 - 3x^2$
$3x^2 = 4 - y$
$x^2 = \dfrac{4-y}{3}$
$x = \sqrt{\dfrac{4-y}{3}}$

tipo I : $\displaystyle\int_0^1 \int_x^{4-3x^2} f(x,y)\, dy\, dx$

tipo II : $\displaystyle\int_0^1 \int_0^y f(x,y)\, dx\, dy + \int_1^4 \int_0^{\sqrt{\frac{4-y}{3}}} f(x,y)\, dx\, dy$

polares : $\displaystyle\int_{\pi/4}^{\pi/2} \int_0^{\frac{-\operatorname{sen}\theta \pm \sqrt{\operatorname{sen}^2\theta + 48\cos^2\theta}}{6\cos^2\theta}} f(r\cos\theta, r\operatorname{sen}\theta)\, dr\, d\theta$

$y = 4 - 3r^2$

$r\operatorname{sen}\theta + 3r^2\cos^2\theta = 4$

$3r^2\cos^2\theta + r\operatorname{sen}\theta - 4 = 0$

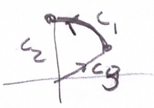
$r = \dfrac{-\operatorname{sen}\theta \pm \sqrt{\operatorname{sen}^2\theta + 48\cos^2\theta}}{6}$

$r > 0$

(b) $\displaystyle\int_C \underbrace{(2y - f(x))}_{P}\, dx + \underbrace{(6x - y^3)}_{Q}\, dy = ?$

$\dfrac{\partial Q}{\partial x} = 6 \qquad \dfrac{\partial P}{\partial y} = 2 \implies \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 6 - 2 = 4$

$\vec{F} = (P, Q)$ é de classe C^1 em $\mathbb{R}^2$ que contém D ; , $\partial D = C_1 \cup C_2 \cup C_3$



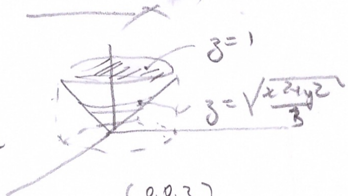
Pelo Teorema de Green,

$\displaystyle\int_{C = \partial D} (2y - f(x))\, dx + (6x - y^3)\, dy = \iint_D 4\, dx\, dy =$

$= 4 \displaystyle\int_0^1 \int_x^{4-3x^2} dy\, dx = 4\int_0^1 (4 - 3x^2 - x)\, dx = 4\left(4x - \frac{3x^3}{3} - \frac{x^2}{2}\right)\Big|_0^1$

$= 4\left(4 - 1 - \frac{1}{2}\right) = 4 \times \frac{5}{2} = 10$ //

(Q2) $z = \dfrac{\sqrt{x^2+y^2}}{3}$



$\rho = k\, \cancel{\text{distância}} = k\sqrt{(x-0)^2 + (y-0)^2 + (z-z)^2}$
$= k\sqrt{x^2+y^2}$

Retangulares

$M = \displaystyle\iiint_W k\sqrt{x^2+y^2}\, dx\, dy\, dz =$

(Q2) continuação 5.

$$M = k \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \int_{\sqrt{\frac{x^2+y^2}{3}}}^{1} \sqrt{x^2+y^2}\, dz\, dy\, dx$$

$z = \sqrt{\frac{x^2+y^2}{3}}$

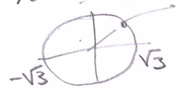
$z = 1$

$\sqrt{\frac{x^2+y^2}{3}} = 1$

$\sqrt{x^2+y^2} = \sqrt{3}$

$x^2+y^2 = 3$

raio $= \sqrt{3}$



$x^2+y^2=3$

$y^2 = 3 - x^2$

$y = \pm\sqrt{3-x^2}$

Cilíndricas:

$$M = k \int_0^{2\pi} \int_0^{\sqrt{3}} \int_{\frac{r}{\sqrt{3}}}^{1} \underbrace{\sqrt{r^2}\, r}_{=r^2}\, dz\, dr\, d\theta$$

$z = \sqrt{\frac{x^2+y^2}{3}} = z = \sqrt{\frac{r^2}{3}}$

$z = \frac{r}{\sqrt{3}}$

Esféricas

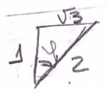
$$M = k \int_0^{2\pi} \int_0^{\pi/3} \int_0^{\sec\varphi} \rho \,\text{sen}\,\varphi\, \rho^2 \,\text{sen}\,\varphi\, d\rho\, d\varphi\, d\theta$$

$$M = k \int_0^{2\pi} \int_0^{\pi/3} \int_0^{\sec\varphi} \rho^3 \,\text{sen}^2\varphi\, d\rho\, d\varphi\, d\theta$$

$z = 1$

$\rho\cos\varphi = 1$

$\rho = \frac{1}{\cos\varphi} = \sec\varphi$



$\tan\varphi = \frac{1}{2}$

$\varphi = \pi/3$

$\sqrt{x^2+y^2} = \sqrt{\rho^2\cos^2\theta\,\text{sen}^2\varphi + \rho^2\,\text{sen}^2\theta\,\text{sen}^2\varphi}$

$= \sqrt{\rho^2\,\text{sen}^2\varphi}$

$= \rho\,|\underbrace{\text{sen}\,\varphi}_{\geq 0}|$

$= \rho\,\text{sen}\,\varphi$

(Q3) $\vec{r}(t) = (\cos t,\ 1+\text{sen}\,t,\ \text{sen}\,t + \cos t)$

$0 \leq t \leq \pi/2$

$\vec{F}(x,y,z) = (2xy - z^2,\ x^2,\ -2xz)$

$$\text{rot}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy - z^2 & x^2 & -2xz \end{vmatrix} =$$

$= (0-0,\ -2z+2z,\ 2x-2x) = \vec{0}$

$\vec{F}$ pode ser conservativo.

$\frac{\partial f}{\partial x} = 2xy - z^2 \xrightarrow{f(x,y,z)} \int (2xy - z^2)\, dx =$

$f(x,y,z) = x^2y - xz^2 + C_1(y,z)$

$\frac{\partial f}{\partial y} = x^2 \Rightarrow f(x,y) = \int x^2\, dy = yx^2 + C_2(x,z)$

$\frac{\partial f}{\partial z} = -2xz \Rightarrow f(x,y,z) = \int -2xz\, dz = -xz^2 + C_3(x,y)$

$C_1(y,z) = 0,\quad C_2(x,z) = -xz^2,\quad C_3(x,y) = x^2y$

$\vec{r}(0) = (1, 1, 1) = A$

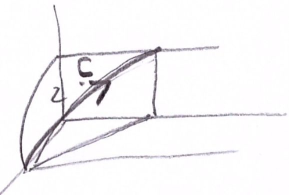
$\vec{r}(\pi/2) = (0, 2, 1) = B$

Como $\vec{F}$ é conservativo

$$\int_C \vec{F}\cdot d\vec{r} = \int_A^B \vec{F}\cdot d\vec{r} = f(B) - f(A) =$$

$$= f(0,2,1) - f(1,1,1) = (0-0) - (1-1) = 0 \;//$$

(Q4) $x^2+z^2=4$, $x+y=1$



$\vec{F} = (2, 3, x+y+z)$

$$\text{rot}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2 & 3 & x+y+z \end{vmatrix} = (1-0,\ 0-(1),\ 0-0) = (1, -1, 0) \neq \vec{0}$$

Logo $\vec{F}$ não é conservativo.

$\vec{r}(t) = (2\cos t,\ 1-2\cos t,\ 2\,\text{sen}\,t)$ $\quad 0 \le t \le \pi/2$

$\vec{F}(\vec{r}(t)) = (2,\ 3,\ 2\cos t + 1 - 2\cos t + 2\,\text{sen}\,t)$

$= (2, 3, 1+2\,\text{sen}\,t)$.

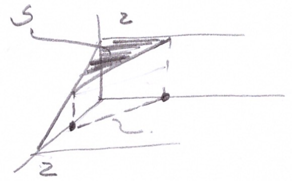
$\vec{r}\,'(t) = (-2\,\text{sen}\,t,\ 2\,\text{sen}\,t,\ 2\cos t)$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = \underbrace{-4\,\text{sen}\,t + 6\,\text{sen}\,t}_{=2\,\text{sen}\,t} + 2\cos t + 4\,\text{sen}\,t\cos t$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\pi/2} (2\,\text{sen}\,t + 4\,\text{sen}\,t\cos t + 2\cos t)\,dt =$$

$$= -2\cos t + \frac{4}{2}\,\text{sen}^2 t + 2\,\text{sen}\,t \Big]_0^{\pi/2}$$

$$= (0+2+2) - (-2+0+0) = 6 \;//$$

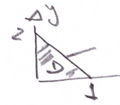


Q5) $\vec{F}(x,y,z) = (x, 2y, 3z)$

$S: \; x+z=2 \quad , \quad 2x+y=2$

(a) $\varphi(x,y) = (x, y, \underbrace{2-x}_{z(x,y)})$

$D \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 2-2x \end{cases}$



$\dfrac{\partial\varphi}{\partial x} \times \dfrac{\partial\varphi}{\partial y} = \left(-\dfrac{\partial z}{\partial x}, -\dfrac{\partial z}{\partial y}, 1\right)$

$= (1, 0, 1)$ — mesmo sentido de $\vec{n}$

$\displaystyle\iint_S \vec{F}\cdot\vec{n}\, dS = \iint_D (x, 2y, 6-3x)\cdot(1,0,1)\, dx\,dy =$

$\displaystyle = \iint_D (x + 0 + 6 - 3x)\, dx\,dy = \iint_D (6-2x)\, dx\,dy =$

$\displaystyle = \int_0^1 \int_0^{2-2x} (6-2x)\, dy\, dx =$

$\displaystyle = \int_0^1 (6-2x)\, y \Big|_0^{2-2x} dx = \int_0^1 (6-2x)(2-2x)\, dx =$

$\displaystyle = \int_0^1 (12 - 12x - 4x + 4x^2)\, dx =$

$\displaystyle = \int_0^1 (12 - 16x + 4x^2)\, dx = 12x - \frac{16x^2}{2} + \frac{4x^3}{3}\Big]_0^1$

$= 12 - 8 + \dfrac{4}{3}$

$= 4 + \dfrac{4}{3} = \dfrac{16}{3}$ //

(b) W é a superfície delimitada por S, S_1, S_2, S_3, S_4,

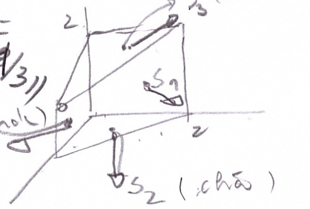
$\partial W = S \cup S_1 \cup S_2 \cup S_3 \cup S_4$

$\vec{F} = (x, 2y, 3z)$ é de classe C^1.

Por Gauss.

$\displaystyle\iint_{S_1 \cup S_2 \cup S_3 \cup S_4} \vec{F}\cdot\vec{n}\, dS + \underbrace{\iint_S \vec{F}\cdot\vec{n}\, dS}_{= \frac{16}{3} \text{ (item a)}} = \boxed{\iiint_W \underbrace{(\operatorname{div} F)}_{1+2+3=6}\, dV} = 8$

$\displaystyle\iint_{S_1 \cup S_2 \cup S_3} \vec{F}\cdot\vec{n}\, dS = 8 - \frac{16}{3} = 8/3$ //



$\displaystyle\iiint_W 6\, dV = 6 \iiint_W dx\,dy\,dz = 6\int_0^1 \int_0^{(2-2x)} \int_0^{2-x} dz\, dy\, dx$