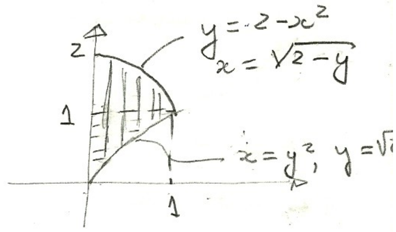
1) a) $\displaystyle\iint_{D_{xy}} f(x,y)\,dx\,dy = I$



Tipo I:

$$I = \int_0^1 \int_{\sqrt{x}}^{2-x^2} f(x,y)\,dy\,dx$$

Tipo II:

$$I = \int_0^1 \int_0^{y^2} f(x,y)\,dx\,dy + \int_1^2 \int_0^{\sqrt{2-y}} f(x,y)\,dx\,dy$$

$\begin{cases} y^2 = x \\ y = 2 - x^2 \end{cases}$

$y = 2 - (y^2)^2$

$y^4 + y - 2 = 0$

possíveis raízes inteiras: $1, -1, 2, -2$

$\underline{y = 1}$, $1 + 1 - 2 = 0$ raiz

$x = 1^2 \Rightarrow x = 1$

$\begin{cases} y = 2 - x^2 \\ x^2 = 2 - y \\ x = \sqrt{2-y} \quad (x \geq 0) \end{cases}$

$\begin{cases} y^2 = x \\ y = \sqrt{x} \quad (y \geq 0) \end{cases}$

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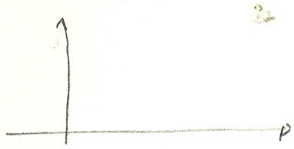
1) (b) $\begin{cases} 3 \leq x + 2y \leq 6 \\ 0 \leq y \leq 1 \end{cases}$



$u = x + 2y$, $v = y$ $\Rightarrow$ $3 \leq u \leq 6$, $0 \leq v \leq 1$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1, \quad \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{1} = 1 = J$$

$$\iint_{D_{xy}} (x+2y)\, e^{xy+2y^2}\,dx\,dy =$$

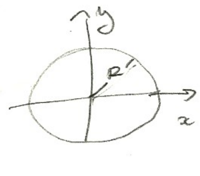
$\begin{cases} xy + 2y^2 = \\ = y(x+2y) = uv \end{cases}$

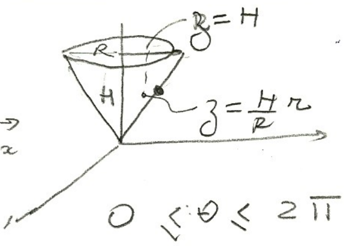
$w = uv$
$dw = u\,dv$

$$= \iint_{D_{uv}} u\, e^{uv} \underbrace{|J|}_{=1}\,du\,dv = \int_3^6 \int_0^1 u\, e^{uv}\,dv\,du$$

$$= \int_3^6 e^{uv}\Big]_0^1 du = \int_3^6 (e^u - e^0)\,du = e^u - u\Big]_3^6 = (e^6 - 6) - (e^3 - 3) = \boxed{e^6 - e^3 - 3}$$

2) $z = \frac{H}{R}\sqrt{x^2+y^2}$





Em cilíndricas:

$z = \frac{H}{R}\, r$

$0 \leq \theta \leq 2\pi$

$0 \leq r \leq R$

$\frac{H}{R} r \leq z \leq H$

$$V = \iiint_{W_{xyz}} dx\,dy\,dz =$$

$$= \iiint_{W_{r\theta z}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{R}\int_{\frac{H}{R}r}^{H} r\,dz\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{R} r\,z\Big]_{\frac{H}{R}r}^{H} dr\,d\theta = \int_0^{2\pi}\int_0^{R} r\left[H - \frac{H}{R}r\right] dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{R} H\left(r - \frac{1}{R}r^2\right) dr\,d\theta = H\int_0^{2\pi} \frac{r^2}{2} - \frac{1}{R}\frac{r^3}{3}\Big]_0^{R} d\theta$$

$$= H\int_0^{2\pi}\left[\frac{R^2}{2} - \frac{1}{R}\frac{R^{\cancel{3}\,2}}{3}\right] d\theta = H\int R^2 \underbrace{\left(\frac{1}{2} - \frac{1}{3}\right)}_{=\frac{1}{6}} \cdot d\theta$$

$$= H \times \frac{R^2}{6} \times \theta\Big]_0^{2\pi} = H \cdot \frac{R^2}{0} \times 2\pi = \frac{1}{3}\pi R^2 H \;//$$

X

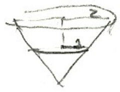
3) $z = \sqrt{x^2+y^2}$, $\rho(x, y, z) = k$

A sólido é simétrica em relação ao eixo z podemos trocar x por $-x$ e y por $-y$ e a densidade é simétrica em relação ao mesmo eixo z logo o centro de massa está no eixo z

Portanto $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \bar{z})$ e só precisamos calcular $\bar{z} = \dfrac{\iiint_W k z\,dx\,dy\,dz}{M}$

$$M = \iiint_W k\,dx\,dy\,dz = k\underbrace{\iiint_W dx\,dy\,dz}_{=V=\text{volume do sólido}}$$

(3) continuação $z = \sqrt{x^2+y^2}$



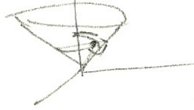
Quando $z = 1$, $H = 1$, $R = 1$

" $z = 2$, $H = 2$, $R = 2$

$$V = \frac{1}{3}\pi(2)^2\cdot 2 - \frac{1}{3}\pi(1)^2\cdot 1 = \frac{1}{3}\pi(8-1) = \frac{7}{3}\pi$$

$$\bar{z} = \frac{k\iiint_W z\,dx\,dy\,dz}{\cancel{k}\times\frac{7}{3}\pi}$$

$\iiint_W z\,dx\,dy\,dz$, em coordenadas esféricas.



$$= \int_0^{2\pi}\int_0^{\pi/4}\int_{\sec\varphi}^{2\sec\varphi}\rho\cos\varphi\,\rho^2\,\text{sen}\varphi\,d\rho\,d\varphi\,d\theta$$

$z = 1 \Rightarrow \rho\cos\varphi = 1 \Rightarrow \rho = \sec\varphi$

$z = 2 \Rightarrow \rho = 2\sec\varphi$, $z = \sqrt{x^2+y^2}$, $\varphi_0 = \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi/4$

$\sec\varphi \le z \le 2\sec\varphi$

$$= \int_0^{2\pi}\int_0^{\pi/4}\rho^3\cos\varphi\,\text{sen}\varphi\,d\rho\,d\varphi\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\frac{\rho^4}{4}\Big|_{\sec\varphi}^{2\sec\varphi}\cos\varphi\,\text{sen}\varphi\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/4}\frac{1}{4}\big(\underbrace{16\sec^4\varphi - \sec^4\varphi}_{15\sec^4\varphi}\big)\cos\varphi\,\text{sen}\varphi\,d\varphi\,d\theta$$

$$= \frac{15}{4}\int_0^{2\pi}\int_0^{\pi/4}(\sec^4\varphi\cdot\cos\varphi\,\text{sen}\varphi)\,d\varphi\,d\theta =$$

$$= \frac{15}{4}\int_0^{2\pi}\int_0^{\pi/4}\Big(\frac{1}{\cos^4\varphi}\cdot\cos\varphi\,\text{sen}\varphi\Big)\,d\varphi\,d\varphi =$$

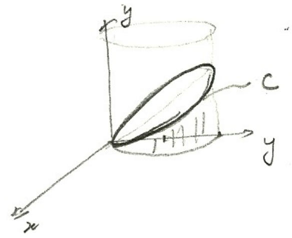
$$= \frac{15}{4}\int_0^{2\pi}\int_0^{\pi/4}\underbrace{\frac{1}{\cos^3\varphi}\cdot\text{sen}\varphi}_{\frac{1}{\cos^2\varphi}\cdot\frac{\text{sen}\varphi}{\cos\varphi}}\,d\varphi\,d\theta = \frac{15}{4}\int_0^{2\pi}\int_0^{\pi/4}\sec^2\varphi\,\tan\varphi\,d\varphi\,d\theta =$$

$$= \frac{15}{4}\int_0^{2\pi}\frac{\tan^2\varphi}{2}\Big|_0^{\pi/4}d\theta = \frac{15}{8}\int_0^{2\pi}(1-0)\,d\theta = \frac{15}{8}\times 2\pi = \frac{15\pi}{4}$$

$$\Rightarrow \bar{z} = \frac{\frac{15\pi}{4}}{\frac{7\pi}{3}} = \frac{15}{4}\times\frac{3}{7} = \frac{45}{28}$$

centro de massa $= \left(0, 0, \frac{45}{28}\right)$

4) $\rho(x,y,z) = \sqrt{4+x^2}$



$x^2 + y^2 = 4y$

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + (y-2)^2 = 4 \quad , \quad z = y$

$$C: \begin{cases} x = 2\cos t \\ y = 2 + 2\,\mathrm{sen}\, t \\ z = 2 + 2\,\mathrm{sen}\, t \end{cases}$$

$\vec{r}(t) = (2\cos t,\ 2 + 2\,\mathrm{sen}\,t,\ 2 + 2\,\mathrm{sen}\,t)\ ,\quad 0 \leq t \leq 2\pi$

$\vec{r}\,'(t) = \underline{(-2\,\mathrm{sen}\,t,\ 2\cos t,\ 2\cos t)}$

$\|\vec{r}\,'(t)\| = \sqrt{\underbrace{4\,\mathrm{sen}^2 t + 4\cos^2 t}_{=4} + 4\cos^2 t} = \sqrt{4 + 4\cos^2 t} = 2\sqrt{1+\cos^2 t}$

$\text{massa} = \int_C \rho\, ds = \int_0^{2\pi} \rho(\vec{r}(t))\, \|\vec{r}\,'(t)\|\, dt$

$\rho(x,y,z) = \sqrt{4+x^2}$

$\rho(r(t)) = \sqrt{4 + 4\cos^2 t} = 2\sqrt{1+\cos^2 t}$

$= \int_0^{2\pi} 2\sqrt{1+\cos^2 t}\; 2\sqrt{1+\cos^2 t}\; dt$

$= 4\int_0^{2\pi} (1 + \cos^2 t)\, dt =$

$= 4\int_0^{2\pi} \left(1 + \frac{1+\cos 2t}{2}\right) dt =$

$= 4\int_0^{2\pi} \frac{3+\cos 2t}{2} = 2\int_0^{2\pi} (3 + \cos 2t)\, dt =$

$= 2\left[3t + \frac{1}{2}\,\mathrm{sen}\, 2t\right]_0^{2\pi} =$

$= 2\left[(3\times 2\pi - 0) - (0 - 0)\right]$

$= 12\pi$ //