1) $x^2+y^2=4y$; $y=z$

$x^2+y^2-4y+4=4$

$x^2+(y-2)^2=4$

$C: \ \vec{r}(t)=(2\cos t,\ 2+2\,\text{sen}\,t,\ 2+2\,\text{sen}\,t)$; $0\le t\le \frac{\pi}{2}$

$2\cos t=2$ $\cos t=1$

$2+2\,\text{sen}\,t=2$ $\text{sen}\,t=0 \Rightarrow t=0$

$2+2\,\text{sen}\,t=2$ $\text{sen}\,t=0$

$2\cos t=0$ $\cos t=0$ $t=\frac{\pi}{2}$

$2+2\,\text{sen}\,t=4$ $2\,\text{sen}\,t=2$

$\vec{r}\,'(t)=(-2\,\text{sen}\,t,\ 2\cos t,\ 2\cos t)$

$\vec{F}(x,y,z)=(x,\ z-y,\ 2y)$

$F(\vec{r}(t))=(2\cos t,\ 0,\ 4+4\,\text{sen}\,t)$

$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)=-4\,\text{sen}\,t\cos t+0+8\,\text{sen}\,t\cos t+8\cos t$

$=4\,\text{sen}\,t\cos t+8\cos t$

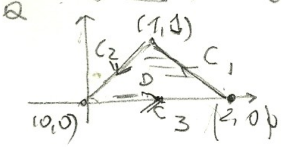
Trabalho $=\displaystyle\int_0^{\pi/2}F(r(t))\cdot r'(t)\,dt=\int_0^{\pi/2}4\,\text{sen}\,t\cos t+8\cos t\,dt=$

$4\,\dfrac{\text{sen}^2 t}{2}+8\,\text{sen}\,t\Big|_0^{\pi/2}=2+8=10$

2) $\displaystyle\int_C \underbrace{(x^2+g(x+y)-g(x))}_{P}\,dx+\underbrace{(2x+g(x+y))}_{Q}\,dy$

$C=C_1\cup C_2$

$\partial D=C_1\cup C_2\cup C_3$



Como g é classe C^1, logo F é de classe C^1 pois é soma de funções de classe C^1 em $\mathbb{R}^2\supset D$; D simplesmente conexo, logo pelo Teorema de Green

$\displaystyle\int_{\partial D}\vec{F}\cdot d\vec{r}=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy$

$\dfrac{\partial Q}{\partial x}-\dfrac{\partial P}{\partial y}=2+g'(x+y)\cdot 1-g'(x+y)\cdot 1=2$

2) continuação

$$\int_{C_1 \cup C_2 \cup C_3} \vec{F} \cdot d\vec{r} = \iint_D 2 \, dx\,dy = 2 \underbrace{\iint_D dx\,dy}_{\text{área de } D} = 2 \times \frac{1}{2} \times 2 \times 1 = 2$$

$$\int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} + \int_{C_3} \vec{F} \cdot d\vec{r} = 2$$

$C_3 : \vec{r}(t) = (t, 0)$

$0 \leq t \leq 2$

$\vec{r}\,'(t) = (1, 0)$

$\vec{F}(t, 0) =$

$= (t^2 + g(t) - g(t), 2t + g(t))$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) =$

$= t^2$

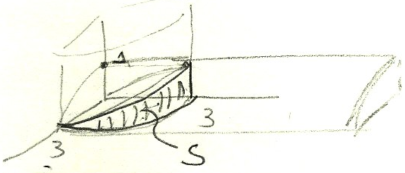
$$\int_{C_3} \vec{F} \cdot d\vec{r} = \int_0^2 t^2 \, dt =$$

$$= \frac{t^3}{3} \Big]_0^2 = \frac{8}{3}$$

$$\int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = 2 - \frac{8}{3} = -\frac{2}{3}$$

3) $x^2 + y^2 = 9$

$x^2 + 9z^2 = 9$



$\rho(x, y, z) = kz^2 \quad , k = 12$

$\rho(x, y, z) = 12 z^2$

$\varphi(\theta, z) = (3\cos\theta, 3\,\text{sen}\,\theta, z)$

$0 \leq \theta \leq \pi/2$

$9\cos^2\theta + 9z^2 = 9$

$z^2 = 1 - \cos^2\theta$

$z^2 = \text{sen}^2\theta$

$z = |\text{sen}\,\theta| \quad 0 \leq \theta \leq \frac{\pi}{2}$

$z = \text{sen}\,\theta$

$D : \begin{cases} 0 \leq \theta \leq \pi/2 \\ 0 \leq z \leq \text{sen}\,\theta \end{cases}$

$$\frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial z} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\,\text{sen}\,\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} =$$

$= (3\cos\theta, 3\,\text{sen}\,\theta, 0)$

$\left\| \frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial z} \right\| = 3.$

$$\text{Massa} = \iint_S \rho \, dS = \iint_S 12 z^2 \, dS =$$

$$= 12 \iint_D z^2 \underbrace{\left\| \frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial z} \right\|}_{=3} dz\, d\theta$$

$$= 36 \iint_D z^2 \, dz \, d\theta =$$

$$= 36 \int_0^{\pi/2} \int_0^{\text{sen}\,\theta} z^2 \, dz \, d\theta$$

$$= 36 \int_0^{\pi/2} \frac{z^3}{3} \Big]_0^{\text{sen}\,\theta} d\theta =$$

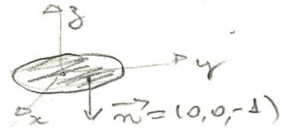
$$= 12 \int_0^{\pi/2} \text{sen}^3\theta \, d\theta = 12 \int_0^{\pi/2} (1 - \cos^2\theta)\,\text{sen}\,\theta \, d\theta$$

$$= 12 \left[-\cos\theta + \frac{\cos^3\theta}{3} \Big]_0^{\pi/2} \right] =$$

$$= 12 \left[(0 + 0) - \left(-1 + \frac{1}{3}\right) \right] = 12 \times \frac{2}{3} = \boxed{8}$$

4) a) $z = 0$

$\varphi(x,y) = (x, y, 0)$ $D: x^2+y^2 \leq 1$



$\left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1) = \underbrace{}_{-\vec{n}}$ unitário

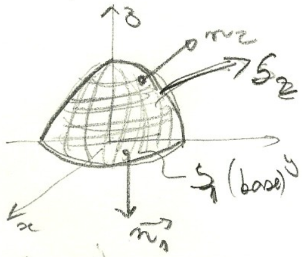
$\iint_{S_1} \vec{F}\cdot\vec{n}\, dS = -\iint_D \vec{F}(\varphi(x,y))\cdot \frac{\partial\varphi}{\partial x}\times\frac{\partial\varphi}{\partial y}\, dx\,dy$ $F(x,y,z) = (3x - e^{y+z}, 2y + e^{x-z}, z)$

$F(x,y,0) = (3x - e^{y}, 2y + e^{x}, 0)$

$= -\iint_D (3x - e^{y}, 2y + e^{x}, 0)\cdot(0,0,1)\, dx\,dy$ F

$= -\iint_D 0\, dx\,dy = 0$ //

b) W – sólido delimitado por S_1 e S_2.



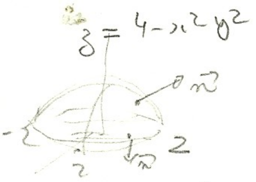
$\partial W = S_1 \cup S_2$

$\vec{F}$ é de classe C^1 em $\mathbb{R}^3$.

Fluxo $= \iint_{S_2} \vec{F}\cdot\vec{n}\, dS$

Pelo Teorema de Gauss $\iint_{\partial W = S_1\cup S_2} \vec{F}\cdot\vec{n}\, dS = \iiint_W \operatorname{div}(\vec{F})\, dx\,dy\,dz$

$\vec{F} = (3x - e^{y-z}, 2y + e^{x-z}, z)$



$\operatorname{div}(\vec{F}) = 3 + 2 + 1 = 6$

$\iint_{S_1\cup S_2} \vec{F}\cdot\vec{n}\, dS = \iiint_W 6\, dx\,dy\,dz$

$\underbrace{\iint_{S_1} \vec{F}\cdot\vec{n}\, dS}_{=0 \text{ (item a)}} + \iint_{S_2} \vec{F}\cdot\vec{n}\, dS = 6 \iiint_W dx\,dy\,dz$

coordenadas cilíndricas.
$0 \leq \theta \leq 2\pi$
$0 \leq r \leq 2$
$0 \leq z \leq 4 - r^2$

$\iint_{S_2} \vec{F}\cdot\vec{n}\, dS = 6 \int_0^{2\pi}\int_0^2\int_0^{4-r^2} r\, dz\, dr\, d\theta$

$= 6 \int_0^{2\pi}\int_0^2 zr\Big|_0^{4-r^2} dr\, d\theta = 6 \int_0^{2\pi}\int_0^2 \underbrace{(4-r^2)r}_{= 4r - r^3}\, dr\, d\theta =$

$= 6 \int_0^{2\pi} 4\cdot\frac{r^2}{2} - \frac{r^4}{4}\Big|_0^2 d\theta = 6\int_0^{2\pi} \underbrace{2\cdot 4 - \frac{16}{4}}_{=4}\, d\theta =$

$= 6 \times 4 \times 2\pi = 48\pi$ //