1) $\left(\underbrace{\frac{x}{4}}_{a} + \underbrace{2y-3}_{b}\right)^{10}$

O termo onde aparecerá $x^4 y^6$ será obtido de

$\binom{10}{6}\left(\frac{x}{4}\right)^4 (2y-3)^6$, onde $\underbrace{\binom{6}{0}}_{=1}(2y)^6(-3)^0 + \cdots$

Logo o termo será $\frac{10!}{6!\,4!} \frac{x^4}{4^4} \cdot 2^6 y^6 =$

$= \frac{6!\, 7\times 8\times 9\times 10}{6!\, 1\times 2\times 3\times 4} \cdot \frac{1}{4^4} \cdot 2^6 x^4 y^6$

$= \frac{210}{(2^2)^4} \times 2^6 x^4 y^6 = \frac{210}{2^8} \times 2^6 \cdot x^4 y^6$

$= \frac{210}{4} \times x^4 y^6 = \frac{105}{2} x^4 y^6$

Logo o coeficiente será $\frac{105}{2} = 50{,}25$.

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2) A razão da série é $r = \frac{2-\sqrt{2}}{\sqrt{3}-2}$

A série será convergente se $|r|<1$
e será divergente se $|r|>1$

$|r| = \left|\frac{2-\sqrt{2}}{\sqrt{3}-2}\right| = < 1 \iff$

$\frac{|2-\sqrt{2}|}{|\sqrt{3}-2|} < 1 \iff |2-\sqrt{2}| < |\sqrt{3}-2|$

como $2-\sqrt{2} > 0 \Rightarrow |2-\sqrt{2}| = 2-\sqrt{2}$

$\sqrt{3}-2 < 0 \Rightarrow |\sqrt{3}-2| = -(\sqrt{3}-2) = 2-\sqrt{3}$

Logo $|r|<1 \iff 2-\sqrt{2} < 2-\sqrt{3} \iff -\sqrt{2} < -\sqrt{3}$

$\iff \sqrt{2} > \sqrt{3}$ como $\sqrt{2} < \sqrt{3}$, concluímos que $|r|>1$

e a série é divergente.

OBS – poderia ter usado valor aproximado de $\sqrt{2}$ e $\sqrt{3}$.

3) $\displaystyle\sum_{i=1}^{\infty}\left(\frac{3}{2x-1}\right)^i = \frac{3}{2x-1} + \frac{3^2}{(2x-1)^2} + \frac{3^3}{(2x-1)^3} + \cdots$

a razão $r = \dfrac{3}{2x-1}$

A série será convergente se $|r| < 1$

$$\left|\frac{3}{2x-1}\right| < 1 \iff \frac{3}{|2x-1|} < 1 \iff 3 < |2x-1|$$

$\iff 2x-1 > 3$ ou $2x-1 < -3$

$2x > 4$ $\qquad$ $2x < -2$

$x > 2$ $\qquad$ $x < -1$

Logo a série será convergente se $x < -1$ ou $x > 2$

$$\sum_{i=1}^{\infty}\left(\frac{3}{2x-1}\right)^i = \frac{a}{1-r}, \quad \text{onde } a = \frac{3}{2x-1} = r$$

$$\frac{a}{1-r} = \frac{\frac{3}{2x-1}}{1-\frac{3}{2x-1}} = \frac{\frac{3}{2x-1}}{\frac{2x-1-3}{2x-1}} = \frac{\frac{3}{2x-1}}{\frac{2x-4}{2x-1}} = \frac{3}{2x-4}$$

Logo precisamos resolver $\dfrac{3}{2x-4} \leq \dfrac{3}{x}$

$$\frac{3}{2x-4} - \frac{3}{x} \leq 0$$

$$\frac{3x-6x+12}{(2x-4)x} \leq 0$$

$$\frac{-3x+12}{(2x-4)x} \leq 0$$

$$\frac{3x-12}{(2x-4)x} \geq 0$$

| | | 0 | | 2 | | 4 | |
|---|---|---|---|---|---|---|---|
| $3x-12$ | $-$ | $-$ | $-$ | $-$ | $-$ | 0 | $+$ |
| $2x-4$ | $-$ | $-$ | $-$ | 0 | $+$ | $+$ | $+$ |
| $x$ | $-$ | 0 | $+$ | $+$ | $+$ | $+$ | $+$ |
| $\frac{3x-12}{(2x-4)x}$ | $-$ | $\nexists$ | $+$ | $\nexists$ | $-$ | 0 | $+$ |

$S_1$ $\qquad$ $S_2$

A solução é $(0,2) \cup [4,\infty)$

Mas é preciso que a série seja convergente, isto é $x < -1$ ou $x > 2$

Logo, a solução é $[4,\infty)$

4) $\underbrace{(8x^5 - 4x^4 - 10x^3 + x^2 + 4x + 1)}_{p(x)} \underbrace{(x^2 + x + 1)}_{q(x)} > 0$

precisamos fatorar $p(x)$ e $q(x)$

possíveis raízes racionais de $p(x)$: $\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{8}\}$

$p(+1) = 8 - 4 - 10 + 1 + 4 + 1 = 0$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 8 | −4 | −10 | 1 | 4 | 1 | → grau 5 → $p(x)$ |
| 1 | 8 | 4 | −6 | −5 | −1 | 0 | → grau 4 → $q_1(x)$ |
| 1 | 8 | 12 | 6 | 1 | 0 | | → grau 3 → $q_2(x)$ |
| $-\frac{1}{2}$ | 8 | 8 | 2 | 0 | | | → grau 2 → $q_3(x)$ |

$q_1(x) = 8x^4 + 4x^3 - 6x^2 - 5x - 1$ → possíveis raízes racionais $\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{8}\}$

$q_1(1) = 8 + 4 - 6 - 5 - 1 = 0$

$q_2(x) = 8x^3 + 12x^2 + 6x + 1$

$q_2(1) \neq 0$

$q_2(-1) = -8 + 12 - 6 + 1 \neq 0$

$q_2(-\frac{1}{2}) = 8 \times (-\frac{1}{8}) + 12(\frac{1}{4}) + 6(-\frac{1}{2}) + 1$

$= -1 + 3 - 3 + 1 = 0$

$q_3(x) = 8x^2 + 8x + 2 = 0$

$4x^2 + 4x + 1 = 0$

$(2x + 1)^2 = 0 \Rightarrow x = -\frac{1}{2}$

logo as raízes de $p(x)$ são $x = 1$, multiplicidade 2
$x = -\frac{1}{2}$, " 3

$p(x) = 8(x-1)^2(x + \frac{1}{2})^3$

$q(x) = x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

não tem solução.

logo $q(x) > 0 \ \forall x$

| | | $-\frac{1}{2}$ | | 1 | |
|---|---|---|---|---|---|
| $8(x-1)^2$ | + | + | + | 0 | + |
| $(x+\frac{1}{2})^3$ | − | 0 | + | + | + |
| $x^2+x+1$ | + | + | + | + | + |
| $p(x)q(x)$ | − | 0 | + | 0 | + |

$\therefore \text{dom } f = (-\frac{1}{2}, 1) \cup (1, \infty)$ //

5)(a) $z = 9 - 3\sqrt{3}\,i$

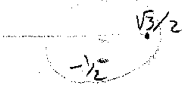
$|z| = \sqrt{81 + 9\times 3} = \sqrt{81+27} = \sqrt{108} = \sqrt{3\times 36} = 6\sqrt{3}$

$a = |z|\cos\theta = 9 \Rightarrow 6\sqrt{3}\cos\theta = 9 \Rightarrow \cos\theta = \frac{9}{6\sqrt{3}} = \frac{9\sqrt{3}}{6\times 3} = \frac{\sqrt{3}}{2}$

$b = |z|\,\text{sen}\,\theta = -3\sqrt{3} \Rightarrow 6\sqrt{3}\,\text{sen}\,\theta = -3\sqrt{3} \Rightarrow \text{sen}\,\theta = -\frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\text{sen}\,\theta = -\frac{1}{2}$




$\Rightarrow \theta = -\frac{\pi}{6} \equiv 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

Logo $\arg z = \frac{11\pi}{6} = \theta$

$8\theta = 8\times\frac{11\pi}{6} = \frac{4\times 11\pi}{3} = \frac{44\pi}{3} = \frac{42\pi + 2\pi}{3} = 14\pi + \frac{2\pi}{3} \equiv \frac{2\pi}{3}$

$z^8 = |z|^8(\cos 8\theta + i\,\text{sen}\,8\theta) =$

$z^8 = (6\sqrt{3})^8\left(\cos\frac{44\pi}{3} + i\,\text{sen}\,\frac{44\pi}{3}\right)$

$z^8 = 6^8\times 3^4\left(\cos\frac{2\pi}{3} + i\,\text{sen}\,\frac{2\pi}{3}\right)$

$w = i \Rightarrow |w| = 1$ e $\arg w = \frac{\pi}{2} \Rightarrow \arg i^7 = 7\arg i = 7\times\frac{\pi}{2} = \frac{4\pi + 3\pi}{2} \equiv \frac{3\pi}{2}$

$\Rightarrow i^7 = -i$ ou $\arg i^7 = \frac{3\pi}{2}$

$z^8 i^7 = 6^8\times 3^4\left(\cos\left(\frac{2\pi}{3} + \frac{3\pi}{2}\right) + i\,\text{sen}\left(\frac{2\pi}{3} + \frac{3\pi}{2}\right)\right)$

$\frac{2\pi}{3} + \frac{3\pi}{2} = \frac{4+9\pi}{6} = \frac{13\pi}{6} = 2\pi + \frac{\pi}{6} \equiv \frac{\pi}{6}$

$z^8 i^7 = 6^8\times 3^4\left(\cos\frac{\pi}{6} + i\,\text{sen}\,\frac{\pi}{6}\right)$

$z^8 i^7 = 6^8\times 3^4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$6^8\times 3^4 = 2^8\times 3^8\times 3^4 = 2^8\times 3^{12}$

$z^8 i^7 = 2^8\times 3^{12}\times\frac{\sqrt{3}}{2} + \frac{2^8\times 3^{12}}{2}i$

$$\boxed{z^8 i^7 = 2^7\times 3^{12}\sqrt{3} + 2^7\times 3^{12}\,i}$$

(5) (b) $z^5 = -i$ $\qquad w = -i$

$|w| = 1$ e $\arg w = \frac{3\pi}{2} = \theta$

$$z = |w|^{1/5}\left(\cos\frac{\theta + 2k\pi}{5} + i\,\text{sen}\,\frac{\theta + 2k\pi}{5}\right),\ k = 0,1,2,3,4$$

$$z = 1\left(\cos\frac{\frac{3\pi}{2} + 2k\pi}{5} + i\,\text{sen}\,\theta + \frac{2k\pi}{5}\right.$$

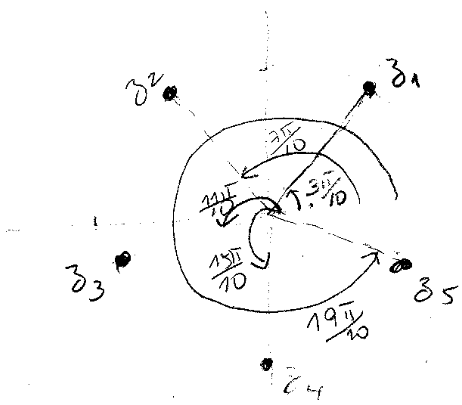
$k = 0$, $\arg z_1 = \dfrac{3\pi/2}{5} = \dfrac{3\pi}{10} \Rightarrow z_1 = \cos\dfrac{3\pi}{10} + i\,\text{sen}\,\dfrac{3\pi}{10}$

$k = 1$, $\arg z_2 = \dfrac{\frac{3\pi}{2} + 2\pi}{5} = \dfrac{7\pi}{10} \Rightarrow z_2 = \cos\dfrac{7\pi}{10} + i\,\text{sen}\,\dfrac{7\pi}{10}$

$k = 2$, $\arg z_3 = \dfrac{\frac{3\pi}{2} + 4\pi}{5} = \dfrac{11\pi}{10} \Rightarrow z_3 = \cos\dfrac{11\pi}{10} + i\,\text{sen}\,\dfrac{11\pi}{10}$

$k = 3$, $\arg z_4 = \dfrac{\frac{3\pi}{2} + 6\pi}{5} = \dfrac{15\pi}{10} = \dfrac{3\pi}{2} \Rightarrow z_4 = \cos\dfrac{3\pi}{2} + i\,\text{sen}\,\dfrac{3\pi}{2} = -i$

$k = 4$, $\arg z_5 = \dfrac{\frac{3\pi}{2} + 8\pi}{5} = \dfrac{19\pi}{10} \Rightarrow z_5 = \cos\dfrac{19\pi}{10} + i\,\text{sen}\,\dfrac{19\pi}{2}$




$\frac{3\pi}{10} \Leftrightarrow 3\times 18^\circ = 54^\circ$

$\frac{7\pi}{10} \rightarrow 7\times 18^\circ = 126^\circ$

$\frac{11\pi}{10} \Leftrightarrow 11\times 18^\circ = 198^\circ$

$\frac{15\pi}{10} = \frac{3\pi}{2}$

$\frac{19\pi}{10} \Leftrightarrow 19\times 18^\circ = 342^\circ$

19
× 18
152
19
342