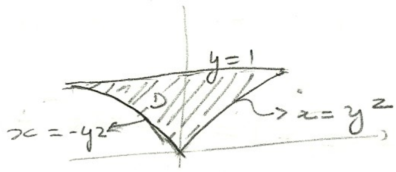
(1) $x = y^2$, $x = -y^2$, $y = 1$

(a) $\displaystyle\iint_D e^{y^3} = \int_0^1 \int_{-y^2}^{y^2} e^{y^3}\,dx\,dy$



$\displaystyle = \int_0^1 x e^{y^3} \Big|_{-y^2}^{y^2} dy =$

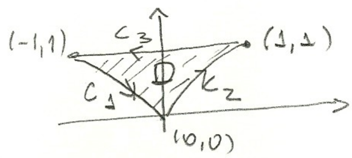
$0 \leq y \leq 1$

$-y^2 \leq x \leq y^2$

$\displaystyle = \int_0^1 (y^2 e^{y^3} - (-y^2 e^{y^3}))\,dy =$

$\displaystyle = 2 \int_0^1 y^2 e^{y^3} = 2 \cdot \frac{1}{3} e^{y^3} \Big|_0^1 = \frac{2}{3}(e^1 - e^0) = \frac{2}{3}(e-1)$

(b) $\displaystyle \text{Trabalho} = \int_{C = C_1 \cup C_2} F \cdot d\vec{r}$



$\partial D = C_1 \cup C_2 \cup C_3$

$\vec{F} = \underbrace{(y+x)}_{P(x,y)}\vec{i} + \underbrace{(x + g(y) + x e^{y^3})}_{Q(x,y)}\vec{j}$

$\vec{F}$ é de classe C^1 em $\mathbb{R}^2$ (aberto contendo D)

D é simplesmente conexo, pelo Teorema de Green $\displaystyle\int_{C_1 \cup C_2 \cup C_3} \vec{F} \cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

$\displaystyle \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 + e^{y^3} - 1 = e^{y^3}$

$\displaystyle \int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} + \int_{C_3} \vec{F} \cdot d\vec{r} = \underbrace{\iint_D e^{y^3} dy}_{= \frac{2}{3}(e-1)}$ (item (a))

$\displaystyle \int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = \frac{2}{3}(e-1) - \int_{C_3} F \cdot d\vec{r} = \frac{2}{3}(e-1) + \int_{C_3^-} \vec{F} \cdot d\vec{r}$

C_3 ; $\vec{r}(t) = (t, 1)$, $-1 \leq t \leq 1$

$\vec{r}\,'(t) = (1, 0)$

$\vec{F}(\vec{r}(t)) = \vec{F}(t, 1) = (1+t,\ t + g(1) + te)$

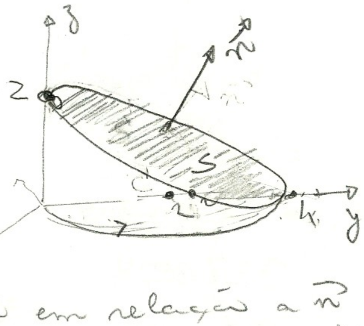
$\displaystyle \int_{C_3^-} \vec{F} \cdot d\vec{r} = \int_{-1}^{1} (1,0) \cdot (1+t,\ t + g(1) + te)\,dt = \int_{-1}^{1} (1+t)\,dt$

$\displaystyle = t + \frac{t^2}{2} \Big|_{-1}^{1} = \left(1 + \frac{1}{2}\right) - \left(-1 + \frac{1}{2}\right) = 2$

$2 - \frac{2}{3} = \frac{4}{3}$

Logo $\displaystyle \int_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = \frac{2}{3}(e-1) + 2 = \frac{4}{3} + \frac{2}{3}e = \frac{2}{3}(e+2)$

2) $S: \quad y + 2z = 4$ no interior de $x^2+y^2=4y$



$\vec{F} = (0, 4x+z, y+3z)$

F de classe C^1 em $\mathbb{R}^3$ (aberto que contém S)

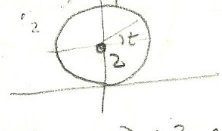
S – simplesmente conexo.

$C = \partial S$ – orientada positivamente em relação a $\vec{n}$ (da figura)

$$\int_{C=\partial S} \vec{F} \cdot d\vec{r} = \iint_S \operatorname{rot}\vec{F} \cdot \vec{n}\, dS.$$

$x^2+y^2 = 4y$

$x^2+(y-2)^2 = 4$



$C: \vec{r}(t) = (2\cos t, 2+2\,\text{sen}\, t, 1-\text{sen}\, t)$

$0 \leq t \leq 2\pi$

$\vec{F}(\vec{r}(t)) = (0, 8\cos t + 1 - \text{sen}\, t, 2+2\,\text{sen}\, t + 3 - 3\,\text{sen}\, t)$

$\vec{F}(\vec{r}(t)) = (0, 1+8\cos t - \text{sen}\, t, 5 - \text{sen}\, t)$

$\vec{r}\,'(t) = (-2\,\text{sen}\, t, 2\cos t, -\cos t)$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) =$

$= 0 + 2\cos t + 16\cos^2 t - 2\,\text{sen}\, t\cos t + \text{sen}\, t \cos t$

$= 2\cos t + 16\cos^2 t - \text{sen}\, t\cos t$

$y = 2 + 2\,\text{sen}\, t$

$y + 2z = 4$

$2z = 4 - y$

$2z = \dfrac{4-2-2\,\text{sen}\, t}{2}$

$z = 1 - \text{sen}\, t$

Logo $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} (2\cos t + 16\underbrace{\cos^2 t}_{\frac{1+\cos 2t}{2}} - \text{sen}\, t\cos t)\, dt$

$= \underbrace{\displaystyle\int_0^{2\pi} 2\cos t\, dt}_{=0} + \int_0^{2\pi} (8 + \underbrace{8\cos 2t)\, dt}_{=\int_0^{2\pi}\cos 2t\, dt = 0} - \underbrace{\int_0^{2\pi} \text{sen}\, t \cos t}_{=0}$

$= 8 \times \displaystyle\int_0^{2\pi} dt = 8 \times 2\pi = 16\pi$ //

$S: \varphi(x,y) = (x, y, 2 - \frac{1}{2}y)$

$\dfrac{\partial\varphi}{\partial x} \times \dfrac{\partial \varphi}{\partial y} = (0, +\frac{1}{2}, 1)$ mesmo sentido de $\vec{n}$.

$y + 2z = 4$

$z = \frac{1}{2}(4-y)$

$= 2 - \frac{1}{2}y$

(x,y): $x^2+y^2 \leq 4y$

$$\operatorname{rot}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 4x+z & y+3z \end{vmatrix} =$$

$= (1-1, 0-0, 4-0) = (0,0,4)$

2) continuação

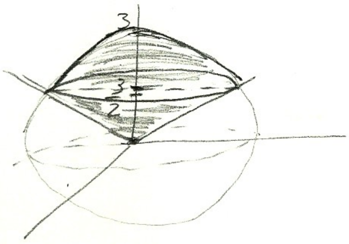
$\vec{rot}\,\vec{F}\,(\varphi(x,y)) = (0,0,4)$.

$$\iint_S \vec{rot}\,\vec{F}\cdot\vec{n}\,dS = \iint_D \vec{rot}\,\vec{F}\cdot\frac{\partial\varphi}{\partial x}\times\frac{\partial\varphi}{\partial y}\,dx\,dy =$$

$$= \iint_D (0,0,4)\cdot\left(0,\tfrac{1}{2},1\right)dx\,dy = \iint_D 4\,dx\,dy = 4\iint_D dx\,dy$$

$$= 4\times \text{área de } D = 4\times\pi\times(2)^2 = 16\pi \;//$$

3) $z = \sqrt{\dfrac{x^2+y^2}{3}}$ — inferiormente

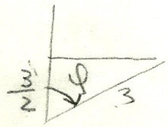


$x^2+y^2+z^2 = 9.$

$z^2 = \dfrac{x^2+y^2}{3} \Rightarrow x^2+y^2 = 3z^2$

$\Rightarrow 3z^2+z^2 = 9 \Rightarrow 4z^2 = 9,\; z>0$

$\Rightarrow z = \dfrac{3}{2}$.

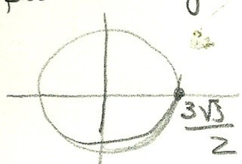


(a) Na interseção: $z = \dfrac{3}{2}$

$r^2 = x^2+y^2 = 3\times\dfrac{9}{4} = \dfrac{27}{4}$.

$r = \dfrac{\sqrt{27}}{2} = \dfrac{3\sqrt{3}}{2}$

projeção no plano xy:



$0 \le \theta \le 2\pi$

$0 \le r \le \dfrac{3\sqrt{3}}{2}$

variação de z:

$z = \sqrt{\dfrac{x^2+y^2}{3}} = \dfrac{\sqrt{x^2+y^2}}{\sqrt{3}} = \dfrac{r}{\sqrt{3}} = \dfrac{r\sqrt{3}}{3}$

$\underbrace{x^2+y^2}_{r^2}+z^2 = 9 \Rightarrow z^2 = 9-r^2,\; z\ge 0$

$z = \sqrt{9-r^2}$

$\Rightarrow \dfrac{r\sqrt{3}}{3} \le z \le \sqrt{9-r^2}$

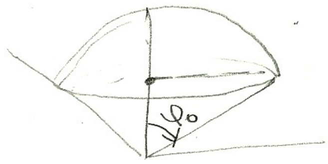
$$\iiint_W (x^2+y^2)\,dx\,dy\,dz = \int_0^{2\pi}\int_0^{\frac{3\sqrt{3}}{2}}\int_{\frac{r\sqrt{3}}{3}}^{\sqrt{9-r^2}} r^2\cdot r\,dz\,dr\,d\theta$$

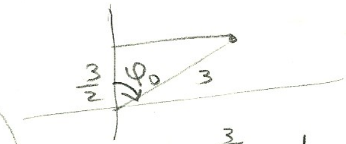
$$= \int_0^{2\pi}\int_0^{\frac{3\sqrt{3}}{2}}\int_{\frac{r\sqrt{3}}{3}}^{\sqrt{9-r^2}} r^3\,dz\,dr\,d\theta$$

3) b) $0 \le \rho \le 3$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \varphi_0 = \pi/3$





$$\begin{cases} x = \rho\cos\theta\,\operatorname{sen}\varphi \\ y = \rho\operatorname{sen}\theta\,\operatorname{sen}\varphi \\ z = \rho\cos\varphi \end{cases}$$

$\cos\varphi_0 = \frac{3/2}{3} = \frac{1}{2}$

$\Rightarrow \varphi_0 = \frac{\pi}{3}$

$$x^2+y^2 = \rho^2\operatorname{sen}^2\varphi\,(\cos^2\theta+\operatorname{sen}^2\theta) = \rho^2\operatorname{sen}^2\varphi$$

$$\iiint_W (x^2+y^2)\,dx\,dy\,dz = \int_0^{2\pi}\int_0^{\pi/3}\int_0^3 \rho^2\operatorname{sen}^2\varphi\cdot\underbrace{\rho^2\operatorname{sen}\varphi}_{=J(T)}\,d\rho\,d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3}\int_0^3 \rho^4\operatorname{sen}^3\varphi\,d\varphi\,d\theta \quad //$$

3) c) Em esféricas.

$$\int_0^{2\pi}\int_0^{\pi/3} \frac{\rho^5}{5}\Big|_0^3 \underbrace{\operatorname{sen}^3\varphi}_{(1-\cos^2\varphi)\operatorname{sen}\varphi}\,d\varphi\,d\theta = \frac{3^5}{5}\int_0^{2\pi}\int_0^{\pi/3}(\operatorname{sen}\varphi-\cos^2\varphi\operatorname{sen}\varphi)\,d\varphi\,d\theta$$

$$= \frac{3^5}{5}\int_0^{2\pi}\left(-\cos\varphi+\frac{\cos^3\varphi}{3}\right)\Big|_0^{\pi/3}d\theta = \frac{3^5}{5}\int_0^{2\pi}\left[\left(-\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{8}\right)-\underbrace{\left(-1+\frac{1}{3}\right)}_{=-2/3}\right]d\theta$$

$$\frac{2}{3}-\frac{1}{2}+\frac{1}{24} = \frac{17-12}{24} = \frac{5}{24}$$

$$= \frac{3^5}{5}\cdot\frac{5}{24}\cdot\int_0^{2\pi}d\theta = \frac{3^5}{24}\times 2\pi = \frac{3^4\cdot 3\pi}{12} = \frac{3^4}{4} = \frac{81\pi}{4} \quad //$$

3) d) $\vec{F}(x,y,z) = \left(\frac{x^3+z^2}{9}, \frac{y^3+z^2}{9}, x^2+y^2\right)$

$\operatorname{div}\vec{F} = \frac{3x^2}{9}+\frac{3y^2}{9}+0 = \frac{1}{3}(x^2+y^2)$

Fluxo $= \iint_S \vec{F}\cdot\vec{n}\,dS$, $\vec{F}$ de classe C^1, $S=\partial W$, $\vec{n}$ para fora de W.

Pelo Teorema de Gauss $\iint_{S=\partial W}\vec{F}\cdot\vec{n}\,dS = \iiint_W \operatorname{div}\vec{F}\,dx\,dy\,dz$

$$= \iiint_W \frac{1}{3}(x^2+y^2)\,dx\,dy\,dz = \frac{1}{3}\underbrace{\iiint_W (x^2+y^2)\,dx\,dy\,dz}_{=\frac{81\pi}{4}\text{ (calculado em c)}}$$

$$= \frac{1}{3}\times\frac{81\pi}{4} = \frac{27\pi}{4} \quad //$$