1) (a) $\lim\limits_{x \to 2} \dfrac{x^x - 4}{x^2 - 4} = \lim\limits_{x \to 2} \dfrac{e^{x \ln x} - 4}{x^2 - 4} \overset{L'H}{=} \lim\limits_{x \to 2} \dfrac{e^{x\ln x}\left(x \cdot \frac{1}{x} + \ln x\right)}{2x}$

$= \dfrac{e^{2\ln 2}(1 + \ln 2)}{4} = \dfrac{4(1 + \ln 2)}{4} = 1 + \ln 2$ //

(b) $\lim\limits_{x \to 1} \dfrac{\sqrt{x+2} - \sqrt{4x-1}}{\sqrt{x} - x} = \dfrac{0}{0}$, indeterminado.

multiplicando pelos conjugados,

$\lim\limits_{x \to 1} \dfrac{(\sqrt{x+2} - \sqrt{4x-1})}{(\sqrt{x} - x)} \cdot \dfrac{\sqrt{x+2} + \sqrt{4x-1}}{\sqrt{x} + x} \cdot \dfrac{\sqrt{x} + x}{\sqrt{x+2} + \sqrt{4x-1}}$

$= \lim\limits_{x \to 1} \dfrac{x + 2 - 4x + 1}{x - x^2} \cdot \dfrac{\sqrt{x+2} + \sqrt{4x-1}}{\sqrt{x} + x} =$

$= \lim\limits_{x \to 1} \dfrac{-3x + 3}{x - x^2} \cdot \dfrac{\sqrt{x+2} + \sqrt{4x+1}}{\sqrt{x} + x} =$

$= \lim\limits_{x \to 1} \dfrac{-3\cancel{(x-1)}}{x\cancel{(1-x)}} \cdot \dfrac{\sqrt{x+2} + \sqrt{4x+1}}{\sqrt{x} + x} = \dfrac{3}{1} \cdot \dfrac{\sqrt{3} + \sqrt{3}}{1+1} = 3\sqrt{3}$ //

2) $f(x) = \begin{cases} \dfrac{\operatorname{sen} x}{|x|} & \text{se } x \neq 0 \\ k & \text{se } x = 0 \end{cases}$

$\lim\limits_{\substack{x \to 0^+ \\ x > 0}} \dfrac{\operatorname{sen} x}{|x|} = \lim\limits_{x \to 0^+} \dfrac{\operatorname{sen} x}{x} = 1$

$\lim\limits_{\substack{x \to 0^- \\ x < 0}} \dfrac{\operatorname{sen} x}{|x|} = \lim\limits_{x \to 0^-} \dfrac{\operatorname{sen} x}{-x} = -1$

Logo $\lim\limits_{x \to 0^+} f(x) \neq \lim\limits_{x \to 0^-} f(x)$ e $\nexists\, k$ tal que f seja contínua em $x = 0$.

3) $xy = 24$ Custo total $= 3 \cdot 2x + 3 \cdot 2y$

$= 6x + 6y$

como $xy = 50$, $y = \dfrac{24}{x}$, $x > 0$

$C(x) = 6x + \dfrac{6 \times 24}{x} = 6\left(x + \dfrac{24}{x}\right)$

$C'(x) = 6\left(1 - \dfrac{24}{x^2}\right) = 6\,\dfrac{(x^2 - 24)}{x^2}$

$x^2 = 24 \quad x = \pm 2\sqrt{6}$

		$-2\sqrt{6}$		0		$2\sqrt{6}$	
$6(x^2-24)$	+	0	-	-	-	0	+
x^2	+	+	+	0	+	+	+
$f'(x)$				$\nexists$	-	0	+
					↘		↗

O custo mais barato é com dimensões $x = y = 2\sqrt{6}$ cm

4) $f(x) = e^{2x-\cos x}$.

$f'(x) = e^{2x-\cos x} \cdot (2+\operatorname{sen} x)$

$-1 < \operatorname{sen} x < 1 \Rightarrow 2-1 < 2+\operatorname{sen} x < 2+1$

$1 < 2+\operatorname{sen} x < 3 \Rightarrow 2+\operatorname{sen} x > 0$

Como $e^{2x-\cos x} > 0 \;\forall x$,

Temos que $f'(x) > 0 \;\forall x$, f é crescente em $(-\infty, \infty)$

Logo f admite inversa.

(Outra justificativa seria $f'(x) \neq 0 \;\forall x$, Pelo Teorema da função Inversa.)

$f(0) = e^{0-\cos 0} = e^{-1} = \frac{1}{e}$

$f'(0) = (e^{0-\cos 0})(2+\operatorname{sen} 0) = e^{-1}(2) = \frac{2}{e}$

$(f^{-1})'\left(\frac{1}{e}\right) = \frac{1}{f'(0)}$

$(f^{-1})'\left(\frac{1}{e}\right) = \frac{1}{\frac{2}{e}} = \frac{e}{2}$ //

5) $f(x) = 1 - \frac{12}{x+3} + \frac{36}{(x+3)^2} = \frac{(x-3)^2}{(x+3)^2}$

(a) Domínio: $x \neq -3 \Rightarrow \operatorname{Dom} f = (-\infty, -3) \cup (-3, \infty)$

Contínua no domínio.

A.V: $\lim_{x \to -3^-} \frac{(x-3)^2 \to 36}{(x+3)^2 \to 0^+} = \infty \Rightarrow$ Eq. de A.V $x = -3$.

$\lim_{x \to -3^+} \frac{(x-3)^2}{(x+3)^2} = \infty$

A.H. $\lim_{x \to \infty} \frac{(x-3)^2}{(x+3)^2} = \lim_{x \to \infty} \frac{2(x-3)}{2(x+3)} = 1$ Eq. de AH $y = 1$

$\lim_{x \to -\infty} \frac{(x-3)^2}{(x+3)^2} = 1$

$f'(x) = \frac{12}{(x+3)^2} - \frac{72}{(x+3)^3} = \frac{12}{(x+3)^2}\left(1 - \frac{6}{x+3}\right)$

$f'(x) = \frac{12}{(x+3)^2} \cdot \frac{(x+3-6)}{(x+3)} = \frac{12(x-3)}{(x+3)^3}$

		-3		3	
$12(x-3)$	$-$	$-$	$-$	0	$+$
$(x+3)^3$	$-$	0	$+$	$+$	$+$
$f'(x)$	$+$ ↗	$\nexists$	$-$ ↘	0	$+$ ↗

crescente em $(-\infty, 3) \cup (3, \infty)$

decrescente em $(-3, 3)$

mín relativo em $x = 3$

não tem máx. relativo pois $-3 \notin \mathrm{dom}\, f$

$$f'(x) = 12\left(\frac{1}{(x+3)^2} - \frac{6}{(x+3)^3}\right)$$

$$f''(x) = 12\left(\frac{-2}{(x+3)^3} + \frac{18}{(x+3)^4}\right) = \frac{12 \times 2}{(x+3)^3}\left(-1 + \frac{9}{x+3}\right)$$

$$= \frac{24}{(x+3)^3}\left(\frac{-x-3+9}{x+3}\right) = \frac{24(-x+6)}{(x+3)^4} = \frac{24(6-x)}{(x+3)^4}$$

		-3		6	
$24(6-x)$	$+$	$+$	$+$	0	$-$
$(x+3)^4$	$+$	0	$+$	$+$	$+$
$f''(x)$	$+$	$\nexists$	$+$	0	$-$
concavidade	p/ cima		p/ cima		p/ baixo

ponto de inflexão em $x = 6$

$$f(3) = 1 - \frac{12}{6} + \frac{36}{36} = 1 - 2 + 1 = 0$$

$$f(6) = 1 - \frac{12}{9} + \frac{36}{81} = 1 - \frac{4}{3} + \frac{4}{3} = \underline{1}$$

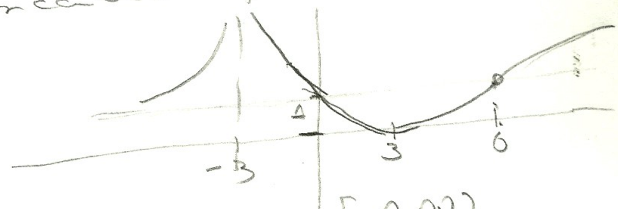

imagem $f = [0, \infty)$

6) $\int\left(6\sqrt[5]{x} - 3x^{-1} + \frac{4}{1+x^2}\right)dx = \int\left(6x^{\frac{1}{5}} - \frac{3}{x} + \frac{4}{1+x^2}\right)dx$

$$= 6\,\frac{x^{\frac{6}{5}}}{\frac{6}{5}} - 3\ln|x| + 4\arctan x + C$$

$$= 5\sqrt[5]{x^6} - 3\ln|x| + 4\arctan x + C$$