RESPOSTAS DA LISTA 4 - Trigonometria

1. $9\sqrt{3}\ m$

2. $h = 6\sqrt{7}\,km$

3. $25,34\,m$

4. (a) $8\,cm$ (b) $\frac{5\sqrt{3}}{3}\,cm$

5. $\frac{3}{2}$

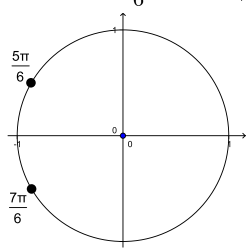
6. $-\frac{2}{3}$

7. $\frac{\sqrt{6}}{2}$

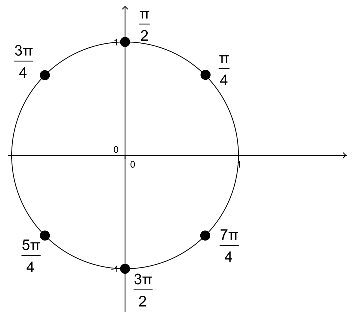
8. (a) $-3 \leq m \leq -\sqrt{7}$
   ou $\sqrt{7} \leq m \leq 7$
   (b) $-\frac{1}{7} \leq m \leq 1$
   (c) $-1 \leq m \leq 3$

10. (a) $\cot x$ (b) $\tan x$

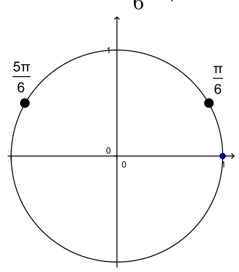
11. (a) $x = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$
    ou $x = \frac{7\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$



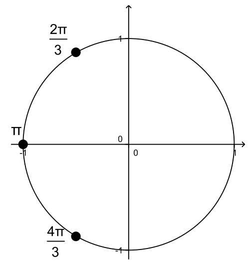
(b) $x = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$
ou $x = \frac{3\pi}{4} + k\pi, \quad k \in \mathbb{Z}$
ou $x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$



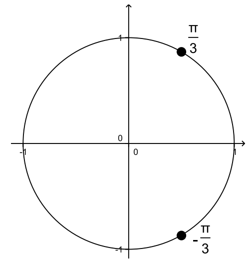
(c) $x = \frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$
ou $x = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$



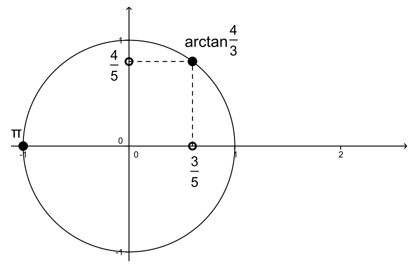
(d) $x = \frac{2\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$
ou $x = \frac{4\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$
ou $x = \pi + 2k\pi, \quad k \in \mathbb{Z}$



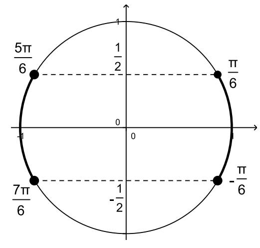
(e) $x = \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$
ou $x = -\frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$



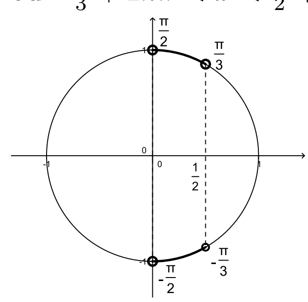
(f) $x = \pi + 2k\pi, \quad k \in \mathbb{Z}$
ou $x = \arctan\frac{4}{3} + 2k\pi, \quad k \in \mathbb{Z}$



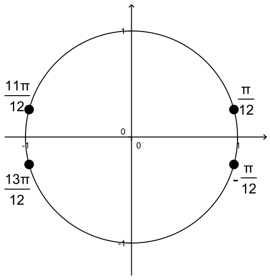
(g) $-\frac{\pi}{6}\pi + 2k\pi < x < \frac{\pi}{6}\pi + 2k\pi, \quad k \in \mathbb{Z}$
ou $\frac{5\pi}{6} + 2k\pi < x < \frac{7\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$



(h) $-\frac{\pi}{2} + 2k\pi < x < -\frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$
ou $\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$



(i) $x = \frac{\pi}{12} + k\pi, \quad k \in \mathbb{Z}$
ou $\quad x = -\frac{\pi}{12} + k\pi, \quad k \in \mathbb{Z}$



(j) Se desenvolver a expresssão usando as identidades

$\text{sen}\,(4x) = 2\,\text{sen}\,(2x)\cos(2x)$

$\text{sen}\,(2x) = 2\,\text{sen}\,(x)\cos(x)$

$\cos(2x) = \cos^2 x - \text{sen}^2 x$

encontra-se a solução na seguinte forma:

$x = k\pi, \quad k \in \mathbb{Z},$

ou $\quad x = \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$

ou $\quad x = -\frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$

ou $\quad x = \theta_1 + 2k\pi, \ k \in \mathbb{Z},$

onde θ_1 é o ângulo tal que:

$\cos(\theta_1) = \frac{-1+\sqrt{5}}{4}$ e $\text{sen}\,(\theta_1) = \frac{\sqrt{10+2\sqrt{5}}}{4}$

isto é, $\quad \theta_1 = \arctan \frac{\sqrt{10+2\sqrt{5}}}{-1+\sqrt{5}}$

ou $\quad x = \theta_2 + 2k\pi, \ k \in \mathbb{Z},$

onde θ_2 é o ângulo tal que:

$\cos(\theta_2) = \frac{-1+\sqrt{5}}{4}$ e $\text{sen}\,(\theta_2) = -\frac{\sqrt{10+2\sqrt{5}}}{4}$

isto é, $\quad \theta_2 = -\arctan \frac{\sqrt{10+2\sqrt{5}}}{-1+\sqrt{5}}$

ou $\quad x = \theta_3 + 2k\pi, \ k \in \mathbb{Z},$

onde θ_3 é o ângulo tal que:

$\cos(\theta_3) = \frac{-1-\sqrt{5}}{4}$ e $\text{sen}\,(\theta_3) = \frac{\sqrt{10-2\sqrt{5}}}{4}$

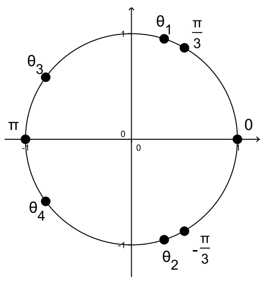
isto é, $\quad \theta_3 = \pi + \arctan \frac{\sqrt{10-2\sqrt{5}}}{-1-\sqrt{5}}$

ou $\quad x = \theta_3 + 2k\pi, \ k \in \mathbb{Z},$

onde θ_4 é o ângulo tal que:

$\cos(\theta_4) = \frac{-1-\sqrt{5}}{4}$ e $\text{sen}\,(\theta_4) = -\frac{\sqrt{10-2\sqrt{5}}}{4}$

isto é, $\quad \theta_4 = \pi - \arctan \frac{\sqrt{10-2\sqrt{5}}}{-1-\sqrt{5}}$



Se desenvolver a expressão usando a identidade:

$\text{sen}\,p + \text{sen}\,q = 2\,\text{sen}\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$

encontra-se a expressão

$\text{sen}\,x + \text{sen}\,(4x) = 2\,\text{sen}\left(\frac{5x}{2}\right)\cos\left(\frac{3x}{2}\right)$

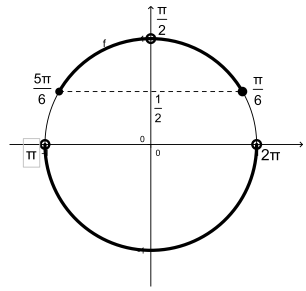
e a mesma solução anterior, escrita de outra forma:

$x = \frac{2k\pi}{5} \quad$ ou $\quad x = \frac{\pi}{3} + \frac{2k\pi}{3}.$

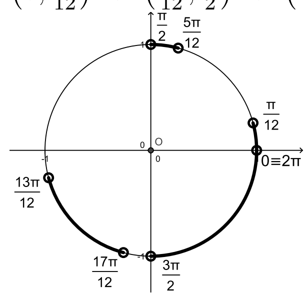
Observe que:

$\theta_1 = \frac{2\pi}{5}; \quad \theta_2 = \frac{4\pi}{5}; \quad \theta_3 = -\frac{4\pi}{5}; \quad \theta_4 = -\frac{2\pi}{5}$

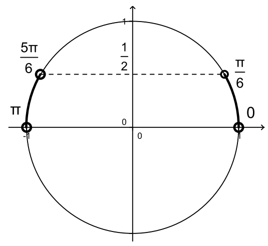
(k) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup (\pi, 2\pi)$



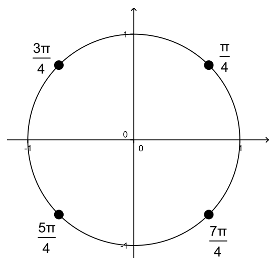
(l) $\left(0, \frac{\pi}{12}\right) \cup \left(\frac{5\pi}{12}, \frac{\pi}{2}\right) \cup \left(\frac{13\pi}{12}, \frac{17\pi}{12}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$



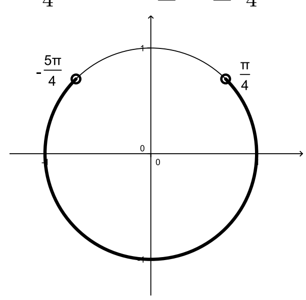
(m) $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$



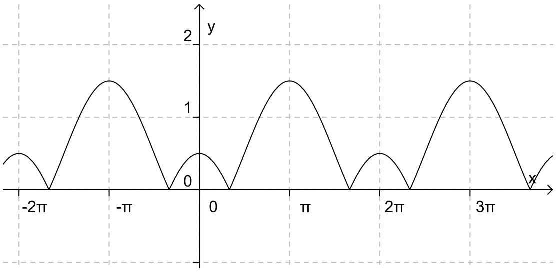
(n) $x = \frac{k\pi}{4}, \quad k \in \mathbb{Z}$



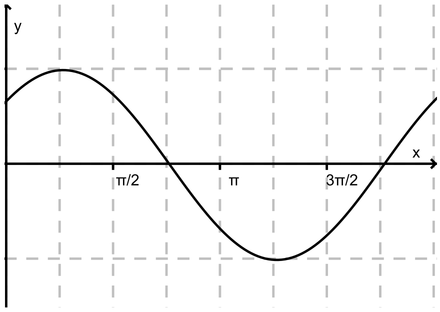
(o) $-\frac{5\pi}{4} + 2k\pi \le x \le \frac{\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$



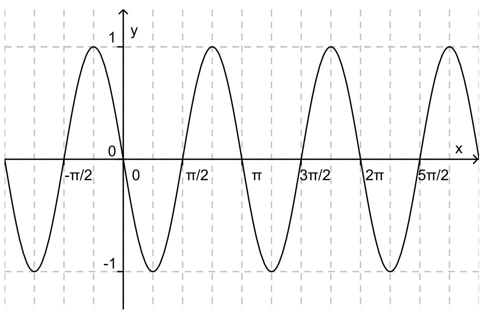
12. (a)



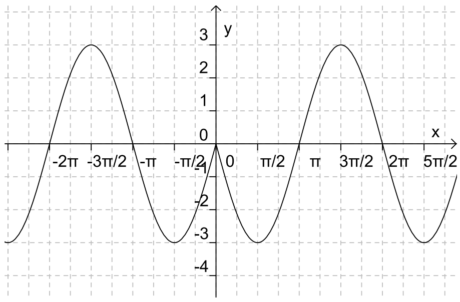
(b)



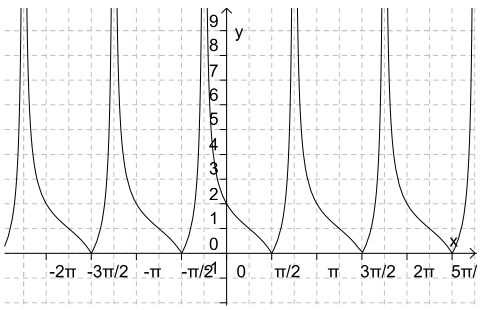
(c)



(d)



(e)



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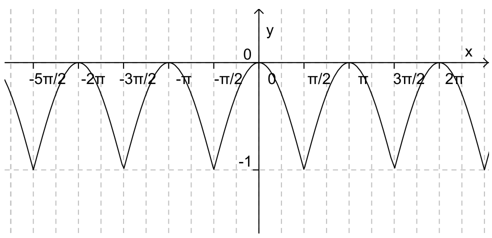
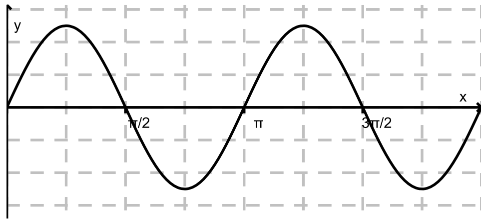
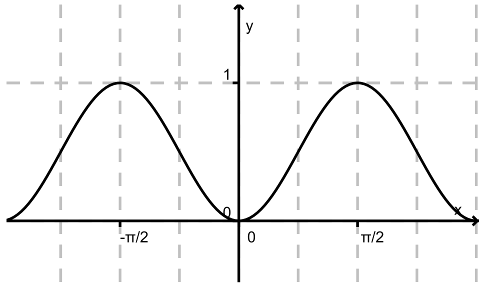


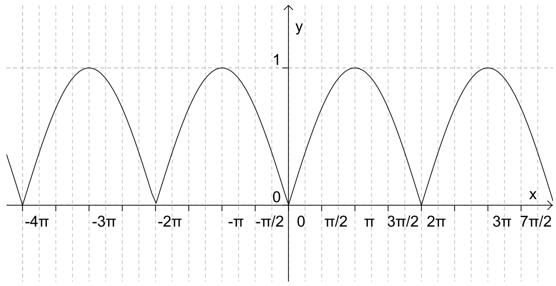
(g)



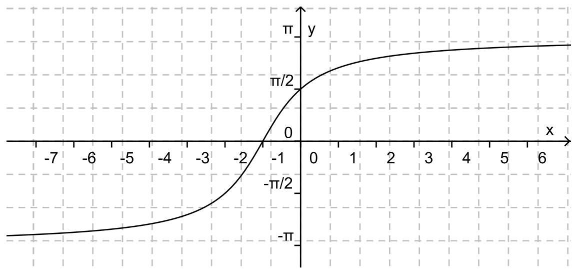
(h)



(i)



(j)



13. (a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{4}$ (c) π

14. Queremos calcular $\cos(\operatorname{arcsen} x)$.

Considere $\theta = \operatorname{arcsen} x$.

Nesse caso, sabemos que

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad \cos\theta \geq 0, \quad x = \operatorname{sen}\theta.$

Queremos calcular $\cos\theta$. Mas,

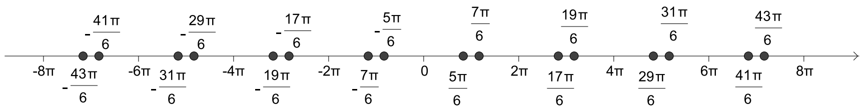
$\cos^2\theta = 1 - \operatorname{sen}^2\theta \Longrightarrow \cos\theta = \pm\sqrt{1 - \operatorname{sen}^2\theta}.$

Como $\cos\theta \geq 0, \quad \cos\theta = \sqrt{1 - \operatorname{sen}^2\theta}$

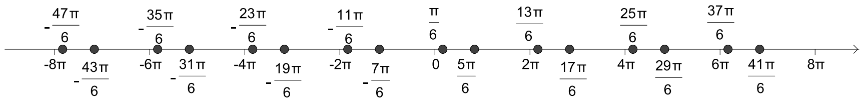
Como $x = \operatorname{sen}\theta, \quad \cos\theta = \sqrt{1 - x^2},$

Como $\theta = \operatorname{arcsen} x, \quad \cos(\operatorname{arcsen} x) = \sqrt{1 - x^2}.$

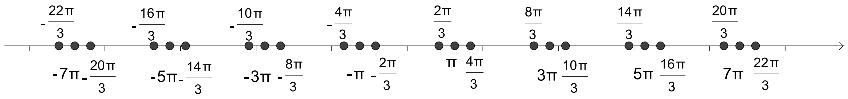
15. (a)



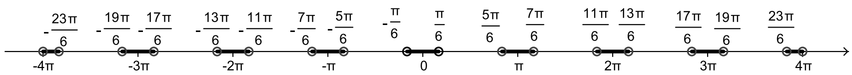
(b)



(c)



(d)



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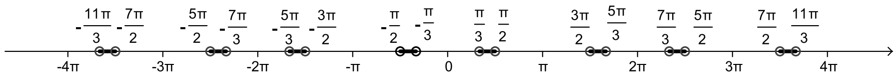
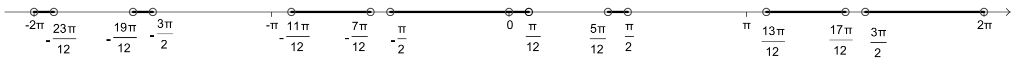
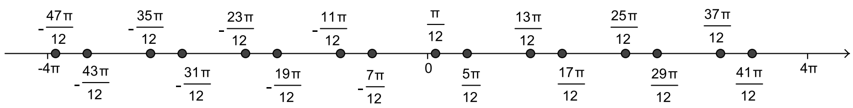


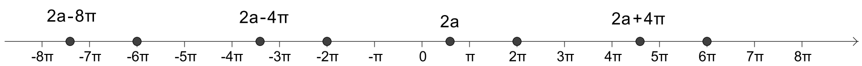
(f)



(g)



(h) $a = \arctan\left(\frac{4}{3}\right) \simeq 0,93$



(i)