Example 2.13 (Heat flux across a surface). The heat flow or heat flux across a surface can be calculated by setting $\boldsymbol{\Phi} = \boldsymbol{q}$, where $\boldsymbol{q}$ is the vector of heat per unit surface per unit time [W/m^2]. Thus, in direction $\boldsymbol{n}$,

$$\dot{Q} = \int_S \boldsymbol{q} \cdot \boldsymbol{n} \, \mathrm{d}S \tag{2.31}$$

Table 2.1. Examples of fluxes across a surface S.

Flux	Symbol	Units	Expression
Volumetric flux	Q	$\mathrm{m^3/s}$	$\int_S \boldsymbol{v} \cdot \boldsymbol{n} \, \mathrm{d}S$
Mass flux	$\dot{m}$, G	kg/s	$\int_S \rho \boldsymbol{v} \cdot \boldsymbol{n} \, \mathrm{d}S$
Flux of a property, with $\phi = $ prop./mass		[prop.]/s	$\int_S \rho \phi \cdot \boldsymbol{n} \, \mathrm{d}S$
Heat flux	$\dot{Q}$	W	$\int_S \boldsymbol{q} \cdot \boldsymbol{n} \, \mathrm{d}S$

Problems

2.1 Given the Eulerian fluid field

$$\boldsymbol{v}(x, y, z, t) = 3t\boldsymbol{i} + xz\boldsymbol{j} + ty^2\boldsymbol{k}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors along the coordinate axis, determine the flow acceleration.

2.2 A fluid field is described by

$$u = \frac{x}{1+t} \; ; \; v = \frac{y}{1+2t} \; ; \; w = 0$$

where t represents time. Calculate the streamlines and trajectories that pass by x_0, y_0 at $t = 0$ and the streakline that passes by x_0, y_0 $\forall t$.

2.3 Using polar coordinates, the fluid field in a tornado can be approximated as

$$\boldsymbol{v} = -\frac{a}{r}\boldsymbol{e}_r + \frac{b}{r}\boldsymbol{e}_\theta$$

where $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are the unit vectors in the directions r and θ. Show that the streamlines obey the logarithmic spiral equation

$$r = C\exp(-\frac{a}{b}\theta)$$

2.4 A two-dimensional transient velocity field is given by

$$u = 5x(1+t) \qquad v = 5y(-1+t)$$

where u is the x velocity component and v, the y component. Find:

(a) The trajectory $x(t)$, $y(t)$ if $x = x_0$, $y = y_0$ at $t = 0$. Is this a Lagrangian or Eulerian flow description?
(b) The streamlines that pass by x_0, y_0.
(c) The streakline that goes by x_0, y_0.

2.5 The ideal flow around a corner placed at the axis origin is given by

$$\begin{aligned} u_x &= ax \\ u_y &= -ay \end{aligned}$$

with a a constant. Determine the streamlines and draw the streamline that goes by the point $(1,1)$ indicating the flow direction for $a > 0$. Calculate the substantial derivative.

2.6 The velocity field in an irrotational vortex, like the ones present in cyclones, is given by

$$\begin{aligned} u_x &= -Ky/(x^2+y^2) \\ u_y &= Kx/(x^2+y^2) \end{aligned}$$

Determine the streamlines and draw a few of them.

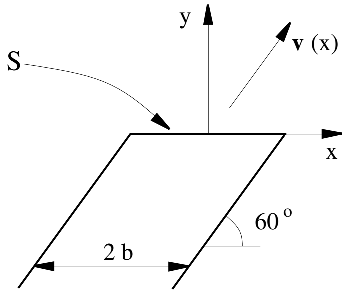
2.7 Check if the velocity field of the above exercise can be expressed in polar coordinates as

$$\begin{aligned} u_r &= 0 \\ u_\theta &= K/r \end{aligned}$$

and calculate the streamlines in polar coordinates.

2.8 Calculate the volumetric and mass flow rate, Q and $\dot{m}$, respectively, across the slit S of width w shown in the Figure, where the velocity vector has an angle of 60° with the x axis and its magnitude is given by $v = v_0(b^2 - x^2)/b^2$. The fluid density is ρ.

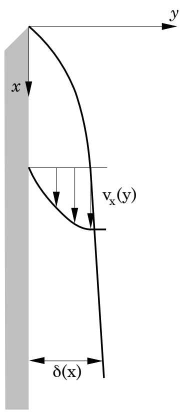
2.9 In film condensation along a vertical plate in a vapor atmosphere (see the Figure), Nusselt found out that in laminar flow the velocity profile at a station x is



Problem 2.8. Slit geometry.

$$v_x(y) = \frac{(\rho_l - \rho_v)g\delta^2}{\mu}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \qquad 0 \le y \le \delta(x)$$

where ρ_l and ρ_v are the density of the fluid in the liquid and vapor phase, respectively, and μ the liquid density. Find the volumetric flow rate per unit width at any value of x.



Problem 2.9. Film condensation on a vertical plate.

2.10 Write the expression of the kinetic energy flux across a surface S.