Problems

9.1 Determine the dimensions of force F, stress σ, power $\dot{W}$, dynamic viscosity μ and thermal conductivity κ.

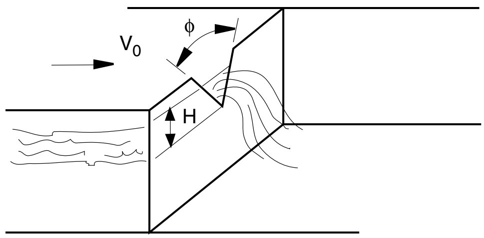
9.2 The variables which control the motion of a boat are the resistance force, F, speed V, length L, density of the liquid ρ and its viscosity μ, as well as gravity acceleration g. Obtain an expression for F using dimensional analysis.

9.3 It is believed that the power P of a fan depends upon the density of the liquid ρ, the volumetric flux Q, the diameter of the propeller D and the angular speed Ω. Using dimensional analysis, determine the dependence of P with respect to the other dimensionless variables.

9.4 In fuel injection systems, a jet of liquid breaks, forming small drops of fuel. The diameter of the resulting drops, d, supposedly depends upon the density of the liquid, the viscosity, surface tension, $[\sigma]$ = force/length, and likewise upon the speed of the stream V and its diameter D. How many dimensionless parameters are required to characterize the process? Find them.

9.5 A disc spins close to a fixed surface. The radius of the disc is R, and the space between the disk and the surface is filled with a liquid of viscosity μ. The distance between the disc and the surface is h and the disc spins at an angular velocity ω. Determine the functional relationship between the torque that acts upon the disc, T, and the other variables.

9.6 A triangular weir is made of a vertical plate with an opening in the shape of a "V" with an angle ϕ cut in the upper part and transversally placed in a channel. The liquid contained in the channel is retained by the plate and obliged to flow through the opening. The discharge flow Q is a function of the raising of the liquid from the vertex of the opening. Furthermore, Q depends upon the gravity and speed at which the flow nears the weir V_0. Determine the expression that will calculate Q. What would the previous expression become if the speed V_0 was not relevant to the problem?



Problem 9.6. A triangular weir can be set up to measure the volumetric flux in a channel.

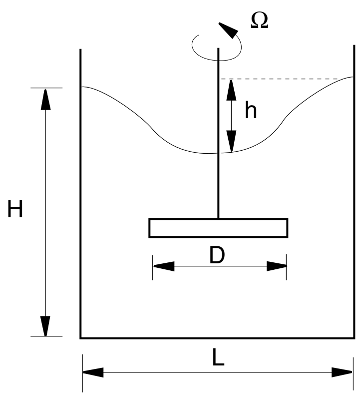
9.7 The drag force, F, experienced by a submarine that moves at a great depth from the surface of the water, is a function of the density ρ, viscosity μ, speed V and the transversal section of the submarine A. An expert suggests that the nondimensional relationship that allows the calculation of F is:

$$\frac{F}{\rho V^2 A} = f\left(\frac{\rho V A}{\mu}\right)$$

(a) Is the number of dimensionless parameters in the expression correct? Why?
(b) Are the parameters correct? If not, correct them.
(c) A geometrically similar model to that of the real submarine has been constructed, so that all the lengths of the model are 1/10 of those corresponding to the submarine. The model is tested in sea water.
  (1) The force of the real submarine when it moves at 5 m/s is to be determined.
  (2) At which speed should the model be tested?

9.8 An automobile must travel through standard air conditions at a speed of 100 km/h. To determine the pressure distribution, a model at a scale of 1/5 of the length of the vehicle is tested in water. Find the speed of water to be used.
$\mu_{\text{water}} = 10^{-3}$ kg/(m s), $\rho_{\text{water}} = 1\,000$ kg/m^3, $\mu_{\text{air}} = 1.8 \times 10^{-5}$ kg/(m s), $\rho_{\text{air}} = 1.2$ kg/m^3.

9.9 The depth of the steady central vortex h in a large tank of oil being stirred by a propeller needs to be predicted. One way is to carry out a study using a reduced scale model. Determine the conditions under which the experiment should be conducted to be considered a valid predictive tool. Note: Consider gh a function of gH, D, L and Ω.



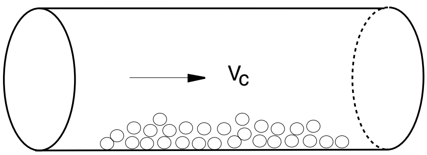
Problem 9.9. Stirring a liquid in a tank produces a vortex.

9.10 A rectangular, thin, flat plate, with dimensions of h (length) and w (width) is placed perpendicularly to a liquid current. Imagine that the drag force D which the liquid has upon the plate is a function of w and h, the density of the liquid ρ and its viscosity μ, as well as the speed V of the liquid coming towards the plate. Determine the set of dimensionless parameters to study the problem experimentally.

9.11 The Reynolds number is a very important parameter for studying transport phenomena and fluid mechanics. Estimate the Reynolds number that would be characteristic of the flow around a car traveling along the highway.

9.12 A thin layer of spherical particles are lying at the bottom of a horizontal tube, as indicated in the Figure. When an incompressible liquid flows along the tube, it can be seen that at a certain critical speed the particles move and are carried along the length of the tube. We wish to study the value of this critical speed V_c. Suppose that V_c is a function of the diameter of the tube D, the particle's diameter D_p, the liquid density ρ, the viscosity of the liquid μ, the density of the particles ρ_p and the gravity acceleration g.

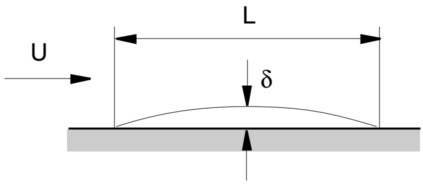
(a) Using ρ, D and g as fundamental variables, obtain the dimensionless parameters of the problem.
(b) Repeat point (a) using ρ, D and μ as fundamental variables.
(c) A laboratory experiment is carried out with the same liquid and particles as the real prototype but at half the size. If a critical speed of 1 m/s is measured, what is the value of the critical speed for the real prototype in cases (a) and (b)? What is happening?
(d) Consider how this problem can be solved and calculate the critical speed in the prototype to get the critical speed of 1 m/s for the model. Which are the properties of the liquid to be used in the experiment?



Problem 9.12. Many industrial methods are based upon passing a liquid current over solid spheres.

9.13 During the drying process of a fine layer of liquid on a surface, the liquid evaporates and the vapor is transported in the air above the surface, as can been seen in the Figure. We are interested in knowing the dependence of the drying time t upon the rest of the variables of the problem (length L, thickness of the layer δ, the liquid's vapor pressure P_v, air speed U, viscosity μ and air density ρ).

(a) Obtain a set of dimensionless variables related to the drying time t with the rest of the variables.
(b) We wish to set up a laboratory experiment to determine the drying time of a soccer field where $P_v = 2\,000$ Pa, $L = 100$ m, $\delta = 0.01$ m and $U = 2$ m/s. In the experiment, the viscosity and the density of the air will be the same as that of the soccer field, but L will be worth 20 m (we don't have a larger laboratory available). Calculate the values of U, δ and P_v in the experiment so that complete similarity exists with the real flow.
(c) If in the experiment the average drying time is $t = 10$ min, calculate the drying time of the soccer field.



Problem 9.13. Drying process of a liquid sheet.