13

Self Evaluation Exercises

Problems

13.1 An axle of diameter d turns inside a fixed concentric bearing of diameter D. The length of the device is L. The space between the axle and the bearing is filled with an oil of viscosity μ. The axle turns at an angular velocity ω so that in the steady state the fluid velocity has only a tangential direction $\boldsymbol{e}_\theta$ and it is a quadratic function of the radius, with a minimum where the velocity is zero.

(a) How much is the heat per unit time $\dot{Q}$ to be eliminated from the device so the fluid is maintained at constant temperature?
(b) If the device is isolated so $\dot{Q} = 0$, assuming the equation of state $\mathrm{d}e = c_v \mathrm{d}T$, what is the rate of variation of the temperature ?

13.2 Given the two-dimensional velocity field

$$\boldsymbol{v} = \left\{ \begin{array}{c} 5y^2 \\ 3y - 3 \end{array} \right\}$$

(a) Calculate the divergence $\boldsymbol{\nabla} \cdot \boldsymbol{v}$.
(b) Is the flow compressible or incompressible? Why?
(c) Determine the viscous dissipation function ϕ_v.

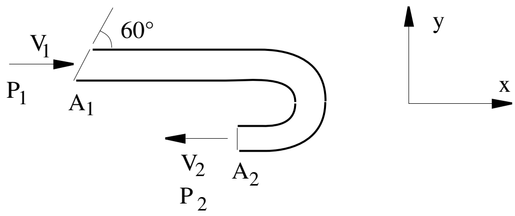
13.3 An approximate method to scale cylindrical stirring tanks for liquids consists of maintaining the power per unit volume $p_v = P/V$ constant. It is considered that the agitation power P is a function of the diameter of the agitator D, its angular velocity ω and the liquid density ρ.

(a) Determine the dimensionless relation of P with respect to the other dimensionless variables.
(b) It is desired to increase the tank volume by 3. What is the scale factor of the diameter and the agitator?

(c) What is the power and the angular velocity for the new tank?

Note. Assume that the tanks are geometrically similar and the flow is turbulent.

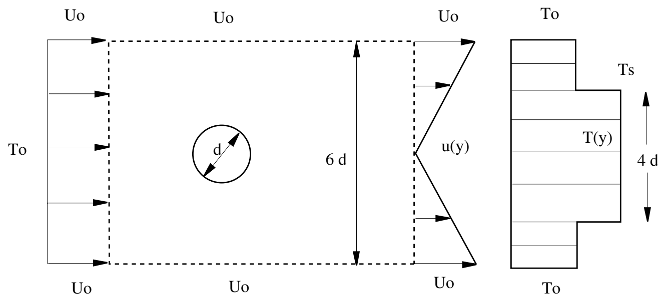
13.4 Determine the horizontal and vertical forces to fix the elbow of the Figure.



Problem 13.4. Force to hold a 180° elbow.

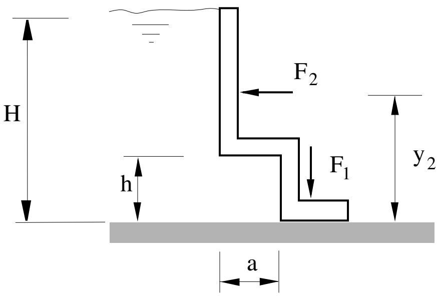
13.5 The Figure shows the velocity v_x and temperature T profiles around a cylinder. The ambient pressure far from the cylinder can be taken constant and equal to 0. If the flow is steady and incompressible, determine the following variables.

(a) The mass flow $\dot{m}$ across the horizontal surfaces of the control volume.
(b) The force F_D necessary to keep the cylinder (of length L) fixed. Calculate the dimensionless friction coefficient, $C_f = \bar{\tau}_0/(\frac{1}{2}\rho U_0^2)$, where the mean stress is defined as $\bar{\tau}_0 = F_D/(2\pi DL)$.
(c) If $de = c_v dT$ and the cylinder is heated at a rate of $\dot{Q}$ calculate T_s for the temperature profiles shown in the Figure. Assume that c_v is constant.



Problem 13.5. Non-isothermal incompressible flow about a cylinder.

13.6 Calculate the vertical F_1 and horizontal F_2 net forces and the point of application y_2 to hold the wall of the tank.



Problem 13.6. Force to hold a tank wall.

13.7 Let a non-Newtonian fluid have the constitutive equation

$$\tau' = \mu \left| \frac{\mathrm{d}u}{\mathrm{d}y} \right|^n \qquad n > 1$$

(a) To what kind of non-Newtonian fluid does it correspond?
(b) If the velocity profile near a solid wall can be expressed as

$$u(y) = a_1 y + a_2 y^2$$

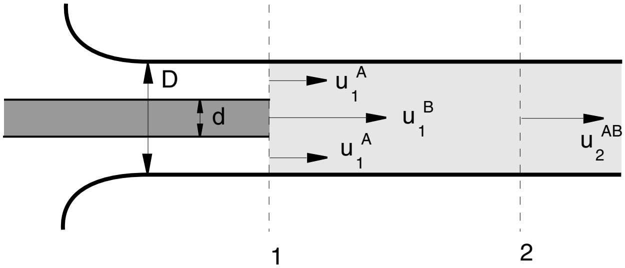
where y is the coordinate orthogonal to the wall, determine τ'_0, the viscous stress at the wall.

13.8 A circular container of 230 mm diameter filled with water at ambient temperature loses mass at a rate of 1.5×10^{-5} kg/s when the ambient is dry and at 22 °C.

(a) Determine the mass transfer coefficient.
(b) Calculate the total heat (by convection and evaporation) which is lost when the ambient air has a relative humidity of 50% and the water temperature is 37 °C.
Gas constant: $R = 8.314$ kJ/(kmol K). *Water properties:* $D_{AB} = 2.3 \times 10^{-5}\mathrm{m}^2/\mathrm{s}$; latent heat of vaporization $L = 2\,257$ kJ/kg. *Air properties:* $\rho = 1.2$ kg/m^3; $\mu = 1.82 \times 10^{-5}$ kg/(m s); $\kappa = 0.026$ W/(m K); Pr $= 0.7$.

13.9 The Figure of the problem shows a common technique to disperse fluid B into fluid A to form the solution AB. The technique consists of mixing both substances through concentric pipes.

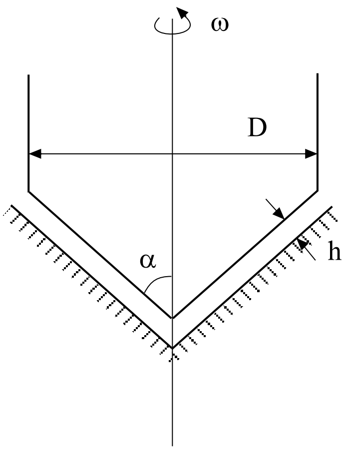
(a) Calculate u_2^{AB} and the pressure loss assuming negligible friction forces and equal densities for all the fluids. Characterize the result for $D = 10$ cm, $d = 2$ cm, $\rho_A = \rho_B = \rho_{AB} = 1$ gr/cm^3, $u_1^A = 1.5$ m/s, $u_1^B = 4.0$ m/s.
(b) Obtain the pressure loss for the case of different densities as a function of d/D, ρ_A/ρ_B, u_1^A/u_1^B and $\rho_A u_1^A/\rho_B u_1^B$.



Problem 13.9. Mixing process through concentric pipes.

(c) What can be concluded from the expression obtained in (b)?

13.10 The conic pivot of the Figure spins at an angular velocity ω and rests over a thin layer of oil with thickness h. Determine the moment due to viscous friction as a function of the angle α, the viscosity μ, the angular velocity, the thickness h and the diameter of the axle D.

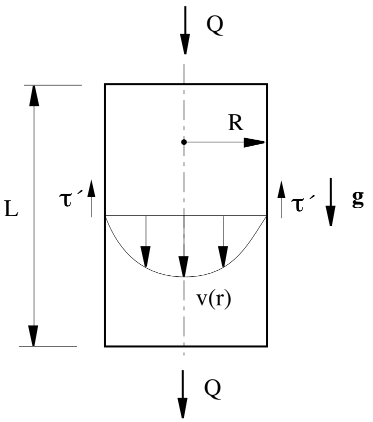


Problem 13.10. Conic bearing to support axial and radial forces.

13.11 From a vertical tube of length L and radius R, a fluid of density ρ and viscosity μ falls. Assuming that the velocity profile is steady, fully developed and parabolic,

$$v(r) = \frac{2Q}{\pi R^4}\,(R^2 - r^2)$$

and that the gravity acts downwards, determine the outgoing volumetric flux Q.



Problem 13.11. Fall of a fluid in a pipe due to gravity.

13.12 The evolution of small perturbations in a fluid can be modeled by the Orr-Sommerfeld equation

$$\nu \frac{\mathrm{d}^4\phi(y)}{\mathrm{d}y^4} - \left[\omega + 2\nu k^2\right] \frac{\mathrm{d}^2\phi(y)}{\mathrm{d}y^2} + k^2 \left[\omega + \nu k^2\right] \phi(y) = 0$$

where ϕ is the stream function [m^2/s], ω the angular velocity of the wave [1/s], k the wavenumber [1/m] and $\nu = \mu/\rho$, the kinematic viscosity. Using ρ, μ, h, U, make the Orr-Sommerfeld equation dimensionless.

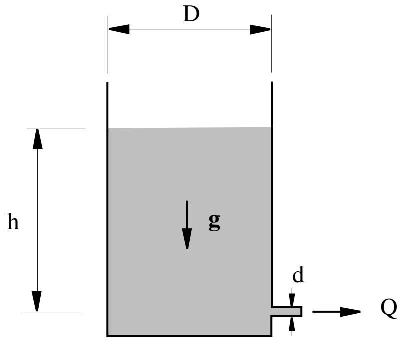
13.13 The time t to discharge a tank depends approximately on its diameter D, the liquid level h, the diameter of the outlet orifice d, the acceleration of the gravity g and the fluid density ρ.

(a) Determine the dimensionless relation for the discharge time.
(b) If a tank is made at a scale four times smaller, what is the discharge time? How much should h be for dimensional analysis to apply?
(c) If the fluid is changed, what is the time of discharge? Justify the answer.
(d) In this section, the viscous effects μ are taken into account. What is the new dimensionless number that appears in the nondimensional relation? Is it possible to have complete similarity when both, the geometric scale of the tank and the fluid are modified? Why?

13.14 A porous cylinder of unknown surface is saturated with water. Dry air is blown perpendicularly to the cylinder at a pressure of 1 atm and velocity 10 m/s, so the air gets humid. The water evaporation rate is 1.684×10^{-5} kg/s and the heat transport coefficient is given by

$$\mathrm{Nu}_D = C\ \mathrm{Re}_D^m\ \mathrm{Pr}^{1/3} \qquad \text{where } C = 0.193 \text{ and } m = 0.618$$

Calculate the surface of the cylinder assuming that both the water and air are at 310 K and that the cylinder diameter is 0.045 m.



Problem 13.13. Draining of a tank through an orifice. Dimensional analysis.

Data for air: $\rho = 1.2$ kg/m^3; $\mu = 1.82 \times 10^{-5}$ kg/(m s); $D_a = 2.3 \times 10^{-5}$ m^2/s; Pr = 0.7. *Gas constant:* $R = 8.314$ kJ/(kmol K). *Water vapor properties:* $M_a = 18$ kg/kmol

13.15 Given the two-dimensional velocity field

$$\boldsymbol{v} = \left\{ \begin{array}{c} -2x^2 t \\ 6yt \end{array} \right\}$$

determine the equation of the streakline that passes by the point (x_0, y_0).

13.16 A planet in formation is made of a fluid of constant density ρ. If at the free surface, located at $r = R$, there is atmospheric pressure p_{atm} and the radial body forces are $f_m = -\frac{4\pi K}{3} r$, determine the hydrostatic pressure distribution $p(r)$ inside the planet.

13.17 In a wind tunnel there is a uniform air flow of 7 m/s (kinematic viscosity $\nu = \mu/\rho = 1.5 \times 10^{-5}$ m^2/s) at 295 K. Aligned with the flow, there is a 4 m long rectangular container, filled with water to a height of 1 cm. If the vapor pressure at the ambient conditions is 2000 Pa and the water is at the air temperature, calculate the time to evaporate the water in the container. The global mass transfer coefficient can be approximated by

$$\overline{\text{Sh}_L} = 0.664\ \text{Re}_L^{1/2}\text{Sc}^{1/3}$$

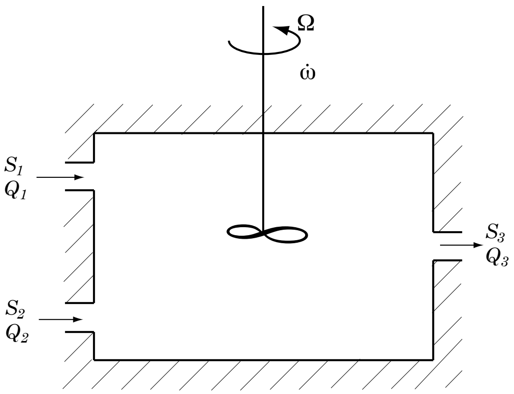
Gas constant: $R = 8\,314.$ J/(kmol K). *Mass diffusivity of water in air:* $D_A = 2.5 \times 10^{-5}$m^2/s.

13.18 The impulsion power of a hydraulic pump is frequently expressed as a function of energy head H [m]. It can be shown that gH (with g the gravity acceleration) depends on the fluid density, ρ [kg/m^3], and viscosity μ [kg/(m s)], the pump angular velocity of rotation N [rad/s], the runner diameter D [m], the volumetric flow rate Q [m^3/s] and the characteristic roughness length ϵ [m].

(a) Determine the characteristic curve gH in dimensionless form, using as fundamental variables ρ, N and D.
(b) Assume that the dissipation effects are negligible, that is, ignore the variables μ and ϵ. In this new situation, called partial similarity, what is the dimensionless relation?
(c) Assume a pump with characteristic curve $H(Q) = 20 - 0.1Q^2$. Assuming the partial similarity of (b), what would the new characteristic curve of the pump $H'(Q')$ be if the rotation speed was doubled?
(d) Again, neglecting the dissipation effects, what is the new characteristic curve if only the fluid density is modified?

13.19 The Figure shows a two-dimensional adiabatic mixing tank. If the flow is steady and incompressible (with density ρ), answer the following questions.

(a) Calculate the exit volumetric flow rate Q_3.
(b) Given p_1, p_2 and p_3, employ the mechanical energy equation to determine the viscous dissipation in the tank D_v.
(c) As a function of the inlet temperatures, T_1, T_2, and the specific heat at constant volume c_v (which can be assumed constant), calculate the exit temperature T_3.



Problem 13.19. Mixing tank.

13.20 A vertical solar panel is $L = 1$ m tall and $w = 2$ m wide. The local Nusselt number follows the correlation

$$\mathrm{Nu}_x = C\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} \qquad C = 0.56$$

where the Grashof number is defined as

$$\mathrm{Gr}_L = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2}$$

with β the expansion coefficient and ΔT the temperature difference between the panel and the environment. Determine the correlation for the global Nusselt number Nu_L.