Lista 1 de EDO
2^o semestre de 2013
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Sequências Numéricas
Séries Numéricas

1. Determine o termo geral das seguintes sequências.

   (a) $0, 2, 0, 2, 0, 2, \ldots$

   (b) $0, 1, 2, 0, 1, 2, 0, 1, 2, \ldots$

2. Calcule, caso exista, o limite das seguintes sequências de números reais.

   (a) $\dfrac{n^3 + 3n + 1}{4n^3 + 2}$;

   (b) $\sqrt{n+1} - \sqrt{n}$;

   (c) $\displaystyle\sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$;

   (d) $\displaystyle\int_1^n \frac{1}{x} dx$;

   (e) $\displaystyle\int_1^n e^{-sx} dx$, com $s > 0$;

   (f) $\displaystyle\int_0^n \frac{1}{1+x^2} dx$;

   (g) $\dfrac{n+1}{\sqrt[3]{n^7 + 2n + 1}}$;

   (h) $n \operatorname{sen} \dfrac{1}{n}$;

   (i) $\dfrac{1}{n} \operatorname{sen} n$;

3. Verifique se as seguintes séries são convergentes ou divergentes.

   (a) $\displaystyle\sum_{k=1}^{n} \frac{1}{\sqrt{k}}$;

(b) $\displaystyle\sum_{k=0}^{n} e^{-k}$;

(c) $\displaystyle\sum_{k=0}^{n} \frac{1}{k!}$;

(d) $\displaystyle\sum_{k=0}^{n} \frac{1}{k^2+1}$;

(e) $\displaystyle\sum_{k=2}^{n} \frac{1}{\ln k}$ (Sugestão: Verifique que $\ln k < k$ para $k \geq 2$);

4. Calcule as seguintes somas.

(a) $\displaystyle\sum_{k=0}^{+\infty} \frac{1}{3^k}$;

(b) $\displaystyle\sum_{k=0}^{+\infty} e^{-k}$;

(c) $\displaystyle\sum_{k=0}^{+\infty} (1+(-1)^k)$;

(d) $\displaystyle\sum_{k=0}^{+\infty} \frac{1}{(4k+1)(4k+5)}$;

(e) $\displaystyle\sum_{k=0}^{+\infty} \frac{1}{k(k+1)(k+2)(k+3)}$;

(f) $\displaystyle\sum_{k=0}^{+\infty} \frac{1}{(4k+1)(4k+3)}$;

5. Mostre que $\displaystyle\sum_{k=0}^{+\infty} \frac{1}{(4k+1)(4k+3)(4k+5)} = \frac{\pi-2}{16}$.

6. Mostre que $\displaystyle\sum_{k=1}^{+\infty} (-1)^{k+1}\frac{\alpha^k}{k} = \ln(1+\alpha)$, com $0 < \alpha \leq 1$, e calcule as seguintes somas.

(a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$;

(b) $\frac{1}{2} - \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} - \frac{1}{4 \cdot 2^4} + \cdots$;

7. Mostre que $-\sum_{k=1}^{+\infty} \frac{\alpha^k}{k} = \ln(1-\alpha)$, com $0 < \alpha < 1$, e calcule $\sum_{k=1}^{+\infty} \frac{1}{k2^k}$.

Respostas

1. (a) $a_n = 1 + (-1)^n$, $n \geq 0$;
   (b)
   (c) $a_n = \frac{n-(-1)^{n+1}}{n}$, $n \geq 1$;
2. (a) $1/4$;
   (b) 0;
   (c) 2;
   (d) $\nexists$
   (e) 0
   (f) $\pi/2$;
   (g) 0
   (h) 1;
   (i) 0.
3. (a) Divergente;
   (b) Convergente;
   (c) Convergente;
   (d) Convergente;
   (e) Divergente para $+\infty$;
4. (a) $3/2$;
   (b) $\frac{e}{e-1}$;
   (c) $+\infty$;
   (d) $\frac{1}{4} = \frac{1}{4} \sum_{k=0}^{+\infty} (b_k - b_{k+1})$, onde $b_k = \frac{1}{4k+1}$;

(e) $\frac{1}{18} = \frac{1}{3}\sum_{k=1}^{+\infty}(b_k - b_{k+1})$, onde $b_k = \frac{1}{k(k+1)(k+2)}$;

(f) $\frac{\pi}{8} = \frac{1}{2}\sum_{k=0}^{+\infty}(\frac{1}{4k+1} - \frac{1}{4k+3}) = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$;

5. $\frac{1}{2}\sum_{k=0}^{+\infty}\left(\frac{1}{(4k+1)(4k+3)} - \frac{1}{(4k+1)(4k+5)}\right) = \frac{\pi - 2}{16}$;

6. (a) $\ln 2$;

   (b) $\ln \frac{3}{2}$;

7. $\ln 2 = -\ln \frac{1}{2} = -\ln(1 - \frac{1}{2}) = \sum_{k=0}^{+\infty} \frac{1}{k2^k}$.