Lista 3 de EDO
2^o semestre de 2013
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Raio de Convergência
Série de Taylor

1. Determine o o raio de convergência das seguintes séries de potências.

   (a) $\displaystyle\sum_{k=0}^{+\infty}(x-3)^n;$

   (b) $\displaystyle\sum_{k=0}^{+\infty}\frac{n}{2^n}x^n;$

   (c) $\displaystyle\sum_{k=0}^{+\infty}\frac{x^{2n}}{n!};$

   (d) $\displaystyle\sum_{k=0}^{+\infty}2^n x^n;$

   (e) $\displaystyle\sum_{k=1}^{+\infty}\frac{(2x+1)^n}{n^2};$

   (f) $\displaystyle\sum_{k=1}^{+\infty}\frac{(x-x_0)^n}{n};$

   (g) $\displaystyle\sum_{k=1}^{+\infty}\frac{(-1)^n n^2(x+2)^n}{3^n};$

   (h) $\displaystyle\sum_{k=1}^{+\infty}\frac{n!x^n}{n^n};$

2. Determine a série de Taylor, em torno de x_0, das seguintes funções.

   (a) $\mathrm{sen} x$, $x_0=0$;
   (b) x^2, $x_0=-1$;
   (c) $\ln x$, $x_0=1$;
   (d) $\frac{1}{1+x}$, $x_0=0$;

(e) $\frac{1}{1-x}$, $x_0 = 2$;

3. Dada $y = \sum_{k=0}^{+\infty} nx^n$, escreva o termo geral de y' e de y''.

4. Dada $y = \sum_{k=0}^{+\infty} a_n x^n$, mostre que se $y'' = y$, então

$$a_{n+2} = a_n/(n+2)(n+1).$$

A partir desta igualdade, determine a_{2n} em função de a_0 e a_{2n+1} em função de a_1

5. Reescreva a expressão dada, escrevendo a série cujo termo geral envolve x^n.

   (a) $\sum_{k=2}^{+\infty} n(n-1)a_n x^{n-2}$;

   (b) $\sum_{k=0}^{+\infty} a_n x^{n+2}$;

   (c) $x \sum_{k=1}^{+\infty} n a_n x^{n-1} + \sum_{k=0}^{+\infty} a_k x^k$;

   (d) $(1-x^2) \sum_{k=2}^{+\infty} n(n-1)a_n x^{n-2}$;

Respostas

1. (a) 1;
   (b) 2;
   (c) ∞;
   (d) $\frac{1}{2}$;
   (e) $\frac{1}{2}$;
   (f) 1;
   (g) 3;

(h) e;

2. (a) $\displaystyle\sum_{n=0}^{+\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$, $R = \infty$;

   (b) $1 - 2(x+1) + (x+1)^2$, $R = \infty$;

   (c) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$, $R = 1$;

   (d) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n x^n$, $R = 1$;

   (e) $\displaystyle\sum_{n=0}^{+\infty} (-1)^{n+1}(x-2)^n$, $R = 1$;

3. $(n+1)^2 x^n$ e $(n+2)^2(n+1)x^n$.

4. $y' = \displaystyle\sum_{n=0}^{+\infty} (n+1)a_{n+1}x^n$ e $y'' = \displaystyle\sum_{n=0}^{+\infty} (n+2)(n+1)a_{n+2}x^n$.

5. (a) $\displaystyle\sum_{n=0}^{+\infty} (n+2)(n+1)a_{n+2}x^n$;

   (b) $\displaystyle\sum_{n=2}^{+\infty} a_{n-2}x^n$;

   (c) $\displaystyle\sum_{n=0}^{+\infty} (n+1)a_n x^n$;

   (d) $\displaystyle\sum_{n=0}^{+\infty} ((n+2)(n+1)a_{n+2} - n(n-1)a_n)x^n$;