Lista de Exercícios

Transformada de Laplace

Nos exercícios 1 a 7, usando a definição de Transformada de Laplace, mostre que:

1. $\mathcal{L}\{1\} = \dfrac{1}{s}$, $s > 0$
2. $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$, $s > a$
3. $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$, $s > 0$, $n \in \mathbb{N}$
4. $\mathcal{L}\{\operatorname{sen}(at)\} = \dfrac{a}{s^2+a^2}$, $s > 0$
5. $\mathcal{L}\{\cos(at)\} = \dfrac{s}{s^2+a^2}$, $s > 0$
6. $\mathcal{L}\{\operatorname{senh}(at)\} = \dfrac{a}{s^2-a^2}$, $s > a$
7. $\mathcal{L}\{\cosh(at)\} = \dfrac{s}{s^2-a^2}$, $s > a$
8. Sabendo que $\displaystyle\int_0^{+\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$, mostre que $\mathcal{L}\{t^{-1/2}\} = \sqrt{\dfrac{\pi}{s}}$, $s > 0$.

Nos exercícios 9 a 14, calcule as transformadas de Laplace.

9. $\mathcal{L}\{e^{3t}\cos(2t)\}$
10. $\mathcal{L}\{\cos^2(at)\}$, $a \in \mathbb{R}$
11. $\mathcal{L}\{\operatorname{sen}(5t)\cos(2t)\}$
12. $\mathcal{L}\{t^2 \operatorname{sen}(t)\}$
13. $\mathcal{L}\{t^3 e^{-t}\}$
14. $\mathcal{L}\{te^{at}\cos(bt)\}$, $a, b \in \mathbb{R}$

Nos exercícios 15 a 17, usando propriedades das transformadas de Laplace, calcule:

15. $\mathcal{L}\{e^t \operatorname{sen}(3t)\}$
16. $\mathcal{L}\{t^5 e^{4t}\}$
17. $\mathcal{L}\left\{e^t \dfrac{d^{100}}{dt^{100}}\left(e^{-t}t^{100}\right)\right\}$
18. Determine a transformada de Laplace $\mathcal{L}\{t^{5/2}\}$.

Mostre que $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{+\infty} F(u)\,du$ e nos exercícios 19 a 21, calcule as transformadas de Laplace.

19. $\mathcal{L}\left\{\dfrac{\operatorname{sen}(t)}{t}\right\}$
20. $\mathcal{L}\left\{\dfrac{\cos(at)-1}{t}\right\}, a \in \mathbb{R}$
21. $\mathcal{L}\left\{\dfrac{e^{at}-e^{-at}}{t}\right\}, a \in \mathbb{R}$

Nos exercícios 22 a 27, calcule as transformadas de Laplace inversa.

22. $\mathcal{L}^{-1}\left\{(s-2)^{-2}\right\}$
23. $\mathcal{L}^{-1}\left\{\dfrac{7}{(s-1)^3} + \dfrac{7}{(s+1)^2-4}\right\}$
24. $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+1)^2(s^2+1)}\right\}$
25. $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+16}\right\}$
26. $\mathcal{L}^{-1}\left\{\arctan\left(\dfrac{4}{s}\right)\right\}$
27. $\mathcal{L}^{-1}\left\{\dfrac{1}{((s^2+1)^2}\right\}$

Nos exercícios 28 a 39, use a transformada de Laplace para resolver os seguintes problemas de valor inicial em $[0, +\infty)$

28. $\begin{cases} y'' + 4y' + 4y = e^{-t} \\ y(0) = 0,\ y'(0) = 1 \end{cases}$

29. $\begin{cases} y'' + 4y' + 3y = 0 \\ y(0) = 0,\ y'(0) = 1 \end{cases}$

30. $\begin{cases} y'' + 6y' - 7y = 0 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

31. $\begin{cases} y'' - y' - 2y = t \\ y(0) = 0,\ y'(0) = 0 \end{cases}$

32. $\begin{cases} y^{iv} - k^4 y = 0 \\ y(0) = 1,\ y'(0) = y''(0) = y'''(0) = 0 \end{cases}$

33. $\begin{cases} y'' - 2y' + 5y = 0 \\ y(0) = 0,\ y'(0) = 1 \end{cases}$

34. $\begin{cases} y'' - 9y' = 5e^{-2t} \\ y(0) = 1,\ y'(0) = 2 \end{cases}$

35. $\begin{cases} y'' - 5y' + 6y = 3e^{3t} \\ y(0) = 0,\ y'(0) = 0 \end{cases}$

36. $\begin{cases} y'' + 4y = 9t \\ y(0) = 0,\ y'(0) = 7 \end{cases}$

37. $\begin{cases} y'' + y = \cos t \\ y(0) = 0,\ y'(0) = -1 \end{cases}$

38. $\begin{cases} y''' - 4y' = \operatorname{senh} t \\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 0 \end{cases}$

39. $\begin{cases} y'' + y' - 2y = 5e^{-t} \operatorname{sen}(2t) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

RESPOSTAS DA LISTA de Transformada de Laplace

9. $F(s) = \frac{s-3}{(s-3)^2+4}$

10. $F(s) = \frac{1}{2s} + \frac{s}{2(s^2+4a)^2}$

11. $F(s) = \frac{1}{2}\left(\frac{7}{s^2+49} + \frac{3}{s^2+9}\right)$

12. $F(s) = \frac{6s^2-2}{(s^2+1)^3}$

13. $F(s) = \frac{6}{(s+1)^4}$

14. $F(s) = \frac{(s-a)^2-b^2}{[(s-a)^2+b^2]^2}$

15. $F(s) = \frac{3}{(s-1)^2+9}$

16. $F(s) = \frac{5!}{(s-4)^6}$

17. $F(s) = \frac{100!(s-1)^{100}}{s^{101}}$

18. $F(s) = \frac{15}{8}\sqrt{\pi}\frac{1}{s^{7/2}}$

19. $F(s) = \frac{\pi}{2} - \arctan s,\ s > 0$

20. $F(s) = -\ln\frac{\sqrt{s^2+a^2}}{s},\ s > 0$

21. $F(s) = \ln\left|\frac{s+a}{s-a}\right|,\ s > |a|$

22. $f(t) = te^{2t}$

23. $f(t) = \frac{7}{2}t^2e^t + \frac{1}{2}e^{-t}\sinh(2t)$

24. $f(t) = \frac{1}{2}(\operatorname{sen}(t) - te^{-t})$

25. $f(t) = \begin{cases} 0 & , 0 \le t < \pi \\ \frac{1}{4}\operatorname{sen}(4t) & , t \ge \pi \end{cases}$

26. $f(t) = \frac{1}{t}\operatorname{sen}(4t)$

27. $f(t) = \frac{1}{2}(\operatorname{sen} t - t\cos t)$

28. $y(t) = e^{-t} - e^{-2t}$

29. $y(t) = \frac{1}{2}\left(e^{-t} - e^{-3t}\right)$

30. $y(t) = \frac{1}{6}\left(7 - \cos(\sqrt{6}t)\right)$

31. $y(t) = \frac{1}{4} - \frac{1}{2}t - \frac{1}{3}e^{-t} + \frac{1}{12}e^{2t}$

32. $y(t) = \frac{1}{4}e^{kt} + \frac{1}{4}e^{-kt} + \frac{1}{2}\cos(kt)$

33. $y(t) = \frac{1}{2}e^t\operatorname{sen}(2t)$

34. $y(t) = \frac{1}{2} + \frac{3}{11}e^{9t} + \frac{5}{22}e^{-2t}$

35. $y(t) = 3te^{3t} - 3e^{3t} + 3e^{2t}$

36. $y(t) = \frac{9}{4}t + \frac{19}{8}\operatorname{sen}(2t)$

37. $y(t) = -\operatorname{sen} t + \frac{1}{2}t\operatorname{sen} t$

38. $y(t) = \frac{1}{4} - \frac{1}{3}\cosh t\frac{1}{12}\cosh(2t)$

39. $y(t) = \frac{13}{12}e^t - \frac{1}{3}e^{-2t} + \frac{1}{4}e^{-t}\cos(2t) - \frac{3}{4}e^{-t}\operatorname{sen}(2t)$