GABARITO - TESTE 3/10/13

(1) $F(s) = \dfrac{6s^2 - 12}{s^3 + s^2 - 4s - 4}$ $\boxed{p = -1 ; \pm 2}$

$$F(s) = \frac{6s^2 - 12}{(s+1)(s+2)(s-2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s-2)}$$

Por Heaviside:

$$A = \left.\frac{6s^2 - 12}{(s+2)(s-2)}\right|_{s=-1} = \frac{-6}{-3} = 2$$

$$B = \left.\frac{6s^2 - 12}{(s+1)(s-2)}\right|_{s=-2} = \frac{12}{4} = 3$$

$$C = \left.\frac{6s^2 - 12}{(s+1)(s+2)}\right|_{s=2} = \frac{12}{12} = 1$$

$$F(s) = \frac{2}{(s+1)} + \frac{3}{(s+2)} + \frac{1}{(s-2)}$$

$$\boxed{F(t) = 2e^{-t} + 3e^{-2t} + e^{2t}} \quad //$$

Por igualdade dos coeficientes:

$$6s^2 - 12 = A[(s+2)(s-2)] + B[(s+1)(s-2)] + C[(s+1)(s+2)]$$

$$6s^2 - 12 = A[s^2 - 4] + B[s^2 - s - 2] + C[s^2 + 3s + 2]$$

$$\begin{cases} (A + B + C) = 6 \Rightarrow A = 6 - B - C = 6 - 3C - C = 6 - 4C \\ -B + 3C = 0 \Rightarrow B = 3C \\ -4A - 2B + 2C = -12 \end{cases}$$

$$-4(6-4C) - 2(3C) + 2C = -12$$

$$-24 + 16C - 6C + 2C = -12$$

$$12C = 12$$

$$\boxed{C = 1}$$

$$B = 3C$$

$$\boxed{B = 3}$$

$$A = 6 - 4C$$

$$\boxed{A = 2}$$

$$F(s) = \frac{2}{(s+1)} + \frac{3}{(s+2)} + \frac{1}{(s-2)}$$

$$\boxed{F(t) = 2e^{-t} + 3e^{-2t} + e^{2t}}$$

(2) $m = 2$ $\qquad Y(0) = 0$

$L = 4$ $\qquad Y'(0) = 0$

$k = 10$

$f(t) = \delta(t) \rightarrow$ função impulso (não tem intervalo tempo).

$$m\,y''(t) + L\,y'(t) + k\,y(t) = f(t)$$

$$2y''(t) + 4y'(t) + 10y(t) = \delta(t)$$

Fazendo laplace:

$$2[s^2Y(s) - \cancel{sY(0)}^{0} - \cancel{Y'(0)}^{0}] + 4[sY(s) - \cancel{Y(0)}^{0}] + 10Y(s) = 1$$

$$2s^2Y(s) + 4sY(s) + 10Y(s) = 1 \qquad (\div 2)$$

$$s^2Y(s) + 2sY(s) + 5Y(s) = 1/2$$

$$Y(s) = \frac{1/2}{s^2 + 2s + 5} = \frac{1/2}{(s+1)^2 + 4} = \frac{1}{2}\,\frac{1}{(s+1)^2 + 4}$$

Aplicando a Laplace inversa:

$$\boxed{Y(t) = \frac{1}{4}\,e^{-t}\,\text{sen}(2t)}$$