OBS: Que podemos dizer de $\int_a^{\infty} \frac{dx}{(x-c)^p}$? O MESMO!

EXERCÍCIOS:

- Encontrar os valor de p para os quais $\int_a^{\infty} \frac{dx}{x(\ln x)^p}$

$$\int_a^{\infty} \frac{dx}{x(\ln x)^p} = \lim_{b \to \infty} \int_a^b \frac{dx}{x(\ln x)^p} \underset{?}{=} \lim_{b\to\infty} \int_{\ln a}^{\ln b} \frac{du}{u^p} = \lim_{b\to\infty} \left. \frac{u^{-p+1}}{-p+1} \right|_{\ln a}^{\ln b} =$$

$u = \ln x$
$du = \frac{dx}{x}$

$$= \lim_{b\to\infty} \frac{(\ln b)^{-p+1}}{-p+1} - \frac{(\ln a)^{-p+1}}{-p+1} = \begin{cases} \infty & -p+1 > 0 \\ 0 & -p+1 < 0 \\ \infty & p = 1 \end{cases}$$

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- Volume do sólido girado pela rotação da região limitada por $f(x) = e^{-x}$ $x > 0$

$$V = \int_0^{\infty} A(x)dx = \int_0^{\infty} \pi e^{-2x} dx = \lim_{a\to\infty} \pi \int_0^a e^{-2x} dx = \lim_{a\to\infty} \left. -\pi \frac{e^{-2x}}{2} \right|_0^a =$$

$$= \lim_{a\to\infty} -\frac{\pi}{2} e^{-2a} + \frac{\pi}{2} = \frac{\pi}{2}$$

- Funções hiperbólicas:

$\operatorname{senh} x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Prop. das derivadas: $(\operatorname{senh} x)' = \cosh x$
$(\cosh x)' = \operatorname{senh} x$
$(\tanh x)' = \frac{1}{\cosh^2 x}$

$$\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}} = \lim_{a\to\infty} \int_a^0 \frac{dx}{e^x+e^{-x}} + \lim_{b\to+\infty} \int_0^b \frac{dx}{e^x+e^{-x}} = \lim_{a\to-\infty} \arctan 1 - \arctan \overset{\to 0}{e^a} + \lim_{b\to\infty} \arctan e^b - \arctan 1$$

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{dx}{e^{-x}(e^{2x}+1)} = \int \frac{e^x dx}{e^{2x}+1} = \int \frac{du}{u^2+1} = \arctan u + C = \arctan(e^x) + C$$

$$= -\arctan(0) + \frac{\pi}{2} = \frac{\pi}{2}$$

3) $\int_{-1}^{1} \frac{e^{1/x}}{x^2}\, dx = \underbrace{\int_{-1}^{0} \frac{e^{1/x}}{x^2}\, dx}_{A} + \int_{0}^{1} \frac{e^{1/x}}{x^2}\, dx$

(A) $\int_{-1}^{0} \frac{e^{1/x}}{x^2}\, dx = \lim_{a \to 0^-} \int_{-1}^{a} \frac{e^{1/x}}{x^2}\, dx = \lim_{a \to 0^-} -\int_{-1}^{-\infty} e^{u}\, du = \lim_{a \to 0} \int_{-\infty}^{-1} e^{u}\, du = e^{-1}$

$u = 1/x \quad du = -dx/x^2$

$-1 < x < 0^- \Rightarrow -1 < u < -\infty$

(2) $\int_{0}^{1} e^{1/x}/x^2\, dx = \lim_{a \to 0^+} \int_{a}^{1} \frac{e^{1/x}}{x^2}\, dx = \lim_{a \to 0^+} -\int_{1/a}^{1} e^{u}\, du = \lim_{a \to 0^+} \int_{1}^{1/a} e^{u}\, du = \infty$

$a < x < 1 \quad \frac{1}{a} < u < 1$

∴ A integral diverge.

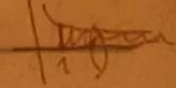
PROPRIEDADE $\int_{0}^{A} \frac{dx}{x^p}$ converge $\Leftrightarrow p < 1$

PROVA $p \neq 1$ $\int_{0}^{A} \frac{dx}{x^p} = \lim_{a \to 0^+} \int_{a}^{A} \frac{dx}{x^p} = \lim_{a \to 0^+} \frac{x^{-p+1}}{-p+1} \Big|_{a}^{A} = \lim_{a \to 0^+} \frac{A^{-p+1}}{-p+1} - \frac{a^{-p+1}}{-p+1} =$

$$= \begin{cases} \frac{A^{-p+1}}{-p+1} & -p+1 > 0 \Rightarrow 1 > p \text{ converge} \\ \infty & -p+1 < 0 \quad 1 < p \text{ diverge} \end{cases}$$

Se $p = 1$ $\int_{0}^{A} \frac{dx}{x}$ diverge.

Ex: Calcular o sólido de revolução gerado pela rotação da região limitada por $f(x) = 1/x^2$ $x > 1$ e o eixo x.



$V = \int_{1}^{\infty} A(x)\, dx = \lim_{a \to \infty} \pi \int_{1}^{a} R(x)\, dx = \lim_{a \to \infty} \pi \int_{1}^{a} \frac{dx}{x^4} = \lim_{a \to \infty} -\pi \cdot \frac{1}{5} \frac{1}{x^5} \Big|_{1}^{a} = \frac{\pi}{5}$

Se L é a mesma região mas $0 < x < 1$:

$V = \int_{0}^{1} A(x)\, dx = \int_{0}^{1} \pi/x^4\, dx$ diverge.